The University of British Columbia: ELEC 221 Final Exam

Thursday 24 April 2025 19:00-21:30

Instructor: Lutz Lampe

Duration: 150 minutes
Resources: formula sheet only (provided separately)
Pages: 4
Problems: 5 (40 points) + 1 bonus question (1 point)

Student conduct during examinations:

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, their UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (a) speaking or communicating with other examination candidates, unless otherwise authorized
   (b) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (c) purposely viewing the written papers of other examination candidates;
   (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (e) using or operating electronic devices including but not limited to telephone, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

0 (1 point)

Bonus question: What did we learn in class related to "hair"?

1 (6 points)

A discrete-time LTI system with the frequency response $H(e^{j\omega})$ produces the output

$$y[n] = \omega_0 \cos(\omega_0 n)$$

when the input

$$x[n] = \cos(\omega_0 n)$$

is applied. This is the case for all $\omega_0 \in [-\pi, \pi)$.

(a) Express $x[n]$ and $y[n]$ in terms of complex exponentials $e^{j\omega_0 n}$ and $e^{-j\omega_0 n}$.

(b) Determine $H(e^{j\omega})$. *(Recall the output of an LTI system when the input is a complex exponential.)*

(c) Sketch $H(e^{j\omega})$ for $\omega \in [-2\pi, 2\pi]$. Label the axes.

Show your work and explain your reasoning to earn full points.

2 (7 points)

We consider sampling and reconstruction of bandlimited discrete-time signals.
Suppose a signal $x[n]$ has a Fourier transform $X(e^{j\omega})$ that is zero for $\frac{\pi}{4} \leq |\omega| \leq \pi$. Sampling $x[n]$ with the pulse-train signal

$$s[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$$

generates the signal

$$y[n] = x[n]s[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n - 4k] .$$

(a) Determine the period of $s[n]$.

(b) Determine the Fourier series coefficients of $s[n]$.

(c) Determine the Fourier transform $S(e^{j\omega})$ of $s[n]$.

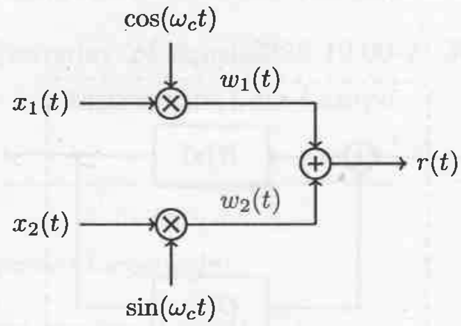
(d) Determine the Fourier transform $Y(e^{j\omega})$ of $y[n]$ as a function of $X(e^{j\omega})$.

(e) Specify the frequency response $H(e^{j\omega})$ of a lowpass filter that produces $x[n]$ as output when $y[n]$ is the input.

Show your work and explain your reasoning to earn full points.

3 (10 points)

We explore quadrature multiplexing using the mutiplexer shown in the following figure:



In this multiplexer, two data signals $x_1(t)$ and $x_2(t)$ are transmitted simultaneously in the same frequency band.

- $x_1(t)$ and $x_2(t)$ are both assumed to be band limited with maximum frequency ω_M so that their Fourier transforms satisfy $X_1(j\omega) = X_2(j\omega) = 0$ for $|\omega| > \omega_M$.
- The two carrier signals $\cos(\omega_c t)$ and $\sin(\omega_c t)$ have a frequency $\omega_c > \omega_M$.

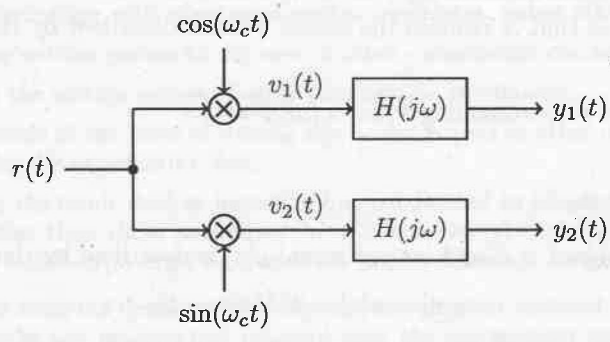
(a) Write the signal $r(t)$ as a function of the data signals and the carrier signals.

