UBC Electrical Engineering and Mathematics
ELEC 211 | MATH 264 - Midterm 2
Wednesday March 19 2025

Duration: 105 Minutes **Start time:** 5:00pm

Materials admitted: Writing supplies, ruler, protractor, compass. Nothing else. **No calculators.** Formula pages are provided with the exam.

This exam has **7** questions on **20 pages**, printed double-sided, including this cover (i.e. 20 pages = 10 sheets of paper). Check that you have them all. Show all your work and solutions on these pages. You may write on both sides of every page.

Write your name and UBC student ID on the front of the paper before the exam ends. Writing anything after the end of the exam period is not allowed.

Turn off and put away all cell phones, and put away course notes and any other learning materials before the exam begins.

Student number								
Section								
Name								
Signature								

Additional instructions

- Please use the spaces indicated.
- If you require extra paper then put up your hand and ask your instructor.
  - You must put your student number on any extra pages.
  - You must indicate the test-number and question-number.
  - Please do this **on both sides** of any extra pages.
- Please do not separate the pages of your test. You must submit all pages.

This page will NOT be marked, but you may use it for rough work.

Part I - ELEC 211 questions

Point values specified for each question. Part marks may be awarded. For all questions, you don't need to do long hand calculations, but simplify your answers where possible.

1. $\boxed{\text{10 marks}}$ Electric Flux & Flux Density

   (a) (2 points) A charge distribution in free space given by $\rho_v = N\ C/m^3$ (where N is a constant) exists in the region defined by $0 \leq \rho \leq b$, $-\infty \leq z \leq \infty$, and $0 \leq \phi \leq 2\pi$. Find the expression for the total electric flux, Ψ, through the surface defined by $\rho = c$, $0 \leq \phi \leq 2\pi$, $0 \leq z \leq 1$, assuming $b < c$.

   (b) (3 points) A charge distribution in free space given by $\rho_v = N\rho^2\ C/m^3$ (where N is a constant) exists in the region defined by $0 \leq \rho \leq b$, $-\infty \leq z \leq \infty$, and $0 \leq \phi \leq 2\pi$. Find the expression for the Electric Flux Density, $\vec{D}$, in the region $\rho \leq b$.

(c) (3 points) If, in spherical coordinates, an Electric Flux Density is defined as $\vec{D} = 2(r-1)^3 \hat{a}_r \ C/m^2$ over the region $2 \leq r \leq 5$ m, $0 \leq \theta \leq \pi$, and $0 \leq \phi \leq 2\pi$, what is the total electric flux, Ψ, through the surface defined by $r = 5$ m, $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq \pi$?

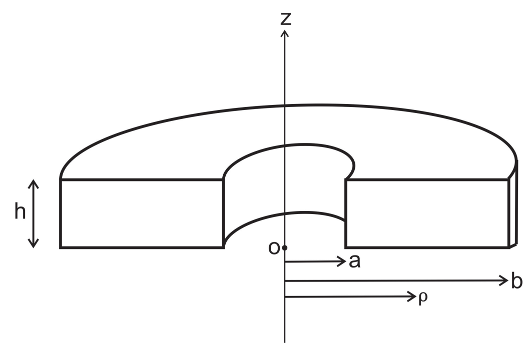
(d) (2 points) For the Electric Flux Density defined in part (c), what is the volume charge density, ρ_v, at radius $r = 4$ m?

This blank page is for your solution to ***Question 1*** *if you need more space.*

2. $\boxed{\text{15 marks}}$ Conductors, Resistance, & Capacitance

NOTE: this description applies to parts (a) to (c). Parts (d) and (e) are independent of each other and of parts (a) to (c).

A metal conductor in the shape of a thick half-circle as shown in the figure has conductivity σ = 1 x 10^6 S/m and dimensions a = 2 cm, b = 4 cm, and h = 1 cm. The two rectangular faces are aligned with the z-axis (i.e., if the conductor was a complete circle, the z-axis would be at it's centre). If a potential difference is applied between the two rectangular faces, the resultant Electric Field Intensity is $\vec{E} = (0.1/\rho)\ \hat{a}_\phi$ V/m.

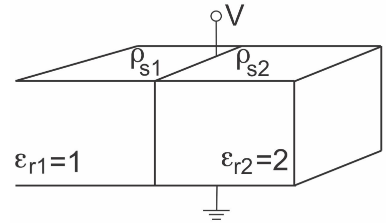


(a) (2 points) What potential difference, V, exists between the two rectangular faces of this structure?

(b) (4 points) What is the total current flowing in the structure?

(c) (1 points) What is the resistance between the two rectangular faces?

(d) (3 points) A parallel plate capacitor is half filled with air and half with a dielectric material as shown. Circle the correct relationship for each quantity in the table provided.



Quantity			
ρ_s	$\rho_{s1} < \rho_{s2}$	$\rho_{s1} > \rho_{s2}$	$\rho_{s1} = \rho_{s2}$
D	$\vec{D}_1 < \vec{D}_2$	$\vec{D}_1 > \vec{D}_2$	$\vec{D}_1 = \vec{D}_2$
E	$\vec{E}_1 < \vec{E}_2$	$\vec{E}_1 > \vec{E}_2$	$\vec{E}_1 = \vec{E}_2$

