UBC Electrical Engineering and Mathematics
ELEC 211 | MATH 264 - Midterm 1
Wednesday February 7 2024

Duration: 90 Minutes **Start time:** 5:15pm

Materials admitted: Writing supplies, ruler, protractor, compass. Nothing else. **No calculators.** Formula pages are provided with the exam.

This exam has **6** questions on **14 pages**, printed double-sided, including this cover (i.e. 14 pages = 7 sheets of paper). Check that you have them all. Show all your work and solutions on these pages. You may write on both sides of every page.

Write your name and UBC student ID on the front of the paper before the exam ends. Writing anything after the end of the exam period is not allowed.

Turn off and put away all cell phones, and put away course notes and any other learning materials before the exam begins.

Student number								
Section								
Name	. . . . .							
Signature								

Additional instructions

- Please use the spaces indicated.
- If you require extra paper then put up your hand and ask your instructor.
  - You must put your student number on any extra pages.
  - You must indicate the test-number and question-number.
  - Please do this **on both sides** of any extra pages.
- Please do not separate the pages of your test. You must submit all pages.

This page will NOT be marked, but you may use it for rough work.

1. [10 marks] Point charge $Q_1 = 1\,\text{nC}$ is situated at $P_1 = (1, 2, 3)$ and point charge $Q_2 = 9\,\text{nC}$ is situated at $P_2 = (2, 4, 5)$, both in Cartesian coordinates. Find the force on Q_2 due to Q_1. Assume units of meters for position and distance measurements. Simplify your answer where possible.

2. [10 marks] An infinite, uniform, line of charge having $\rho_\ell = 1\,\text{nC/m}$ lies on the z-axis. A point charge $Q = -1\,\text{nC}$ is situated at $(1, 2, 3)$ in Cartesian coordinates. Find the Electric Field Intensity, $\vec{E}$, at the point $P = (0, 2, 0)$. Simplify your answer where possible.

3. [25 marks] An infinite line of charge having a uniform linear charge of density ρ_ℓ passes through the point $P_0 = (2, 6, 4)$. We know that the electric field intensity at the origin due to the line is $\vec{E}(0, 0, 0) = \langle 1, 2, -2 \rangle$.

   (a) (15 points) Find the equation of the line.

   (b) (10 points) Find ρ_ℓ (you don't need to include units).

*This blank page is for your solution to **Question 3** if you need more space.*

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4. [30 marks] Let $\mathcal{C}_1$ be the curve from $(0,0,0)$ to $(1,1,1)$ along the intersection of the surfaces $y = x$ and $z = x^3$ with the parametrization $\vec{L}_1(t)$.

   (a) (10 points) Find the total charge on the wire given the charge density function $\rho(x,y,z) = 12xy^2 + 24z$.

   (b) (10 points) Find $\int \vec{F} \cdot d\vec{L}_1$ given $\vec{F}(x,y,z) = (xz - y)\hat{a_x} + (z + x)\hat{a_y} + y\hat{a_z}$.

   (c) (10 points) Find $\int \vec{F} \cdot d\vec{L}_2$ where $\vec{L}_2(t)$ is the parametrization of the straight line C_2, from the point $(0,0,0)$ to $(1,1,1)$.

*This blank page is for your solution to **Question 4** if you need more space.*

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5. 10 marks A system of three point charges each having the same charge, Q, and placed at $(0,-1,0)$, $(-1,0,0)$, and $(0,1,0)$ in free space results in an Electric Field Intensity distribution along the positive z-axis given by:

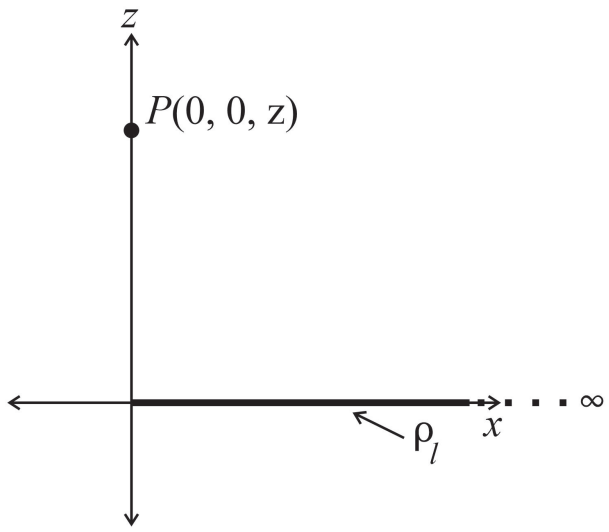
$$\vec{E} = \frac{Q}{4\pi\varepsilon_o(1^2+z^2)^{3/2}}\hat{a}_x + \frac{3Qz}{4\pi\varepsilon_o(1^2+z^2)^{3/2}}\hat{a}_z \text{ V/m}$$

(a) (5 points) What is the potential difference between the point A = (0,0,10) and the point B = (0,0,1)?

(b) (5 points) Can one use this same expression for $\vec{E}$ to calculate the potential difference between C = (10,10,10) and B = (0,0,1)? Why or why not? (note: you do not need to calculate a potential difference for this part)

This blank page is for your solution to ***Question 5*** *if you need more space.*

6. 15 marks A semi-infinite line of charge having a uniform line charge density ρ_ℓ (C/m) exists on the positive x-axis ($0 \leq x \leq \infty$). Find the expression for the Electric Field Intensity, $\vec{E}$, at any point $P = (0, 0, z)$ on the positive z-axis, i.e., ($0 < z \leq \infty$). See the figure shown below. Simplify your answer where possible.



This blank page is for your solution to ***Question 6*** *if you need more space.*