UBC Electrical Engineering and Mathematics
ELEC 211 | MATH 264 - Midterm 2
Wednesday March 20 2024

Duration: 90 Minutes **Start time:** 5:15pm

Materials admitted: Writing supplies, ruler, protractor, compass. Nothing else. **No calculators.** Formula pages are provided with the exam.

This exam has **18 pages**, printed double-sided, including this cover (i.e. 20 pages = 10 sheets of paper). Check that you have them all. Show all your work and solutions on these pages. You may write on both sides of every page.

Write your name and UBC student ID on the front of the paper before the exam ends. Writing anything after the end of the exam period is not allowed.

Turn off and put away all cell phones, pagers, alarms, etc. before the exam begins. Put all course notes or other learning materials away.

Student number								
Section								
Name								
Signature								

Additional instructions

- Please use the spaces indicated.
- If you require extra paper then put up your hand and ask your instructor.
  - You must put your student number on any extra pages.
  - You must indicate the test-number and question-number.
  - Please do this **on both sides** of any extra pages.
- Please do not separate the pages of your test. You must submit all pages.

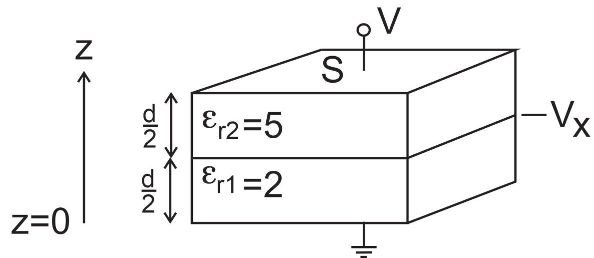
This page will NOT be marked, but you may use it for rough work.

1. $\boxed{\text{25 marks}}$ Note: the individual parts of this question are unrelated to each other.

   (a) (7 points) An infinite line of charge having line charge density ρ_ℓ is situated on the z-axis. Find the total electric flux through a surface defined by $\rho = a$, $0 \le \phi \le \pi/2$, $-1 \le z \le 1$.

(b) (7 points) A sphere of charge centered at the origin has the volume charge density $\rho_v = Nr$, where N is a constant (assume $N > 0$). Find the total electric flux through the surface defined by $r = a$, $0 \leq \theta \leq \pi/3$, $0 \leq \phi \leq 2\pi$.

(c) (6 points) For the capacitor shown in the figure, find the value of V_x, (the potential at the boundary between the two dielectrics) in terms of V.



(d) (5 points) An air-filled parallel plate capacitor has plate surface area S, plate separation d, and a potential, V, supplied by a battery between the two plates. If the separation between the plates is increased to 2d while leaving the battery connected, how will each of the following values change (place a check mark in the appropriate column for each quantity)?

Quantity	Up	Down	Unchanged
E			
C			
V			
ρ_s			
D			

2. $\boxed{\text{20 marks}}$ Consider the ellipsoid S given by

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

with the unit normal pointing outward.

(a) (5 points) Parameterize S.

(b) (12 points) Compute the flux $\iint_S \vec{D} \cdot \hat{n}\, \mathrm{d}S$, directly where $\vec{D}$ is electric flux density

$$\vec{D}(x, y, z) = \langle x, y, z \rangle$$

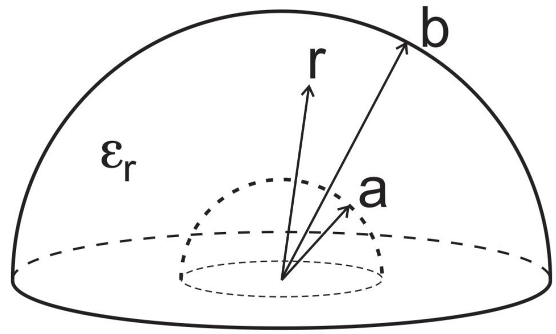
This blank page is for your solution to ***Question 2*** *if you need more space.*

(c) (3 points) Verify your answer in (b) using the divergence theorem.

*This blank page is for your solution to **Question 2** if you need more space.*

3. $\boxed{\text{15 marks}}$ Note: Parts a and b are unrelated.

(a) (10 points) A hemispherical capacitor has an inner conductor defined by $r = a$, $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$, and an outer conductor defined by $r = b$, $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$ (in other words, two thin hemispherical shells of radius a and b). The space between the two conductors is filled with a uniform dielectric having relative permittivity ε_r. Find the expression for the capacitance of this structure.



(b) (5 points) A spherical conductor has radius a and a total charge Q deposited on it's surface. The conductor is surrounded by a dielectric whose relative permittivity is a function of radius. Find an equation for $\varepsilon_r(r)$ that would result in a constant value of $\vec{E}$ in the dielectric.

4. $\boxed{\text{20 marks}}$ Consider the surface S, defined by

$$x^2 + y^2 + (z-3)^2 = 16, \quad z \geq 1,$$

oriented outward (away from the point $(0, 0, 3)$).

(a) (3 points) Parametrize S.

(b) (6 points) Write down, but do not evaluate the integral that calculates the flux $\iint_S \vec{D} \cdot \hat{n}\, \mathrm{d}S$, directly, using the parametrization of S in part a) where $\vec{D}$ is electric flux density

$$\vec{D}(x, y, z) = \langle x + z^2, -2y + sin(z), 2z \rangle$$

(You can leave the integral that calculates the flux in its dot product form, you don't need to simplify)

*This blank page is for your solution to **Question 4** if you need more space.*

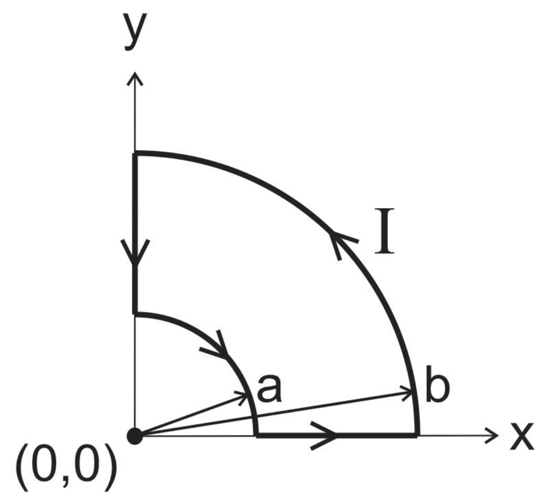
(c) (4 points) Find a capping surface T so that $S \cup T$ is a closed surface oriented outward, and calculate the flux $\iint_T \vec{D} \cdot \hat{n} \, \mathrm{d}T$, using this surface T in the correct orientation.

(d) (7 points) Calculate the flux in part (b) using the divergence theorem.

This blank page is for your solution to ***Question 4*** *if you need more space.*

5. $\boxed{\text{20 marks}}$ Note: parts a and b are unrelated.

(a) (8 points) For the current carrying filament as shown in the figure, find H at (0,0) using the Biot-Savart law.



(b) (12 points) A hollow cylindrical conductor centred about the z-axis has inner radius $\rho = a$, outer radius $\rho = b$, and length l. It has a radially dependent conductivity given by $\sigma(\rho) = \sigma_o(1 + \rho)$. A potential V exists between the two ends of the conductor.

i) Find the expression for $\vec{J}$ in the region $a \leq \rho \leq b$

ii) Find the expression for $\vec{H}$ in the region $a \leq \rho \leq b$

iii) Find the expression for $\vec{H}$ in the region $\rho > b$

iv) Find the expression for the resistance of the conductor

This blank page is for your solution to ***Question 5*** *if you need more space.*