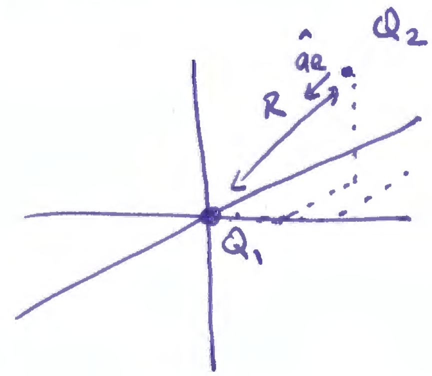
Part I - ELEC Questions. Point values specified for each question. Part marks may be awarded.

1. ~~10~~ 8 marks Electric Forces

   (a) (4 points) Point charge $Q_1 = 3\,\mathrm{C}$ is situated at $P_1 = (0,0,0)$ and point charge $Q_2 = 4\pi\,\mathrm{C}$ is situated at $P_2 = (1,1,1)$, both in Cartesian coordinates. Find the force on Q_1 due to Q_2. Assume units of meters for position and distance measurements. Simplify your answer where possible.



distance between 2 charges is

$$R = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

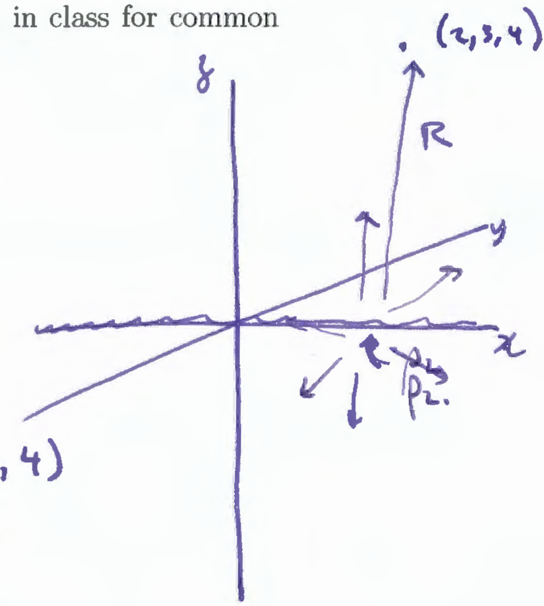
$$\bar{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\,\hat{a}_R \text{ N}$$

where $\hat{a}_R = \dfrac{\langle -1,-1,-1\rangle}{\sqrt{1^2+1^2+1^2}} = \dfrac{\langle -1,-1,-1\rangle}{\sqrt{3}}$

$$\therefore \bar{F}_{21} = \frac{3(4\pi)}{4\pi\varepsilon_0(\sqrt{3})^2}\,\frac{\langle -1,-1,-1\rangle}{\sqrt{3}} \text{ N}$$

$$\therefore \bar{F}_{21} = \frac{\langle -1,-1,-1\rangle}{\sqrt{3}\,\varepsilon_0} \text{ N}$$

(b) (~~6~~ 4 points) An infinite, uniform, line of charge having $\rho_\ell = 1\,\text{nC/m}$ lies on the x-axis. A point charge $Q = 1\,\text{C}$ is situated at $(2, 3, 4)$ in Cartesian coordinates. Find the force on the point charge, Q, due to the infinite line of charge. You may use equations developed in class for common field distributions in your solution.



Field due to ρ_L is radially outward from x-axis

The distance that matters is from $(2,0,0)$ to $(2,3,4)$

$$R = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$E = \frac{F}{Q}$$

Electric field is force per unit charge, & our charge is 1 C. (a unit charge)

