① 2 marks

$$H(j\omega) \stackrel{F}{\updownarrow}$$

ⓐ $y(t) = x(t) * h(t) - x(t)$

ⓘ Let $x_1(t) \longrightarrow y_1(t) = x_1(t) * h(t) - x_1(t)$

$x_2(t) \longrightarrow y_2(t) = x_2(t) * h(t) - x_2(t)$

Then $a x_1(t) + b x_2(t) \longrightarrow \left(a x_1(t) + b x_2(t)\right) * h(t)$

$- a x_1(t) - b x_2(t)$

$= a x_1(t) * h(t) - x_1(t)$

$+ b x_2(t) * h(t) - x_2(t)$

$= a y_1(t) + b y_2(t)$

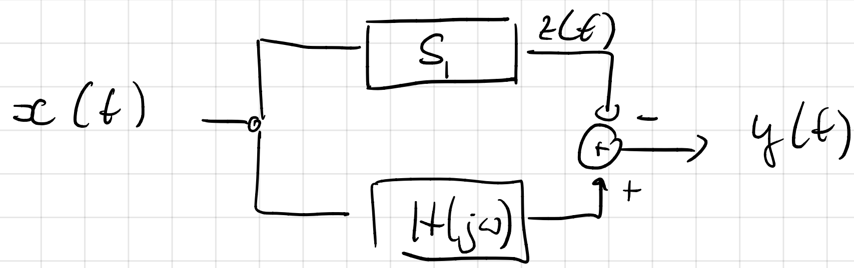
$\Rightarrow$ Linear //

ⓘⓘ $x_1(t-\tau) \longrightarrow x_1(t-\tau) * h(t) + x_1(t-\tau)$

$= y_1(t-\tau)$

$\Rightarrow$ time invariant //

OR :



where system S_1: $z(t) = x(t)$

System S_1 is obviously LTI.

The SoI is parallel concatenation of 2 LTI systems. Therefore, it is LTI. //

(b) $$y(t) = x(t) * h(t) - x(t)$$

2 marks

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) - X(j\omega)$$

$$H_{SoI}(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = H(j\omega) - 1 \quad //$$

(c) Ideal highpass filter:

2 marks

$$H_{SoI}(j\omega) = \begin{cases} 1 & \text{for } |\omega| > \omega_c \\ 0 & \text{for } |\omega| < \omega_c \end{cases}$$

$$\Rightarrow \quad H(j\omega) = \begin{cases} 2 & \text{for } |\omega| > \omega_c \\ 1 & \text{for } |\omega| < \omega_c \end{cases}$$

also ok:

$$H_{SoI}(j\omega) = \begin{cases} -1 & |\omega| > \omega_c \\ 0 & |\omega| < \omega_c \end{cases}$$

equal sign can be anywhere

$$H(j\omega) = \begin{cases} 0 & |\omega| > \omega_c \\ 1 & |\omega| < \omega_c \end{cases}$$

2

1.5 marks each

(a) $y[n]$ is independent of $x[2]$ $\Rightarrow$ not invertible

(def $x_1[n] = x_2[n]$ $\forall n \neq 2$
and $x_1[2] \neq x_2[2]$.

Then $y_1[n] = y_2[n]$.)

(b) def $x_1(t) = x_2(t) + a$,
where $a \neq 0$.

Then, $y_1(t) = y_2(t)$ (but $x_1(t) \neq x_2(t)$)

$\Rightarrow$ not invertible

(c) def $x_1[n] = 2$ $\forall n$

and $x_2[n] = -2$ $\forall n$

Then $y_1[n] = y_2[n] = 4$ $\forall n$

$\Rightarrow$ not invertible

(d) $S: \; y[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

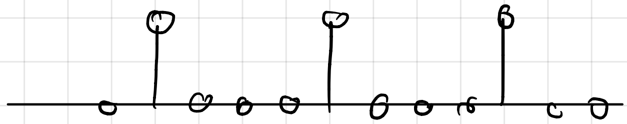
We can find an inverse system: $S^{-1}: \; y[n] = x[2n]$

