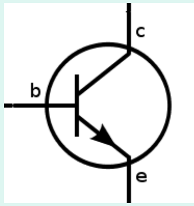
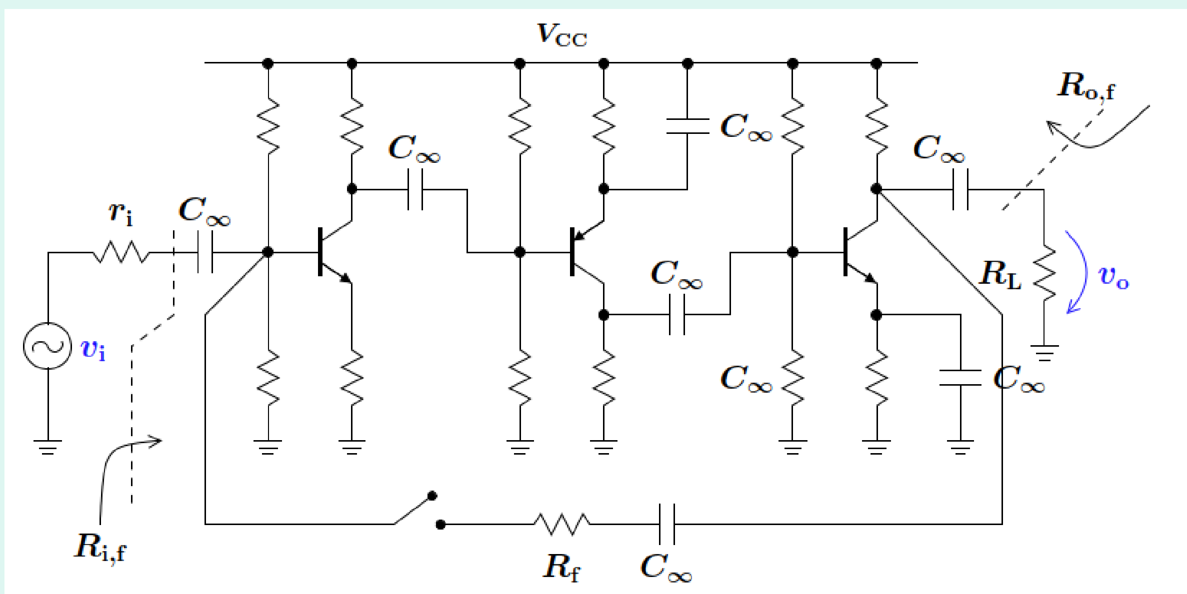


