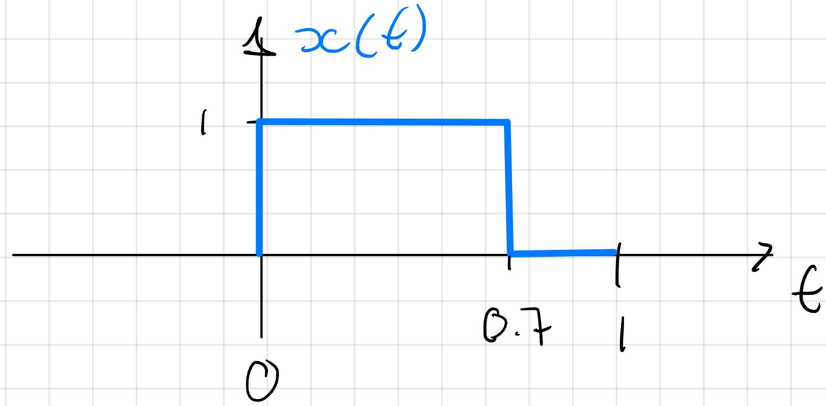
Q4 Fourier series signal approximation

(a)



$$P_x = \frac{1}{T}\int_0^T |x(t)|^2\, dt = \int_0^{0.7} 1^2\, dt = 0.7$$

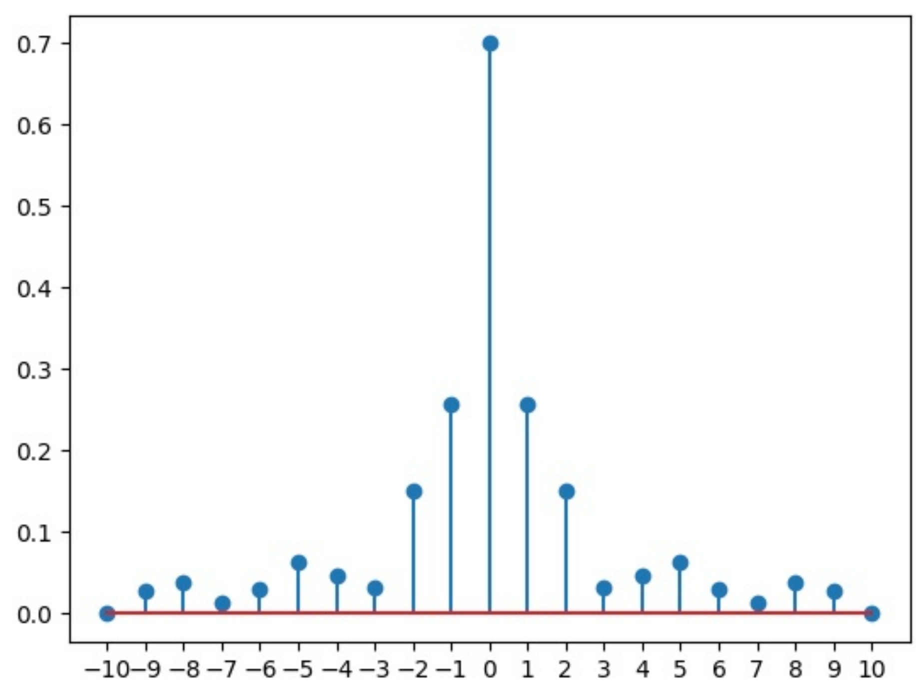
(b)

$$c_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt = \int_0^{0.7} e^{-jk\omega_0 t}\, dt$$

$$= \begin{cases} 0.7 & k=0 \\ \frac{1}{-jk\omega_0}\, e^{-jk\omega_0 t}\Big|_0^{0.7} & k\neq 0 \end{cases} = \begin{cases} 0.7 & k=0 \\ \frac{j}{2\pi k}\left[e^{-j1.4\pi k} - 1\right] & k\neq 0 \end{cases}$$

1 mark

$|c_k|$ ↑



k ⟶

0.5 marks

Low frequencies are contributing most.

(c) $e(t) = x(t) - y(t)$

$$P_x = \sum_{k=-\infty}^{\infty} |c_k|^2 , \quad P_y = \sum_{k=-K}^{K} |c_k|^2$$

1 mark

$$P_e = \sum_{k=-\infty}^{-K-1} |c_k|^2 + \sum_{k=K+1}^{\infty} |c_k|^2 = P_x - P_y$$

