② Equalization

$$h(t) = \alpha \cdot e^{-\beta t} u(t) = \gamma \cdot \frac{1}{\tau} e^{-t/\tau} u(t)$$

where $\tau = \frac{1}{\beta}$, $\gamma = \alpha \cdot \tau = \alpha/\beta$

see Section 6.5.1 in textbook

1) 3 dB point : $\omega = \frac{1}{\tau} = \beta$

2) $R(j\omega) = a \cdot Y(j\omega) + b\omega\, Y(j\omega)$

$$= a\, H(j\omega)\, X(j\omega) + bj\omega\, H(j\omega)\, X(j\omega)$$

$$= (a + bj\omega)\, H(j\omega)\, X(j\omega)$$

$$\Rightarrow (a + bj\omega)\, H(j\omega) = 1 \qquad H(j\omega) = \frac{\gamma}{j\omega\tau + 1}$$

$$\gamma \frac{a + bj\omega}{1 + j\omega\tau} = 1$$

$$\Rightarrow \quad a = 1/\gamma = \beta/\alpha$$

$$b = \tau/\gamma = 1/\alpha$$

③ AM

$x_1(t)$ has bandwidth $\omega_{m,1}$

$x_2(t)$ has bandwidth $\omega_{m,2}$

SSB AM + FDM : $\omega_{m,1} + \omega_{m,2}$

DSB AM + FDM : $2(\omega_{m,1} + \omega_{m,2})$

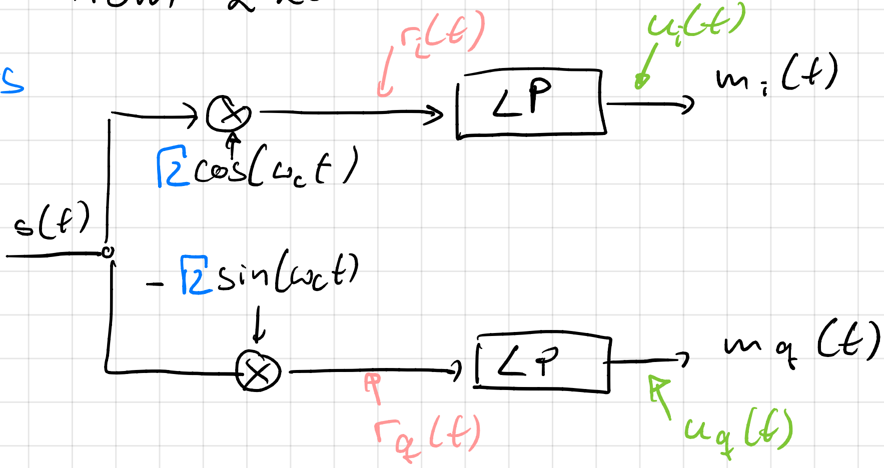
DSB AM w/ carrier + FDM : $2(\omega_{m,1} + \omega_{m,2})$

QAM : $\max(\omega_{m,1}, \omega_{m,2})$

4 QAM

(a) From L20:

2 points



LP with cut-off frequency $\omega_M < \omega_{co} < 2\omega_c - \omega_M$

For $m_i(t)$:
$$s(t) = \sqrt{2}\, m_i(t)\cos(\omega_c t) - \sqrt{2}\, m_q(t)\sin(\omega_c t)$$
$$r_i(t) = s(t)\sqrt{2}\cos(\omega_c t)$$
$$= 2\, m_i(t)\cdot \tfrac{1}{2}\left[1+\cos(2\omega_c t)\right]$$
$$- 2\, m_q(t)\cdot \sin(\omega_c t)\cos(\omega_c t)$$
$$u_i(t) = r_i(t) * h_{LP}(t)$$
slide 13 on SSB in L20
$$= m_i(t)$$

For $m_q(t)$:
$$r_q(t) = -s(t)\sqrt{2}\sin(\omega_c t)$$
$$= 2\, m_q(t)\ \tfrac{1}{2}\left[1-\cos(2\omega_c t)\right]$$
$$- 2 m_i(t)\cos(\omega_c t)\sin(\omega_c t)$$
$$u_q(t) = r_q(t) * h_{LP}(t)$$
$$= m_q(t)$$

(b) 1 point

$$s(t) = \sqrt{2}\,[m_i(t)\cos(\omega_c t) - m_q(t)\cos(\theta_c)\sin(\omega_c t) - m_q(t)\sin(\theta_c)\cos(\omega_c t)]$$

$$= \sqrt{2}\,[\underbrace{(m_i(t) - m_q(t)\sin\theta_c)}_{\text{cross-talk}}\cos(\omega_c t) - m_q(t)\cos(\theta_c)\sin(\omega_c t)]$$