ELEC 301 - Electronic Circuits

L04 - Sep 11

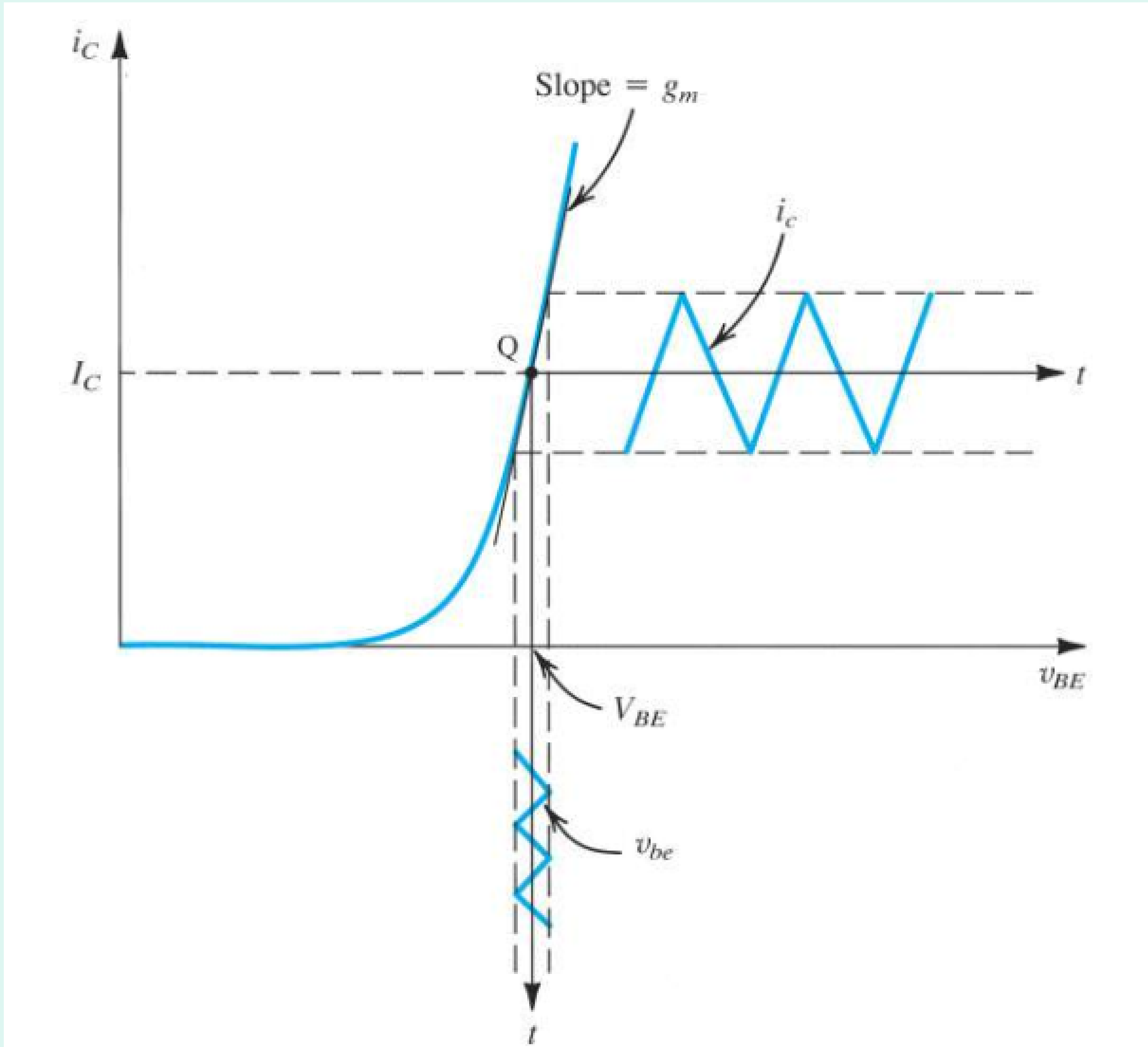
Instructor: Edmond Cretu

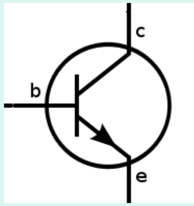


Last time

- Trends and design strategy: PCB vs. ASIC
- Amplification principles: diports, nonlinear device transfer characteristics
- Energy transfer from DC (power supply) to signal energy

Amplification



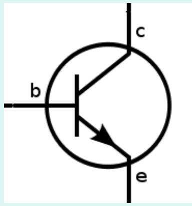


Amplification principle

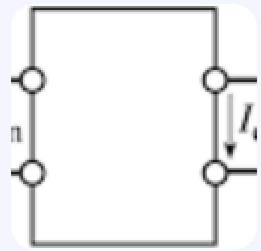
- We need separate input output ports
- Energy is transferred from DC (battery) to signal through the inherent nonlinearity of the device
- **Question**: is this the best we can do? DC levels are very prone to noise
- Comparison BJT vs MOSFET:

$$i_C(t) = I_S e^{v_{BE}/V_T} \Rightarrow i_C \approx \underbrace{I_S e^{V_{BE}/V_T}}_{I_{C0}} + \underbrace{\frac{I_S e^{V_{BE}/V_T}}{V_T}}_{g_m = I_{C0}/V_T} v_{be}(t)$$

$$i_D(t) = \frac{1}{2}k'\frac{W}{L}\left(v_{GS}(t) - V_T\right)^2 \approx \underbrace{\frac{1}{2}k'\frac{W}{L}\left(V_{GS} - V_T\right)^2}_{I_{D0}} + \underbrace{k'\frac{W}{L}\left(V_{GS} - V_T\right)}_{g_m} \cdot v_{gs}(t)$$