That is: $x[n] \longrightarrow \boxed{S} \longrightarrow y[n] \longrightarrow \boxed{S^{-1}} \longrightarrow x[n]$

$\Rightarrow$ invertible

3

$x[n]$:



a) Period $N = 4$ // 1 mark

$$c_k = \frac{1}{4} \sum_{n=0}^{3} \delta[n] e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{1}{4} \quad \forall k$$ // 2 marks

b) $$y[n] = x[n] * h[n]$$

3 marks

$$\updownarrow$$

$$\sum a_k e^{j\frac{2\pi}{N}k} = \sum c_k e^{j\frac{2\pi}{N}k} H(e^{j\frac{2\pi}{N}k})$$

$$a_k = c_k H(e^{j\frac{2\pi}{N}k})$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-jk\omega}$$

$$= e^{-j\omega} + 1 + e^{j\omega}$$

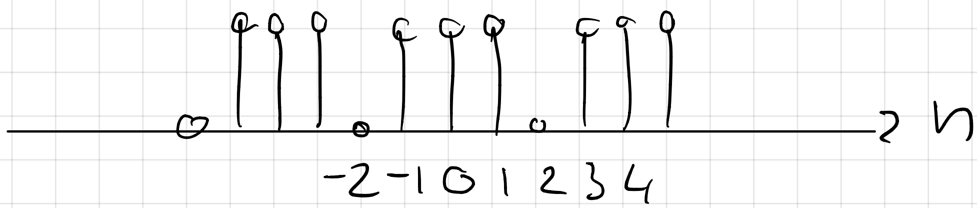
Can also leave it as exponential $= 1 + 2\cos(\omega)$

$$H(e^{j\frac{\pi}{2}k}) = 1 + 2\cos(k\tfrac{\pi}{2})$$

$$a_k = \frac{1}{4}\left(1 + 2\cos(k\tfrac{\pi}{2})\right)$$

alternatively: $y[n] = x[n] * h[n]$

$$= \sum \Big(\delta[n-4k]$$
$$\delta[n-4k-1]$$
$$\delta[n-4k+1] \Big)$$



$$a_k = \frac{1}{4} \sum_{n=-1}^{1} e^{j\frac{2\pi}{4}k} = \frac{1}{4}\left(1 + e^{-j\frac{\pi}{2}k} + e^{j\frac{\pi}{2}k}\right)$$
$$= \frac{1}{4}\left(1 + 2\cos\left(\frac{\pi}{2}k\right)\right)$$

also ok:
$$= \frac{1}{4}\left(1 + e^{j\frac{\pi}{2}k} + e^{j\frac{3}{2}\pi k}\right)$$

(4) $Y_i(j\omega) = H_i(j\omega)\, X(j\omega)$

4 marks

$$X(j\omega) = \pi\left[\delta(\omega-1) + \delta(\omega+1)\right]$$

(i) $H_1(j\omega) = \dfrac{1}{j\omega} + \pi\delta(\omega) - \dfrac{1}{2}2\pi\delta(\omega)$

$$= \frac{1}{j\omega}$$

$$\Rightarrow Y_1(j\omega) = \frac{\pi}{j\omega}\left[\delta(\omega-1) + \delta(\omega+1)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right]$$

$$\Rightarrow y_1(t) = \sin(t)$$

(ii) $H_2(j\omega) = -2 + \dfrac{5}{2+j\omega}$

$$\Rightarrow Y_2(j\omega) = \pi\left[\underbrace{\left(-2 + \frac{5}{2+j}\right)}_{1/j}\delta(\omega-1) + \underbrace{\left(-2 + \frac{5}{2-j}\right)}_{-1/j}\delta(\omega+1)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right]$$

$$\Rightarrow y_1(t) = \sin(t)$$

(b) e.g. $h_3(t) = \frac{1}{2} h_1(t) + h_2(t)$

2marks