UBC ELEC 211/MATH 264 FORMULA PAGES - FULL COURSE

PHYSICAL CONSTANTS

Permittivity of free space:	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m	Permeability of free space:	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Electron charge:	$e = 1.602 \times 10^{-19}$ C	Electron mass:	$m = 9.109 \times 10^{-31}$ kg
Speed of light in vacuum:	$c = 2.998 \times 10^8$ m/s		

ELECTROSTATIC PRINCIPLES

Coulomb's Law:	$\mathbf{F}_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{R_{12}^2}\mathbf{a}_{12}$	$\mathbf{a}_{12} = \dfrac{\mathbf{R}_{12}}{\lvert\mathbf{R}_{12}\rvert} = \dfrac{\mathbf{R}_{12}}{R_{12}} = \dfrac{\mathbf{r}_2 - \mathbf{r}_1}{\lvert\mathbf{r}_2 - \mathbf{r}_1\rvert}$
Point Charge Q at O:	$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\mathbf{a}_r,\ V = \dfrac{Q}{4\pi\varepsilon_0 r}$	(r comes from spherical coords)
Line Charge, density ρ_L, on z-axis:	$\mathbf{E} = \dfrac{\rho_L}{2\pi\varepsilon_0}\left(\dfrac{\mathbf{a}_\rho}{\rho}\right),\ V = \dfrac{\rho_L}{2\pi\varepsilon_0}\ln\left(\dfrac{1}{\rho}\right)$	(ρ comes from cylindrical coords)
Sheet Charge, density ρ_S, on $z = 0$:	$\mathbf{E} = \pm\dfrac{\rho_S}{2\varepsilon_0}\mathbf{a}_z,\ V = -\dfrac{\rho_S\lvert z\rvert}{2\varepsilon_0}$	(Both ρ_S and ρ_L must be constant here.)
Electric Flux Density:	$\mathbf{D} = \varepsilon\mathbf{E}$	($\varepsilon = \varepsilon_0\varepsilon_r$ in general; $\varepsilon_r = 1$ in free space)
Gauss's Law, I:	$Q_{\text{enc}} = \Psi$, where	$\Psi = \oiint_{\mathcal{S}} \mathbf{D} \bullet \widehat{\mathbf{n}}\, dS$ is net outward flux
Gauss's Law, II:	$Q_{\text{enc}} = \iiint_{\mathcal{V}} \rho_v\, dv$, where	$\rho_v = \nabla \bullet \mathbf{D}$ gives charge density
Electric field and potential:	$\mathbf{E} = -\nabla V$	$V(B) - V(A) = -\int_A^B \mathbf{E} \bullet d\mathbf{L}$ (path indep)
Generalized Poisson Equation:	$\nabla \bullet (\varepsilon \nabla V) = -\rho_v$	(Case $\rho_v = 0$, $\varepsilon =$ const is Laplace's Equation.)
Energy in Electrostatic Field:	$W_E = \frac{1}{2}\iiint_{\mathcal{R}} \mathbf{D}\bullet\mathbf{E}\, dv = \frac{1}{2}\iiint_{\mathcal{R}} \varepsilon\lvert\mathbf{E}\rvert^2\, dv$	

CONDUCTORS, CURRENT, RESISTANCE

Ideal conductor ("$\sigma \to \infty$"):	$\mathbf{E} = \mathbf{0}$	$V =$ const.
Ideal conductor boundary:	$\mathbf{E} \parallel \widehat{\mathbf{n}}$	$\rho_S = \mathbf{D} \bullet \widehat{\mathbf{n}}$
Current and conductivity:	$\mathbf{J} = \sigma\mathbf{E}$ "Ohm's Law I"	$I = \iint_{\mathcal{S}} \mathbf{J} \bullet \widehat{\mathbf{n}}\, dS$
	$\mathbf{J} = \rho_v \mathbf{v}$	$\nabla \bullet \mathbf{J} = -\dfrac{\partial \rho_v}{\partial t}$

Simple Resistor (length L, constant cross-section S, constant conductivity σ): $R = \dfrac{L}{\sigma S}$

Fancy Resistor (all current from A to B crosses surface $\mathcal{S}$—"Ohm's Law II"):

$$R = \frac{\lvert\Delta V\rvert}{\lvert I\rvert} = \frac{\left\lvert -\int_A^B \mathbf{E} \bullet d\mathbf{L}\right\rvert}{\left\lvert \iint_{\mathcal{S}} \mathbf{J} \bullet \widehat{\mathbf{n}}\, dS\right\rvert}$$