BJT vs. MOSFET



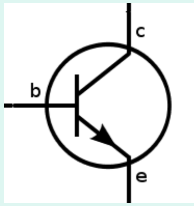
BJT

- Transconductance g_m=1..400mS (exponential relation i_c-v_{be}) (40mS for I_C=1uA, 385mS for I_C=10mA), linear dependence on I_C
- **higher gain applications in discrete circuits**
- higher output power than MOS
- HF performance f_T>300GHz
- lower low-frequency noise than MOS

MOSFET

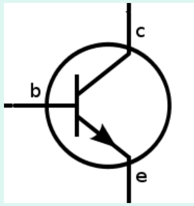
Transconductance gm=10uS…30mS, Square-root dependency on I_D

- higher input impedance than BJT
- better geometric control (W/L), more predictable performance
- fT>200GHz for <20nm technologies
- higher 1/f noise (surface effects)



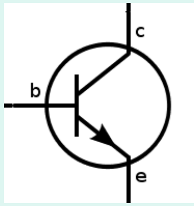
Problems in electronic circuit

- **Design**: given a functional specification, design an appropriate electronic circuit
- **Analysis**: given a circuit, provide thorough analysis of its performance
- **Optimization**: combine design and analysis in iterative cycles to improve the operation w.r.t. defined set of goals
- **Synthesis**: use EDA tools to generate an electronic circuit from a set of specifications
- **Approximation**: use approximations for doing fast analytical computation in analysis

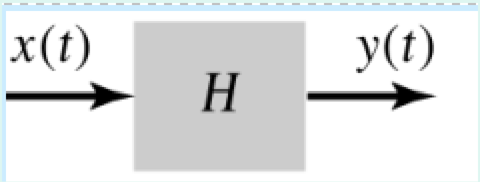


Review - Linear circuits

- Goal: simplification techniques for circuit analysis - reduced representations through iterative/global transformations
- Linear time-invariant (LTI) circuits/systems
- **Linearity** = superposition + homogeneity
- **Time-invariant** system - the structure of the system does not change in time (no matter when we apply an input, we get the same output response)
- LTI allows the design and analysis in either time or frequency domain
- Often the right feedback allows the **linearization** of a system

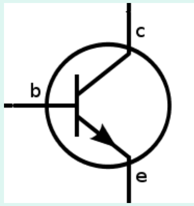


Time-invariant systems



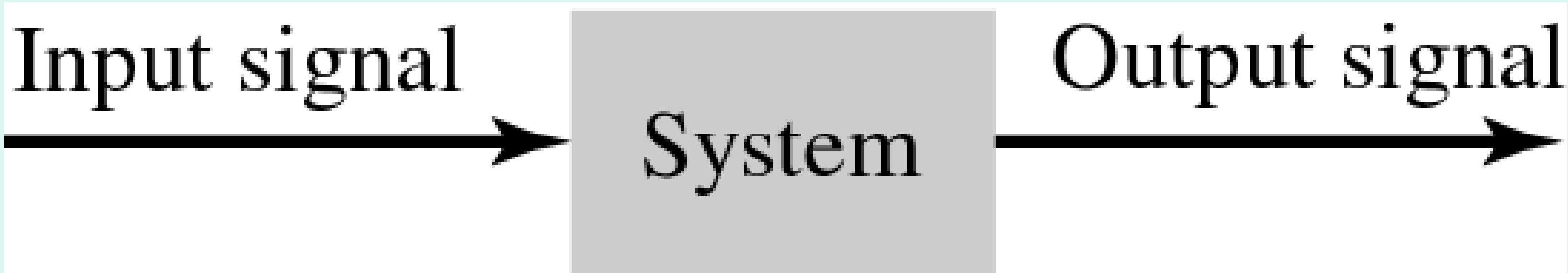
- A system is **time invariant** ↔ a time shift in the input signal produces an identical time shift in the output
- Heuristics: the structure of the system remains the same, it responds identically no matter when the input signal is applied
- What counts is the relative difference of time between the input signal and the output one