E for a line of uniform charge is

$$E = \frac{\rho_L}{2\pi\varepsilon_0 R}\,\hat{a}_R$$

∴ E will be

$$\frac{1\,\text{nC}}{2\pi\varepsilon_0(5)}\;\frac{\langle 0,3,4\rangle}{5}\ \text{V/m}$$

and F will be

$$QE = \frac{(1\,\text{C})(1\,\text{nC})\,\langle 0,3,4\rangle}{2\pi\varepsilon_0(5)(5)}\ \text{N}$$

$$\boxed{F = \frac{1\times10^{-9}}{\cancel{2\pi}\;50\pi\varepsilon_0}\,\langle 0,3,4\rangle\ \text{N}}$$

or

$$\frac{\langle 0,3,4\rangle}{50\pi\varepsilon_0}\ \text{nN}$$

2. $\boxed{\text{10 marks}}$ 9 Charge distributions

   (a) (~~4~~ 2 points) Calculate the total amount of charge in a filament located along the x-axis having a line charge density defined by $\rho_\ell = Nx^2\,\mu\text{C/m}$, where $N > 0$ and $1 \leq x \leq 2$ m.

$$Q_{TOT} = \int_1^2 Nx^2\,dx$$

$$= \left.\frac{Nx^3}{3}\right|_1^2$$

$$= \frac{N(8-1)}{3}\ \mu C.$$

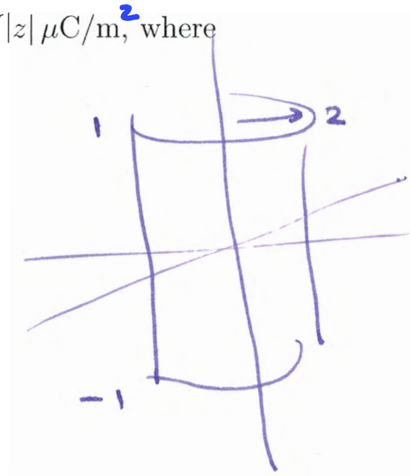
$$= \frac{7N}{3}\ \mu C$$

(b) (~~6~~ 3 points) Calculate the total amount of charge on a surface defined by

$$\rho = 2 \qquad -1 \le z \le 1 \qquad 0 \le \phi \le 2\pi$$

cylindrical shell

and having a surface charge density defined by $\rho_s = N|z|\ \mu\text{C/m}^2$, where $N > 0$ and all distance units are in meters.



$$Q_{TOT} = 2\int_0^{2\pi}\int_0^1 N z\, \underbrace{dz\, \rho\, d\phi}_{dS}$$

Take $2 \times \int_{z=0}^{1} dz$

$$= 2\int_0^{2\pi}\int_0^1 N z\, dz\, (2)\, d\phi$$

$\rho = 2$

$$= (2)(2)(2\pi) N \left.\frac{z^2}{2}\right|_0^1$$

$$Q_{TOT} = 4\pi N \ \mu C$$

(c) (~~5~~ 4 points) Calculate the total amount of charge in a volume defined by

$$0 \le \theta \le \pi/2 \qquad 0 \le \phi \le 2\pi \qquad a \le r \le b$$

and having a volume charge density defined by $\rho_v = \frac{N}{r^2}\,\mu\text{C/m}^3$. Assume that $b > a$ and $N > 0$.

$$Q_{TOT} = \iiint_{\text{volume}} \rho_v \, dv$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\int_a^b \frac{N}{r^2} \underbrace{r^2 \sin\theta \, dr \, d\theta \, d\phi}_{dv}$$

$$= N[2\pi]\,[r]_a^b\,[-\cos\theta]_0^{\pi/2}$$

ϕ $\qquad$ θ

r

$$= N(2\pi)(b-a)(-0 - -1)$$

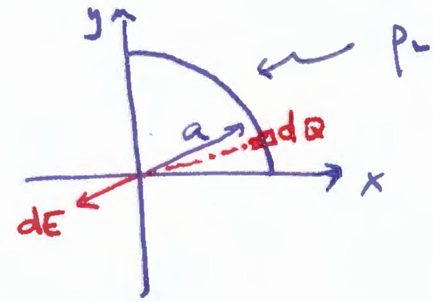
$$= N 2\pi (b-a) \ \mu C$$

3. $\boxed{\text{10 marks}}$ Electric Field Intensity

A finite filament of charge exists along the line defined by

$$\rho = a \qquad z = 0 \qquad 0 \leq \phi \leq \pi/2$$

The line charge density for this filament is given by $\rho_\ell = N\cos(\phi)$ C/m where $N > 0$. Find the Electric Field Intensity, $\vec{E}$, at the origin (0,0,0).