CAPACITORS AND DIELECTRICS

Permittivity:	$\varepsilon = \varepsilon_r \varepsilon_0$	(Gauss's Law still works, as above)
Polarization:	$\mathbf{P} = \mathbf{D} - \varepsilon_0\mathbf{E}$	

Simple Capacitor (parallel plates of area S, separation d): $C = \dfrac{\varepsilon S}{d}$ stores $W_E = \frac{1}{2}CV^2$ Joules

Fancy Capacitor (surface $\mathcal{S}$ is one plate; points A, B on opposite plates):

$$C = \frac{\lvert Q\rvert}{\lvert\Delta V\rvert} = \frac{\left\lvert \iint_{\mathcal{S}} \mathbf{D} \bullet \widehat{\mathbf{n}}\, dS\right\rvert}{\left\lvert -\int_A^B \mathbf{E} \bullet d\mathbf{L}\right\rvert}$$

Dielectric interface with normal $\mathbf{n}$: $\mathbf{D}_1 \bullet \mathbf{n} = \mathbf{D}_2 \bullet \mathbf{n}$ AND $\mathbf{E}_1 \times \mathbf{n} = \mathbf{E}_2 \times \mathbf{n}$

MAGNETOSTATICS

Biot-Savart Law:	$d\mathbf{H} = \dfrac{I\,d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$	$\mathbf{H} = \displaystyle\int \frac{I\,d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$
Current I flowing in filament $\rho = 0$, direction $\mathbf{a}_z$:	$\mathbf{H} = \dfrac{I}{2\pi\rho}\mathbf{a}_\phi$; or, for segment,	$\mathbf{H} = \dfrac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\mathbf{a}_\phi$
Current sheet with density $\mathbf{K}$ [A/m], normal $\widehat{\mathbf{n}}$:	$\mathbf{H} = \dfrac{1}{2}\mathbf{K} \times \widehat{\mathbf{n}}$	$I = \displaystyle\int \mathbf{K} \bullet d\mathbf{w}$
Current crossing surface $\mathcal{S}$, from current density $\mathbf{J}$:	$I = \displaystyle\iint_{\mathcal{S}} \mathbf{J} \bullet d\mathbf{S}$	$\mathbf{J} = \nabla \times \mathbf{H}$
Ampère's Circuital Law (ACL):	$I = \displaystyle\oint \mathbf{H} \bullet d\mathbf{L}$	(compare Stokes's Theorem)
Magnetic Flux Density:	$\mathbf{B} = \mu\mathbf{H}$	$\mu = \mu_r\mu_0$
Magnetic Flux (Wb):	$\Phi = \displaystyle\iint_{\mathcal{S}} \mathbf{B} \bullet d\mathbf{S}$	$\displaystyle\oiint_{\mathcal{S}} \mathbf{B} \bullet d\mathbf{S} = 0$
Energy in Steady Magnetic Field:	$W_H = \frac{1}{2}\displaystyle\iiint_{\mathcal{R}} \mathbf{B}\bullet\mathbf{H}\,dv = \frac{1}{2}\iiint_{\mathcal{R}} \mu\,\lvert\mathbf{H}\rvert^2\,dv$	
Magnetic Force on Moving Charge:	$\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$	$\mathbf{J} = \mathbf{v}\rho_v$
Magnetic Force on Current Filament:	$d\mathbf{F} = I\,d\mathbf{L} \times \mathbf{B}$	$\mathbf{F} = \displaystyle\int_{\mathcal{C}} I\,d\mathbf{L}\times\mathbf{B} = -\int_{\mathcal{C}} I\mathbf{B}\times d\mathbf{L}$
Magnetic Force on Current Sheet or Cloud:	$d\mathbf{F} = (\mathbf{K}\,dS) \times \mathbf{B}$	$d\mathbf{F} = (\mathbf{J}\,dv) \times \mathbf{B}$
Magnetic Dipole Moment ($\mathbf{m} = \mathbf{p}_m$):	$d\mathbf{m} = I\,d\mathbf{S}$	$\mathbf{m} = NIS\widehat{\mathbf{n}}$
Magnetic Torque on Given Dipole:	$\vec{\tau} = \mathbf{m} \times \mathbf{B}$	$\lvert\vec{\tau}\rvert = NI\,\lvert\mathbf{B}\rvert\,\lvert\mathbf{S}\rvert$, if $\mathbf{B} \perp \mathbf{S}$
Review: Force $\mathbf{F}$ with moment arm $\mathbf{R}$ gives torque:	$\vec{\tau} = \mathbf{R} \times \mathbf{F}$	