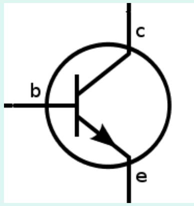
$$H\{\ \}\ \text{is time invariant iff}\ \left[y(t)=H\{x(t)\}\Rightarrow y(t-t_0)=H\{x(t-t_0)\},\ any\ t_0\right]$$



Causality in systems

- A physical system is causal: the effect of an input signal comes only after the signal occurs!
- **Causal system** = a system where the output y(t) at some specific instant t_0 only depends on the input x(t) for values of t less than or equal to t_0. Therefore these kinds of systems have outputs and internal states that depends only on the current and previous input values.
- Modern physics have shaken the image of a strictly causal nature (quantum entanglement)

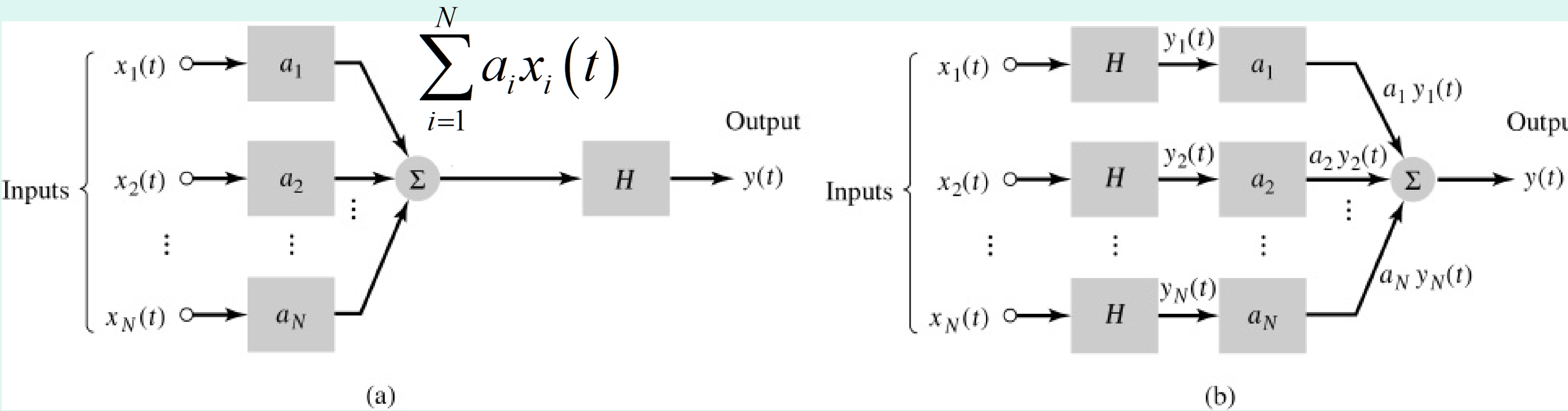




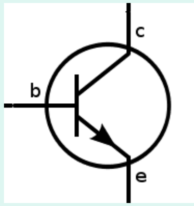
Linear systems

- Linearity of a system means that its action can be described by a **linear operator H**
- **Superposition + homogeneity properties**

$$H\left\{a_1 x_1(t) + a_2 x_2(t)\right\} = a_1 H\left\{x_1(t)\right\} + a_2 H\left\{x_2(t)\right\}$$