$dQ = \rho_L \rho \, d\phi$

Σ contributions from point charges

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} \hat{a}_R \quad \text{V/m}$$

$R = a \qquad R^2 = a^2$

Each dQ will make a contribution to a $-\hat{a}_x$ and $-\hat{a}_y$ component of $\bar{E}$.

$$\therefore E = \int_0^{\pi/2} \frac{Na\cos\phi \, d\phi}{4\pi\varepsilon_0 a^2} \left(\cos\phi(-\hat{a}_x) + \sin\phi(-\hat{a}_y)\right)$$

$$= \frac{N}{4\pi\varepsilon_0 a} \int_0^{\pi/2} \left(\cos^2\phi(-\hat{a}_x) + \cos\phi\sin\phi(-\hat{a}_y)\right) d\phi$$

$\int_0^{\pi/2} \cos^2\phi \, d\phi = \pi/4$ from formula page

$$= \frac{N}{4\pi\varepsilon_0 a}\left[\frac{\pi}{4}(-\hat{a}_x) + \frac{1}{2}(-\hat{a}_y)\right]$$

$\int \cos\phi \sin\theta$ from formula page

4. $\boxed{13 \text{ marks}}$

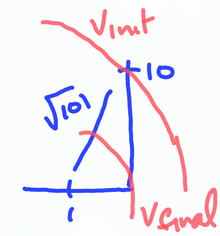
$\boxed{3}$ a) $E = \dfrac{A}{2\varepsilon_0}\hat{a}_z \text{ V/m}$ $\qquad V = -\displaystyle\int_{\text{init}}^{\text{final}} \frac{A}{2\varepsilon_0}\, dz\, \hat{a}_z$

for an infinite sheet

$$= -\frac{Az}{2\varepsilon_0}\Bigg|_{10}^{0} = \frac{5A}{\varepsilon_0}$$

$\boxed{3}$ b) radius at $(0,0,10)$ is $\sqrt{101}$

radius at $(0,0,0)$ is 1



$$\therefore \quad V = \frac{B}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{101}}{1}\right) \approx \frac{B}{2\pi\varepsilon_0} \ln 10 \;.\; V$$

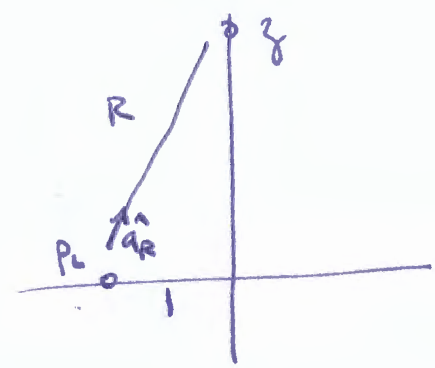
either OK.

$\boxed{4}$ c) $R = \sqrt{1^2 + z^2}$

$$\hat{a}_R = \frac{1\hat{a}_x + z\hat{a}_z}{\sqrt{1^2+z^2}}$$

For an infinite line

$E = \dfrac{\rho_L}{2\pi\varepsilon_0 R}\hat{a}_R \text{ V/m}$



$$E = \frac{\cancel{\rho_L}\, B}{2\pi\varepsilon_0\sqrt{1^2+z^2}}(\hat{a}_R)$$

$$= \frac{B}{2\pi\varepsilon_0\sqrt{1^2+z^2}\,\sqrt{1^2+z^2}}(\hat{a}_x + z\hat{a}_z)$$

$$\bar{E} = \frac{B(\hat{a}_x + z\hat{a}_z)}{2\pi\varepsilon_0(1^2+z^2)} \text{ V/m}$$

$\boxed{3}$ ~~d~~.)

$$E = \frac{B(\hat{a_x} + z\hat{a_z})}{2\pi\varepsilon_0(1^2+z^2)}$$

$$V = -\int_{init}^{final} \frac{B(\hat{a}_x + z\hat{a}_z)}{2\pi\varepsilon_0(1^2+z^2)} \cdot \hat{a}_z dz$$

$$V = -\int_{init}^{final} \frac{B z\, dz}{2\pi\varepsilon_0(1+z^2)} \quad V.$$