Q4

$$x(t) = \cos(2t)$$

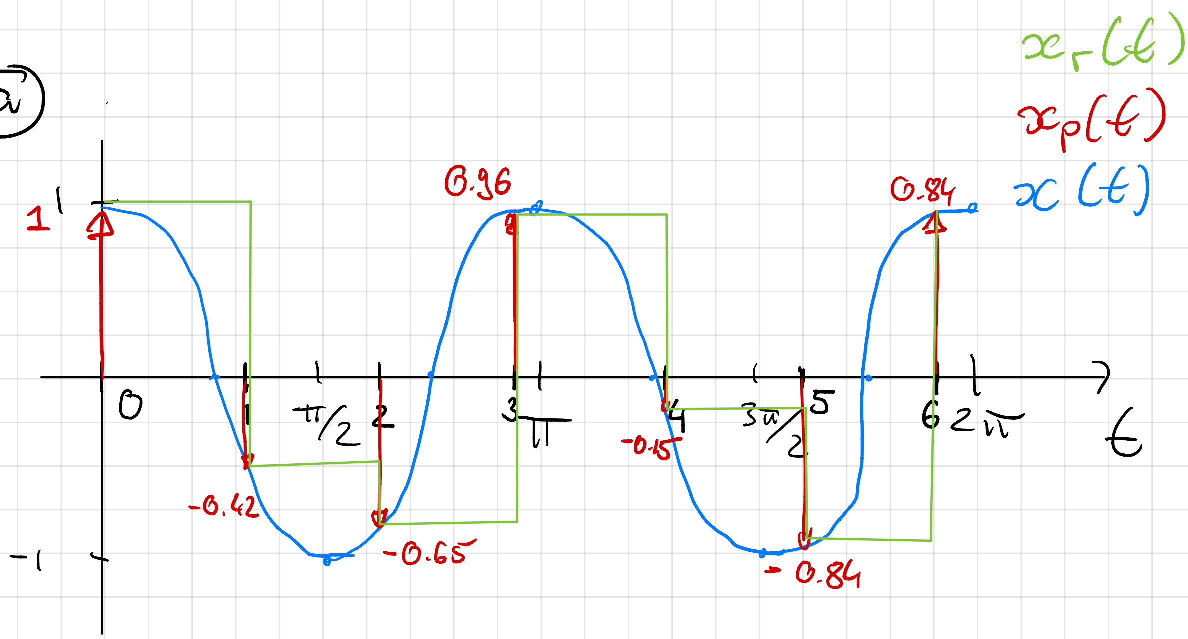
$$\Rightarrow \omega_M = 2$$

$$\Rightarrow \omega_s > 2\omega_M = 4$$

$$\Rightarrow T = \frac{2\pi}{\omega_s} < \frac{\pi}{2}$$

$$\Rightarrow T = 1$$

(a)



(b)

$$X_r(j\omega) = X_p(j\omega) \cdot H_0(j\omega)$$

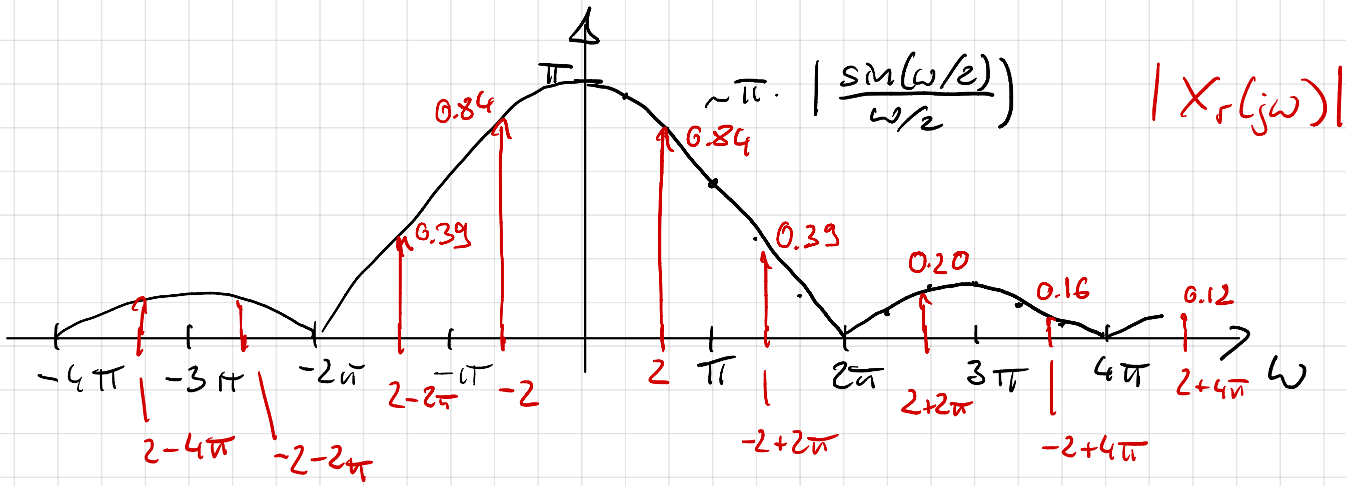
$$H_0(j\omega) = e^{-j\omega\frac{T}{2}} \frac{2\sin\omega\frac{T}{2}}{\omega} \overset{T=1}{=} e^{-j\frac{\omega}{2}} \frac{2\sin\left(\frac{\omega}{2}\right)}{\omega}$$