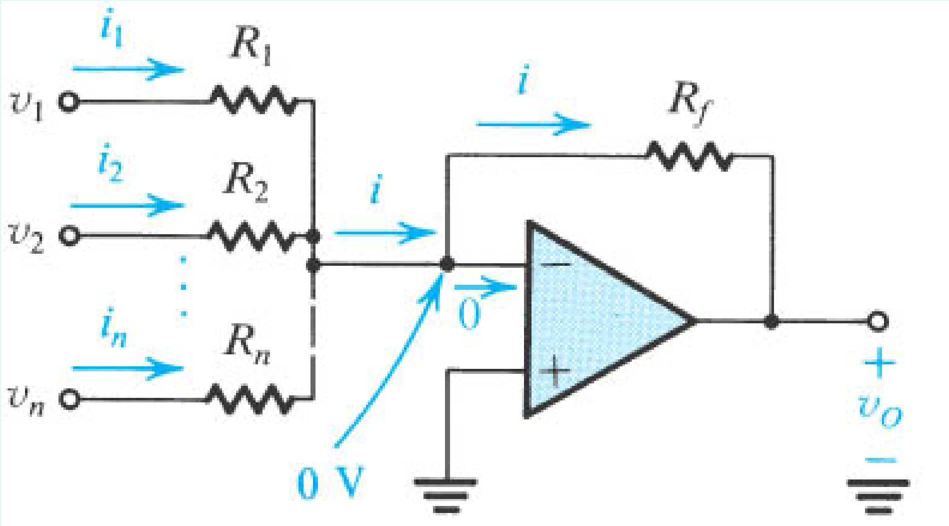


If these two configurations produce the same output $y(t)$, the operator H is linear.



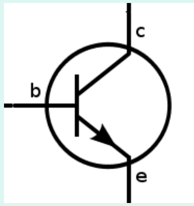
Use of superposition as an analysis tool

- Exm: the weighted summer



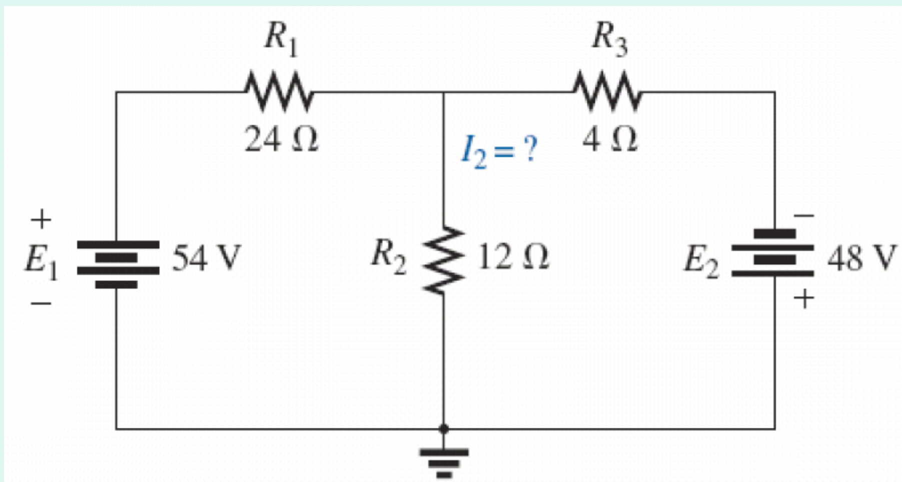
$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

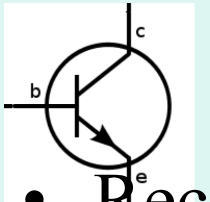
Total effect = sum of partial effects



Exm - homework

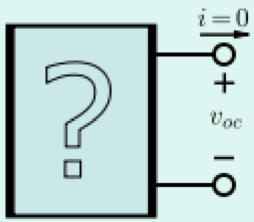
- Use superposition to determine the current through the 12Ω resistor

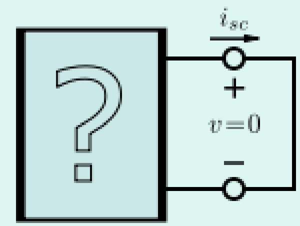




Circuit transformations

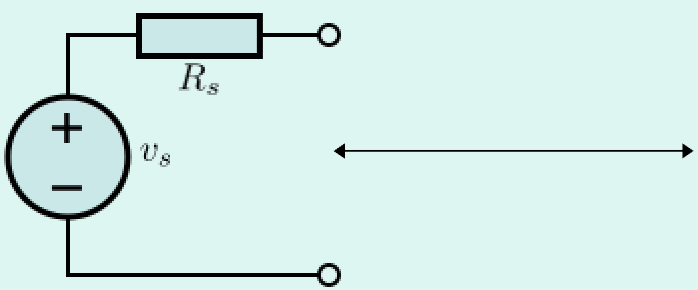
- Recall Thevenin/Norton equivalent circuit theorems for linear uniports
- Application: graphical reduction of branches