INDUCTORS AND MAGNETIC MATERIALS

Permeability:	$\mu = \mu_r\mu_0$	
Simple inductor (N filaments, current I in each):	$L = \dfrac{N\Phi}{I}$	stores $W_H = \frac{1}{2}LI^2$ Joules
Mutual Inductance:	$M_{12} = \dfrac{N_2\Phi_{12}}{I_1} = \dfrac{N_1\Phi_{21}}{I_2} = M_{21}$	
Material interface with normal $\mathbf{n}$:	$\mathbf{B}_1 \bullet \mathbf{n} = \mathbf{B}_2 \bullet \mathbf{n}$	$\mathbf{H}_1 \times \mathbf{n} = \mathbf{H}_2 \times \mathbf{n}$

MAGNETIC CIRCUITS

Magnetomotive force (simple setup—N turns, current I):	$V_m = NI$	
Magnetomotive force (general—filament from A to B):	$V_m(B) - V_m(A) = -\displaystyle\int_A^B \mathbf{H} \bullet d\mathbf{L}$	(path restrictions apply)
Reluctance (cross-section S, length ℓ):	$\mathcal{R} = \dfrac{V_m}{\Phi} = \dfrac{\ell}{\mu S}$	(integral defining Φ shown above)
Air-gap force (cross-section S):	$\mathbf{F} = \dfrac{1}{2\mu_0}\lvert\mathbf{B}\rvert^2 S\,\widehat{\mathbf{n}}$	

MAXWELL'S EQUATIONS (POINT FORM, GENERAL CASE—set $\frac{\partial\mathbf{B}}{\partial t} = \mathbf{0}$ and $\frac{\partial\mathbf{D}}{\partial t} = \mathbf{0}$ in static situations)

$\nabla \bullet \mathbf{D} = \rho_v$	$\nabla \bullet \mathbf{B} = 0$	$\nabla \times \mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial\mathbf{D}}{\partial t}$

TIME-VARYING FIELDS

Faraday's Law (case of $N = 1$ current filament):	$\text{emf} = -\dfrac{d\Phi}{dt} = -\displaystyle\iint_{\mathcal{S}} \frac{\partial\mathbf{B}}{\partial t} \bullet \widehat{\mathbf{n}}\,dS$	(units: Volts)
	$\text{emf} = \displaystyle\oint_{\mathcal{C}} \mathbf{E} \bullet d\mathbf{L}$	(loop shape matters!)

VECTOR IDENTITIES

For $\mathbf{u} = u_x\mathbf{a}_x + u_y\mathbf{a}_y + u_z\mathbf{a}_z,\ \mathbf{v} = v_x\mathbf{a}_x + v_y\mathbf{a}_y + v_z\mathbf{a}_z,\ \mathbf{w} = w_x\mathbf{a}_x + w_y\mathbf{a}_y + w_z\mathbf{a}_z,$

$$\mathbf{u}\bullet\mathbf{v} = u_xv_x + u_yv_y + u_zv_z = |\mathbf{u}|\,|\mathbf{v}|\cos(\theta), \quad 0 \le \theta \le \pi$$

$$|\mathbf{u}| = \sqrt{\mathbf{u}\bullet\mathbf{u}} = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = \langle u_yv_z - u_zv_y,\ u_zv_x - u_xv_z,\ u_xv_y - u_yv_x\rangle$$

$$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta$$

$$\mathbf{u}\bullet(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\bullet(\mathbf{w}\times\mathbf{u}) = \mathbf{w}\bullet(\mathbf{u}\times\mathbf{v})$$

$$\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\bullet\mathbf{w})\mathbf{v} - (\mathbf{u}\bullet\mathbf{v})\mathbf{w}$$

DISTANCES AND PROJECTIONS

From point (x_0, y_0, z_0) to plane $Ax + By + Cz = D$: $\quad s = \dfrac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$

$$\mathbf{F} = \mathbf{proj_u}(\mathbf{F}) + \mathbf{orth_u}(\mathbf{F}) \qquad \mathbf{proj_u}(\mathbf{F}) = \left(\frac{\mathbf{F}\bullet\mathbf{u}}{\mathbf{u}\bullet\mathbf{u}}\right)\mathbf{u}$$