$$\Rightarrow \quad \frac{P_e}{P_x} = \frac{P_x - P_y}{P_x} \leq 0.05$$

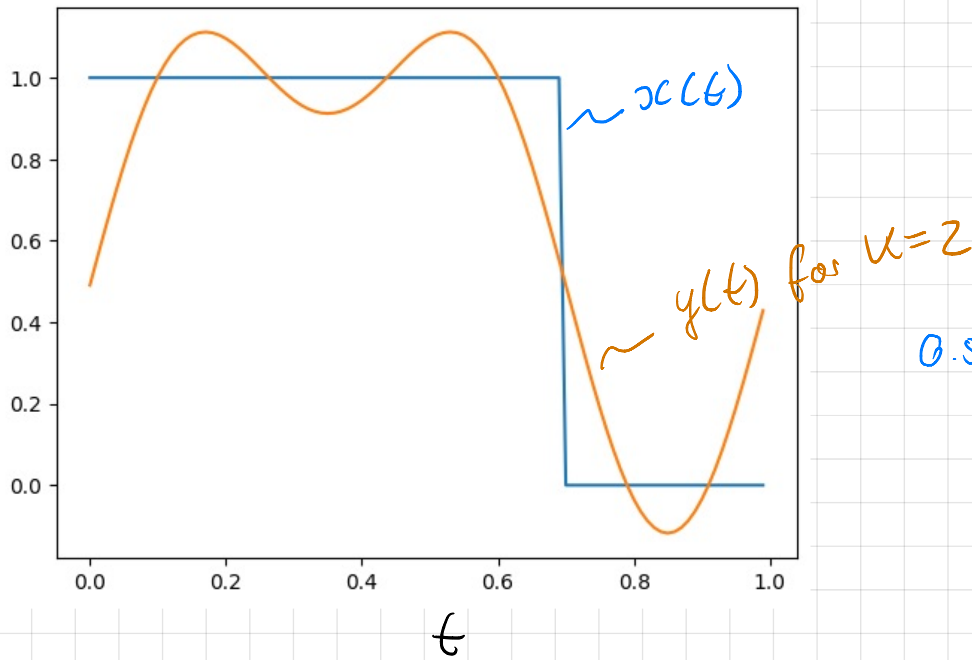
$$\frac{P_y}{P_x} \geq 0.95$$

1 mark

computer program
$\Rightarrow$ $\underline{K=2}$ (

K	P_y/P_x	P_e/P_x
0	0.70	0.30
1	0.89	0.11
2	0.95	0.05

)



0.5 marks

We observe that discontinuities are smoothed out. 0.5 marks

Q5: Echo system

(a) stability $\underset{LTI}{\longleftrightarrow}$ $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

S1: $\int_{-\infty}^{\infty} |h_1(t)|\, dt = \int_{-\infty}^{\infty} \left| \sum_{k=0}^{\infty} a^k \delta(t-kT) \right| dt = \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} |a^k|\, \delta(t-kT)\, dt$

$= \sum_{k=0}^{\infty} |a|^k \overbrace{\int_{-\infty}^{\infty} \delta(t-kT)\, dt}^{=1\ \forall k} = \sum_{k=0}^{\infty} |a|^k = \begin{cases} \frac{1}{1-a} & \text{if } |a|<1 \\ \infty & \text{if } |a| \geq 1 \end{cases}$

$\Rightarrow$ stable for a with $|a|<1$ 1mark

S2: $\int_{-\infty}^{\infty} |h_2(t)|\, dt = \int \delta(t) + |a|\delta(t-T)\, dt$

$= 1 + |a|$

$\Rightarrow$ stable for all a with $|a| < \infty$

0.5 marks

(b) $H_1(j\omega) = \int_{-\infty}^{\infty} h_1(t)\, e^{-j\omega t}\, dt = \sum_{k=0}^{\infty} a^k \int \delta(t-kT) e^{-j\omega t}\, dt$

$= \sum_{k=0}^{\infty} a^k e^{-j\omega kT}$

$H_2(j\omega) = \int_{-\infty}^{\infty} h_2(t)\, e^{-j\omega t}\, dt = 1 - a\, e^{-j\omega T}$

(c) $h(t) = h_1(t) * h_2(t)$

$$= h_1(t) * \delta(t) - a\, h_1(t) * \delta(t-T)$$

$$= h_1(t) - a\, h_1(t-T)$$

$$= \sum_{k=0}^{\infty} a^k \delta(t-kT) - \underbrace{\sum_{k=0}^{\infty} a \cdot a^k \delta(t-T-kT)}_{\sum_{k=0}^{\infty} a^{k+1} \delta(t-(k+1)T)}$$

$$= \sum_{k=1}^{\infty} a^k \delta(t-kT)$$

$$= \delta(t)$$ 1 mark

$\Rightarrow$ system S1 is invertible as system S2 is its inverse system 0.5 mark

(d) $$H_1(j\omega) H_2(j\omega) = \sum_{k=0}^{\infty} a^k e^{-j\omega kT} \left(1 - a e^{-j\omega T}\right)$$

$$= \sum_{k=0}^{\infty} a^k e^{-j\omega kT} - \sum_{k=0}^{\infty} a^{k+1} e^{-j\omega(k+1)T}$$

$$= \sum_{k=0}^{\infty} a^k e^{-j\omega kT} - \sum_{k=1}^{\infty} a^k e^{-j\omega kT}$$

$$= 1$$ 0.5 mark

$\Rightarrow$ makes sense, as $H(j\omega) = H_1(j\omega) H_2(j\omega)$ is frequency response of system with impulse response $h(t) = \delta(t)$

$$\Rightarrow H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = 1 \; //$$

0.5 mark