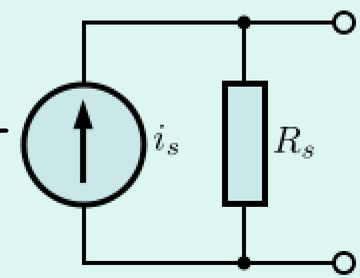


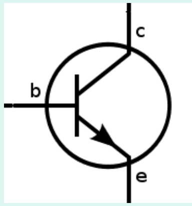
$$v_s = v_{oc}$$

$$i_s = i_{sc}$$



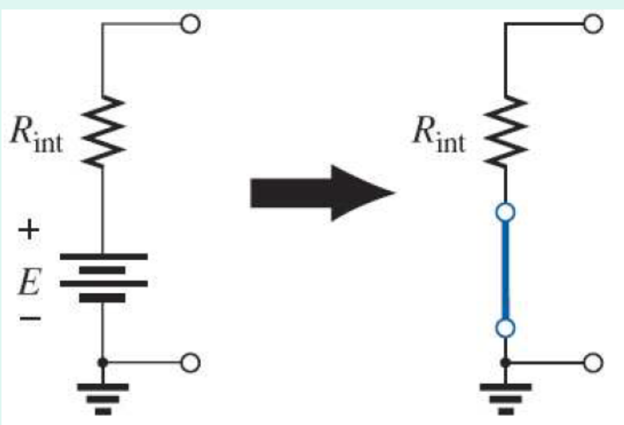
$$i_s = \frac{v_s}{R_s}$$



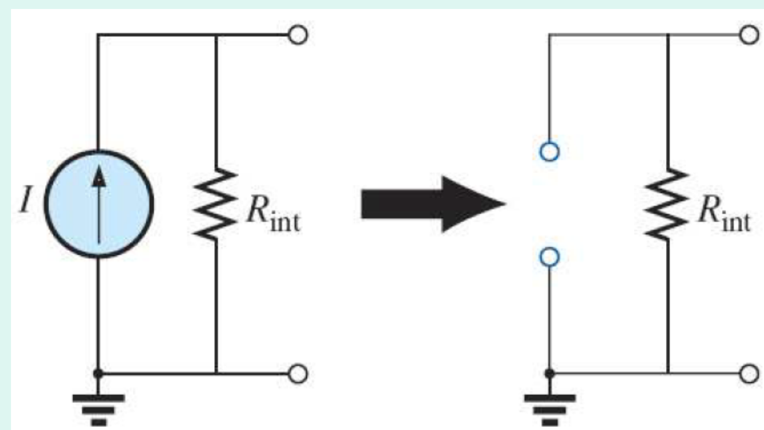


Circuit reduction techniques (1)

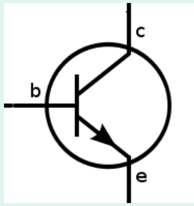
- Superposition theorem: the current through, or voltage across, any element of a network is equal to the algebraic sum of the currents and voltages produced independently by each source (with all other sources removed)



Removing a voltage

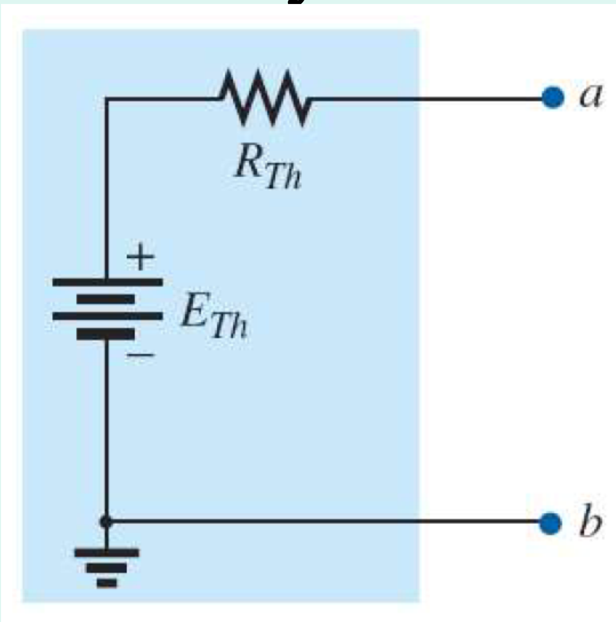


Removing a current



Thevenin's theorem (internal view)