(b) Determine the Fourier transform $W_1(j\omega)$ of the signal $w_1(t) = x_1(t)\cos(\omega_c t)$ as a function of $X_1(j\omega)$ and ω_c.

(c) Sketch an $X_1(j\omega)$ of your choice, and sketch the $W_1(j\omega)$ that corresponds to your $X_1(j\omega)$. Label the axes. In particular, identify location, width, and height of spectra.

(d) Determine the Fourier transform $R(j\omega)$ of $r(t)$ as a function of $X_1(j\omega)$ and $X_2(j\omega)$ and ω_c.

We now wish to recover the data signals using the demultiplexing system shown below.



We only consider the upper branch of the demultiplexer in the following.

(e) Write the Fourier transform $V_1(j\omega)$ of $v_1(t) = r(t)\cos(\omega_c t)$ as a function of the Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$ and the carrier frequency ω_c.

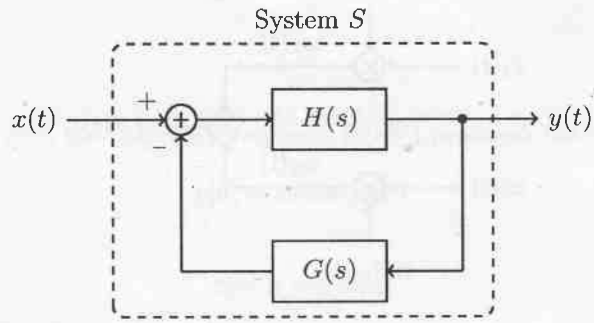
(f) Let $H(j\omega)$ be an ideal lowpass filter. Determine its cutoff frequency and amplitude so that $y_1(t) = x_1(t)$.

Show your work and explain your reasoning to earn full points.

As some of you may be looking for this: $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, $\cos^2(a) = \frac{1}{2}(\cos(2a)+1)$. *It is not needed though.*

4 (10 points)

We use the feedback system configuration shown in the figure below to implement a system S. The two component systems are LTI systems with system functions $H(s)$ and $G(s)$.



(a) Let $H(s) = 1/(s-1)$ and $G(s) = b$ be causal system functions, where $b \in \mathbb{R}$. Note that S is a also causal system

 (i) Determine the impulse responses $h(t)$ and $g(t)$ of the component systems.

 (ii) Determine the system function $Q(s)$ of the overall system S.
 State the poles and region of convergence of $Q(s)$.

 (iii) For what values of b is the system S causal and stable?

(b) We now wish to use the feedback system S with $H(s) = 1/(s+1)$ to implement a causal LTI system that is represented by the differential equation

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

for input $x(t)$ and output $y(t)$.

 (i) Determine the system function $D(s) = Y(s)/X(s)$ corresponding to the differential equation.

 (ii) Determine the system function $Q(s)$ of the system S as a function of $G(s)$.

 (iii) Determine $G(s)$ so that S realizes the causal system described by the differential equation, i.e., $Q(s) = D(s)$.

Show your work and explain your reasoning to earn full points.

5 (7 points)

An LTI system with input signal $x[n]$ and output signal $y[n]$ is described by the difference equation

$$y[n] = x[n] - 2^{-4a}x[n-4] ,$$

where $a \in \mathbb{R}$ with $0 < a < 1$. We wish to build an "equalizer" for this "dispersive" system.

(a) Find the system function $H(z) = \frac{Y(z)}{X(z)}$.
 Determine all poles and zeros of $H(z)$, and indicate the region of convergence.

(b) We now wish to recover $x[n]$ from $y[n]$.
 Find the system function $H_2(z)$ of the LTI system that will output $x[n]$ for input $y[n]$.
 State all possible regions of convergence for $H_2(z)$.
 For each of region of convergence, state whether or not the system is causal or stable.

Show your work and explain your reasoning to earn full points.