DERIVATIVE IDENTITIES – valid for smooth scalar-valued ϕ, ψ and smooth vector-valued $\mathbf{F}$, $\mathbf{G}$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla\bullet(\mathbf{F}\times\mathbf{G}) = (\nabla\times\mathbf{F})\bullet\mathbf{G} - \mathbf{F}\bullet(\nabla\times\mathbf{G})$$

$$\nabla\bullet(\phi\mathbf{F}) = (\nabla\phi)\bullet\mathbf{F} + \phi\,(\nabla\bullet\mathbf{F})$$

$$\nabla\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\bullet\mathbf{G}) - \mathbf{G}\,(\nabla\bullet\mathbf{F}) - (\mathbf{F}\bullet\nabla)\mathbf{G} + (\mathbf{G}\bullet\nabla)\mathbf{F}$$

$$\nabla\times(\phi\mathbf{F}) = (\nabla\phi)\times\mathbf{F} + \phi\,(\nabla\times\mathbf{F})$$

$$\nabla(\mathbf{F}\bullet\mathbf{G}) = \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F}) + (\mathbf{F}\bullet\nabla)\mathbf{G} + (\mathbf{G}\bullet\nabla)\mathbf{F}$$

$$\nabla\times(\nabla\phi) = \mathbf{0} \qquad (\mathbf{curl}\,\mathbf{grad} = \mathbf{0})$$

$$\nabla\bullet(\nabla\times\mathbf{F}) = 0 \qquad (\mathrm{div}\,\mathbf{curl} = 0)$$

$$\nabla^2\phi(x,y,z) = \nabla\bullet\nabla\phi(x,y,z) = \mathrm{div}\,\mathbf{grad}\,\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

SURFACE NORMALS AND AREA ELEMENTS

For any oriented surface normal $\mathbf{n} \ne \mathbf{0}$, $\quad d\mathbf{S} = \widehat{\mathbf{n}}\,dS = \dfrac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{a}_z|}\,dx\,dy = \dfrac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{a}_y|}\,dx\,dz = \dfrac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{a}_x|}\,dy\,dz, \qquad dS = |d\mathbf{S}|$

Graph Surface $z = f(x,y)$: normal $\mathbf{n} = \pm\left\langle \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, -1\right\rangle$ $\quad \widehat{\mathbf{n}}\,dS = \pm\left\langle \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, -1\right\rangle dx\,dy$

Level Surface $G(x,y,z) = 0$: normal $\mathbf{n} = \pm\nabla G(x,y,z)$ (choose sign to orient)

Parametric Surface $\langle x,y,z\rangle = \mathbf{R}(u,v)$: $\quad d\mathbf{S} = \pm\left(\dfrac{\partial\mathbf{R}}{\partial u}\times\dfrac{\partial\mathbf{R}}{\partial v}\right)du\,dv$ $\quad$ (choose sign to orient; $\widehat{\mathbf{n}} = \dfrac{d\mathbf{S}}{|d\mathbf{S}|}$)

CARTESIAN COORDINATES (x,y,z)

Line Element: $d\mathbf{L} = \mathbf{a}_x\,dx + \mathbf{a}_y\,dy + \mathbf{a}_z\,dz$

Volume Element: $dv = dx\,dy\,dz$

Scalar field: $f(x,y,z)$

Vector field: $\mathbf{F}(x,y,z) = F_x\,\mathbf{a}_x + F_y\,\mathbf{a}_y + F_z\,\mathbf{a}_z$

Differential operator ∇: $\quad \nabla = \mathbf{a}_x\dfrac{\partial}{\partial x} + \mathbf{a}_y\dfrac{\partial}{\partial y} + \mathbf{a}_z\dfrac{\partial}{\partial z} = \left\langle\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right\rangle$

Gradient: $\nabla f = \dfrac{\partial f}{\partial x}\,\mathbf{a}_x + \dfrac{\partial f}{\partial y}\,\mathbf{a}_y + \dfrac{\partial f}{\partial z}\,\mathbf{a}_z$

Divergence: $\nabla\bullet\mathbf{F}(x,y,z) = \dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} + \dfrac{\partial F_z}{\partial z}$

Curl: $\nabla\times\mathbf{F} = \mathbf{curl}\,\mathbf{F} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

Laplacian: $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

POLAR AND CYLINDRICAL COORDINATES (ρ, ϕ, z)

Transformation: $x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad z = z$

Local basis: $\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y, \quad \mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y, \quad \mathbf{a}_z = \mathbf{a}_z$

Surface element (on $\rho = a$): $d\mathbf{S} = \pm a\,\mathbf{a}_\rho\,d\phi\,dz$

Line Element: $d\mathbf{L} = \mathbf{a}_\rho\,d\rho + \rho\mathbf{a}_\phi\,d\phi + \mathbf{a}_z\,dz$

Surface element (on $z =$const.): $d\mathbf{S} = \pm\rho\,\mathbf{a}_z\,d\rho\,d\phi$