$$X_p(j\omega) = \frac{1}{T}\sum_k X\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \overset{T=1}{=} \sum_k X(j(\omega - k2\pi))$$

$$= \pi\sum_k \left[\delta(\omega - 2 - k2\pi) + \delta(\omega + 2 - k2\pi)\right]$$

$$\Rightarrow X_r(j\omega) = \pi \sum_k e^{-j\omega\frac{T}{2}} \frac{2\sin(\frac{\omega}{2})}{\omega} \left[\delta(\omega-2-k2\pi) + \delta(\omega+2-k2\pi)\right]$$

$$\Rightarrow |X_r(j\omega)| = \pi \sum_k \left| \frac{\sin(\omega/2)}{\omega/2} \right| \left[\delta(\omega-2-k2\pi) + \delta(\omega+2-k2\pi)\right]$$



(c) need a filter $H_f(j\omega)$ such that

$$H_0(j\omega) \cdot H_f(j\omega) = \begin{cases} T & |\omega| < \omega_s/2 = \pi \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow H_f(j\omega) = \begin{cases} e^{j\frac{\omega}{2}} \dfrac{\omega/2}{\sin(\frac{\omega}{2})} & |\omega| < \pi \\ 0 & \text{otherwise} \end{cases}$$

Q5

(a) Check that $x_c(t)$ is sufficiently band limited:

$$X_c(j\omega) = \begin{cases} 1, & |\omega| < \frac{\pi}{T} = \frac{\omega_s}{2} = \omega_c \\ 0, & \text{otherwise} \end{cases}$$ ✓

(b) $$x_d[n] = x(nT) = \frac{\sin(\pi n)}{\pi n T} = \frac{1}{T}\delta[n]$$ //

$$y_c(t) = x_c(t-\Delta)$$

$$= \frac{\sin(\pi(t-\Delta)/T)}{\pi(t-\Delta)}$$

(c) $$y_d[n] = y_c(nT) = \frac{\sin(\pi(nT-\Delta)/T)}{\pi(nT-\Delta)}$$

$$= \frac{1}{T}\frac{\sin(\pi(n-\frac{\Delta}{T}))}{\pi(n-\Delta/T)}$$ //

(d) $$\frac{1}{T}\delta[n] \xrightarrow{h_d[n]} \frac{1}{T}\frac{\sin(\pi(n-\Delta/T))}{\pi(n-\Delta/T)}$$

$$\Rightarrow h_d[n] = \frac{\sin(\pi(n-\frac{\Delta}{T}))}{\pi(n-\Delta/T))}$$ //

(e) $$h_d[n] = \frac{\sin(\pi(n-\frac{1}{2}))}{\pi(n-\frac{1}{2})}$$

Since $y_c(t) = x(t-T/2)$, $y_d[n]$ is a "half-sample shifted version" of $x_d[n]$ through the process of interpolation:

$$x_d[n] \rightarrow x_c(t) \rightarrow y_c(t) = x_c(t-\tfrac{T}{2}) \rightarrow y_d[n]$$