(c) 1 point

(factor $\sqrt{2}$ has not changed, but ok if misunderstood from problem formulation)

$$r_i(t) = s(t)\sqrt{2}\cos(\omega_c t + \varphi_c)$$

$$= s(t)\sqrt{2}\cos(\varphi_c)\cos(\omega_c t) - s(t)\sqrt{2}\sin\varphi_c\sin(\omega_c t)$$

$$= 2\,m_i(t)\cos(\varphi_c)\tfrac{1}{2}[1+\cos(2\omega_c t)]$$

$$- 2\,m_q(t)\cos(\varphi_c)\sin(\omega_c t)\cos(\omega_c t)$$

$$+ 2\,m_q(t)\sin(\varphi_c)\tfrac{1}{2}[1-\cos(2\omega_c t)]$$

$$- 2\,m_i(t)\sin(\varphi_c)\cos(\omega_c t)\sin(\omega_c t)$$

$$u_i(t) = r_i(t) * h_{LP}(t)$$

$$= \underbrace{m_i(t)\cos(\varphi_c) + m_q(t)\sin(\varphi_c)}_{\text{Cross-talk}} = \mathrm{Re}\{(m_i(t) + j m_q(t))e^{-j\varphi_c}\}$$

analogous: $$u_q(t) = \underbrace{m_q(t)\cos(\varphi_c) - m_i(t)\sin(\varphi_c)}_{\text{Cross-talk}} = \mathrm{Im}\{(m_i(t) + j m_q(t))e^{-j\varphi_c}\}$$

$$\phi_c = \frac{\pi}{2} \Rightarrow \begin{array}{l} u_i(t) = m_q(t) \\ u_q(t) = m_i(t) \end{array} \Rightarrow \text{signals are swapped}$$

5 2nd order system

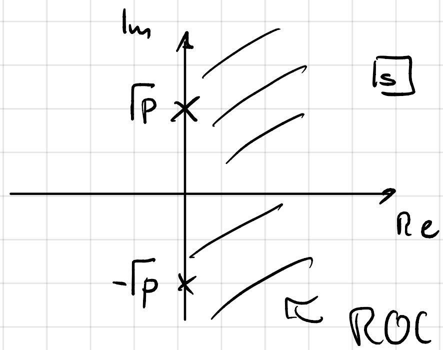
(a) $H_1(s) = \frac{1}{s^2+p}$

2 points

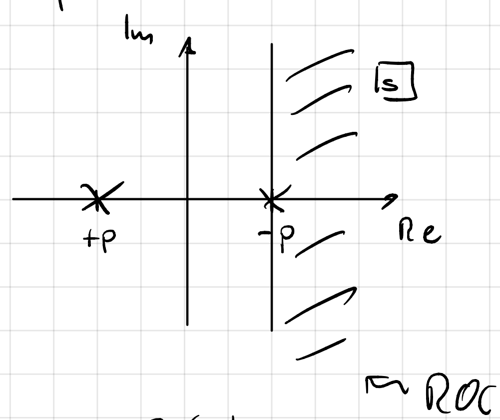
① $p>0$: $H_1(s) = \frac{1}{(s+j\sqrt{p})(s-j\sqrt{p})}$

② $p<0$: $a^2=-p$ $\quad H_1(s) = \frac{1}{(s+a)(s-a)}$

Causal system



$Re\{s\} > 0$



$Re\{s\} > -p$

$\Rightarrow$ $Re\{s\} = 0$ is not part of ROC

$\Rightarrow$ always unstable //

(b) $Y(s) = [X(s) - Y(s)\,G(s)]\,H_1(s)$

1 point

$Y(s)\,[1 + G(s)\,H_1(s)] = X(s)\,H_1(s)$

$$H_2(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1+G(s)\,H_1(s)} = \frac{1}{s^2+p+a+bs}$$

(c) Poles: $s^2 + bs + (a+p) = 0$

2 points

$$s_{1,2} = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - \underbrace{(a+p)}_{c}}$$

(i) if $c \leq 0$, then $\sqrt{\frac{b^2}{4} - c} \geq \frac{b}{2}$

$\Rightarrow$ $\max[Re\{s_{1,2}\}] \geq 0 \Rightarrow$ unstable

$\Rightarrow$ require $c > 0$ ✓

(ii) if $c > \frac{b^2}{4}$, then $\sqrt{\frac{b^2}{4} - c}$ imaginary

$\Rightarrow$ $Re\{s_{1,2}\} = -\frac{b}{2} \Rightarrow$ require $b > 0$ ✓

$\Rightarrow$ $b > 0$ guarantee stability of LTI2

$a + p > 0$ //