Volume element: $dv = \rho\,d\rho\,d\phi\,dz$

Scalar field: $f(\rho, \phi, z)$

$$\nabla f = \frac{\partial f}{\partial\rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\phi}\mathbf{a}_\phi + \frac{\partial f}{\partial z}\mathbf{a}_z$$

$$\nabla\times\mathbf{F} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \dfrac{\partial}{\partial\rho} & \dfrac{\partial}{\partial\phi} & \dfrac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$$

Vector field: $\mathbf{F}(\rho, \phi, z) = F_\rho\,\mathbf{a}_\rho + F_\phi\,\mathbf{a}_\phi + F_z\,\mathbf{a}_z$

$$\nabla\bullet\mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

SPHERICAL COORDINATES (r, θ, ϕ)

Transformation: $x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$

Local basis: $\mathbf{a}_r = \sin\theta\cos\phi\,\mathbf{a}_x + \sin\theta\sin\phi\,\mathbf{a}_y + \cos\theta\,\mathbf{a}_z$, $\quad \mathbf{a}_\theta = \cos\theta\cos\phi\,\mathbf{a}_x + \cos\theta\sin\phi\,\mathbf{a}_y - \sin\theta\,\mathbf{a}_z$,
$\mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y$

Volume element: $dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

Line Element: $d\mathbf{L} = \mathbf{a}_r\,dr + r\mathbf{a}_\theta\,d\theta + r\sin\theta\mathbf{a}_\phi\,d\phi$

Surface area element (on $r = a$): $d\mathbf{S} = \pm a^2\sin\theta\,\mathbf{a}_r\,d\theta\,d\phi$

Scalar field: $f(r, \theta, \phi)$

$$\nabla f = \frac{\partial f}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\mathbf{a}_\phi$$

$$\nabla\times\mathbf{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & r\sin\theta\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ F_r & rF_\theta & r\sin\theta F_\phi \end{vmatrix}$$

Vector field: $\mathbf{F}(r, \theta, \phi) = F_r\,\mathbf{a}_r + F_\theta\,\mathbf{a}_\theta + F_\phi\,\mathbf{a}_\phi$

$$\nabla\bullet\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(F_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

INTEGRATING DERIVATIVES: THE FUNDAMENTAL THEOREM OF CALCULUS (FTC)

Line-integral form: $$\int_{\mathcal{C}}\nabla g\bullet d\mathbf{L} = \int_{\mathcal{C}}\frac{\partial g}{\partial x}\,dx + \frac{\partial g}{\partial y}\,dy + \frac{\partial g}{\partial z}\,dz = g_{\text{final}} - g_{\text{initial}}$$

Stokes's Theorem: $$\iint_{\mathcal{S}}(\nabla\times\mathbf{G})\bullet d\mathbf{S} = \oint_{\mathcal{C}}\mathbf{G}\bullet d\mathbf{L} = \oint_{\mathcal{C}}G_x\,dx + G_y\,dy + G_z\,dz$$

Divergence Theorem: $$\iiint_{\mathcal{R}}\nabla\bullet\mathbf{G}\,dv = \oiint_{\mathcal{S}}\mathbf{G}\bullet\hat{\mathbf{n}}\,dS$$

DEFINITE INTEGRALS

$$\int_0^{\pi/2}\sin x\,dx = \int_0^{\pi/2}\cos x\,dx = 1 \qquad \int_0^{\pi/2}\sin^3 x\,dx = \int_0^{\pi/2}\cos^3 x\,dx = \frac{2}{3} \qquad \int_0^{\pi/2}\sin^5 x\,dx = \int_0^{\pi/2}\cos^5 x\,dx = \frac{8}{15}$$

$$\int_0^{\pi/2}\sin^2 x\,dx = \int_0^{\pi/2}\cos^2 x\,dx = \frac{\pi}{4} \qquad \int_0^{\pi/2}\sin^4 x\,dx = \int_0^{\pi/2}\cos^4 x\,dx = \frac{3\pi}{16} \qquad \int_0^{\pi/2}\sin^6 x\,dx = \int_0^{\pi/2}\cos^6 x\,dx = \frac{5\pi}{32}$$

INDEFINITE INTEGRALS

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1) \qquad \int\tan x\,dx = \ln|\sec x| \qquad \int\frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} \quad (a > 0)$$

$$\int\sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x \qquad \int\cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x \qquad \int\frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \quad (a > 0)$$

$$\int\frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \quad (a > 0) \qquad \int\frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 - x^2}} \qquad \int\frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \quad (a > 0) \qquad \int\frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

Adapted from R. A. Adams, *Calculus, A Complete Course,* Addison-Wesley, 2003.