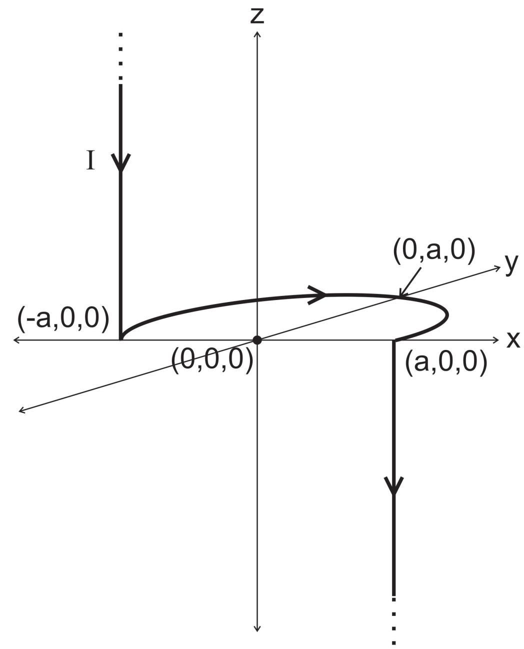
(e) (5 points) An air-filled parallel plate capacitor has plate surface area S, plate separation d, and a potential, V, supplied by a battery connected between the two plates. If the battery is disconnected and then the separation between the plates is increased to 2d <u>all without</u> discharging the capacitor, how will each of the following values change? Place a check mark in the appropriate column for each quantity.

Quantity	Up	Down	Unchanged
E			
C			
V			
ρ_s			
D			

3. $\boxed{\text{15 marks}}$ Magnetic Fields, Magnetic Flux & Flux Densities

NOTE: Part (a) is independent of parts (b) through (d).

(a) (5 points) An infinite current filament carrying a current, I, is bent into the shape shown in the figure. The section lying in the $x-y$ plane is a semicircle of radius a. Find the expression for the Magnetic Field Intensity, $\vec{H}$, at the origin.



(b) (5 points) A long cylindrical conductor of radius a is coincident with the $\hat{a}_z$ axis and has a current density defined by

$$\vec{J} = \frac{A}{a}\rho \ \hat{a}_z \ \ A/m^2$$

where A is a constant. Find expressions for the Magnetic Field Intensity, $\vec{H}$, for all ρ.

(c) (3 points) For the conductor in part (b), find the total Magnetic Flux, Φ, through the surface defined by $b \leq \rho \leq c$, $0 \leq z \leq h$, and $\phi = 0$ (assume $a < b < c$ and $h > 0$).

(d) (2 points) Assume that $\phi = 0$ corresponds with the x-axis of a Cartesian coordinate system. Consider the point $P(2a, 0, 0)$ in Cartesian coordinates. Now assume that in addition to the conductor in part (b) there is an infinite sheet current located in the $x = -2a$ plane. Determine the magnitude and direction of the sheet current density, $\vec{K}$, needed to achieve $\vec{H} = 0$ A/m at the point P.

This blank page is for your solution to ***Question 3*** *if you need more space.*

Part II: Math 264 Questions

Mastery system grading. Learning Objectives are noted on each question.

4. (Learning Objective 5.3)

   Let M be the part of the cone $x^2+y^2=z^2$, for $0 \leq z \leq 1$, and $y \geq 0$, oriented away from the z-axis and $\vec{F}(x,y,z) = \langle x+e^{yz^2}, y+\cos(x), z+xy\rangle$. Calculate

   $$\iint_M \vec{F}\cdot\hat{n}\mathrm{d}S,$$

   using Divergence Theorem.

*This blank page is for your solution to **Question 4** if you need more space.*

5. (Learning Objective 6.1)

Let $\mathcal{S}$ be the part of the paraboloid $z = 4 - x^2 - y^2$ that lies inside the cylinder $x^2 + (y-1)^2 = 4$; and let $\mathcal{C}$ be the curve at the intersection of the paraboloid and the cylinder, parametrized by

$$L(t) = \langle 2\cos(t), 2\sin(t) + 1, -4\sin(t) - 1 \rangle \qquad \text{where } 0 \leq t \leq 2\pi.$$

Moreover, let $\vec{F}(x, y, z) = \langle xz, x, yz \rangle$.

Use Stokes' Theorem to calculate $\iint_{\mathcal{S}} (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \mathrm{d}S$ in the downward direction.

Note: Be careful with the orientations.

You need to show your work, and use Stokes' theorem to get the mastery mark for this question.

*This blank page is for your solution to **Question 5** if you need more space.*

6. (Learning Objective 6.2)

   In the following parts, you are given non-closed curves. Find a closing curve with the correct direction to them so that we can apply Stokes' theorem.

   (a) C_1 is the curve parametrized by

   $$\vec{L}_1(t) = \langle \cos(t), 2\sin(t), 0 \rangle,$$

   where $0 \le t \le 3\pi/2$.

   (b) C_2 is the curve parametrized by

   $$\vec{L}_2(t) = \begin{cases} \langle t, 2t, 0 \rangle & 0 \le t \le 1 \\ \langle 2-t, 4-2t, t^2-1 \rangle & 1 \le t \le 2 \end{cases}$$

   You need to get both of these parts correct to get the mastery mark for this question.

This blank page is for your solution to ***Question 6*** *if you need more space.*

7. (Learning Objective 6.3)

Use Stokes' Theorem to calculate the line integral

$$\int_C \vec{F} \cdot \mathrm{d}\vec{L},$$

where C is the part of the circle of radius 2 centred at the origin with parametrization

$$L(t) = \langle \cos(t), \sin(t), 0 \rangle,$$

where t goes from $\pi/2$ to $3\pi/2$, and

$$\vec{F}(x, y, z) = \langle x^2 + \cos(x), e^y + y^3, z + e^{z^2} \rangle.$$

You need to show your work, and use Stokes' theorem to get the mastery mark for this question.

*This blank page is for your solution to **Question 7** if you need more space.*