- Any 2-terminal (uniport) linear network can be replaced by an equivalent circuit consisting solely of a voltage source and series impedance

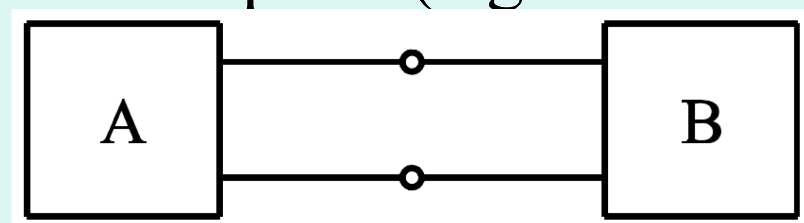


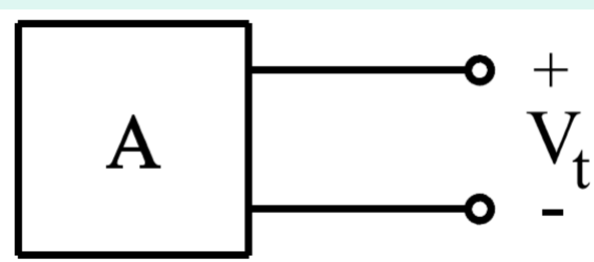
1. Zth(s) is calculated setting all sources to zero (including initial conditions on L,C)
2. Eth(s) is calculated by returning all sources to their original position and finding the open-circuit voltage between a-b

Thevenin equivalent circuit (DC case)

Thevenin's theorem - external view

- Any portion of a linear circuit can be reduced, w.r.t. to a port, to an equivalent Thevenin equivalent source V_t and an equivalent impedance Z_t
- Attention to hidden coupling between ports (e.g. magnetic energy)





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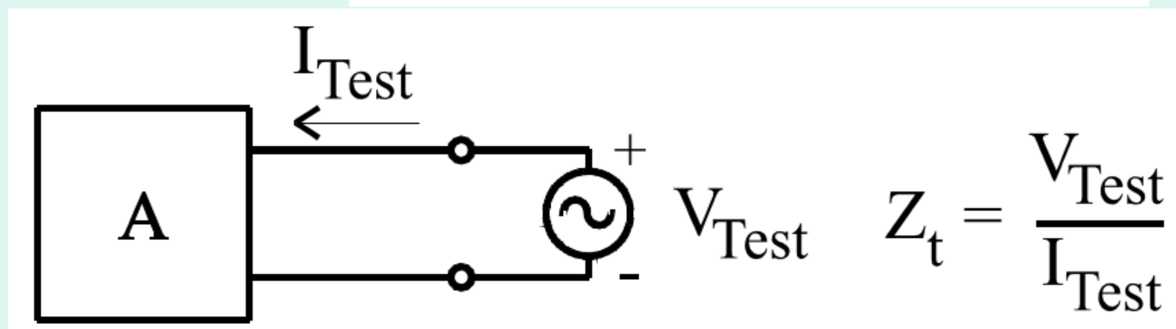
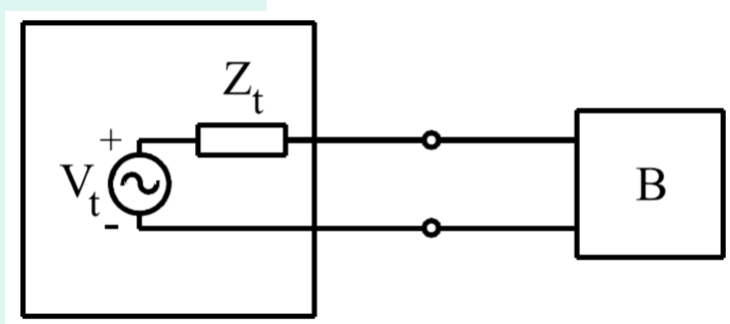
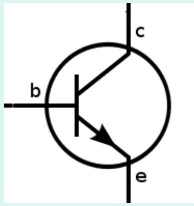


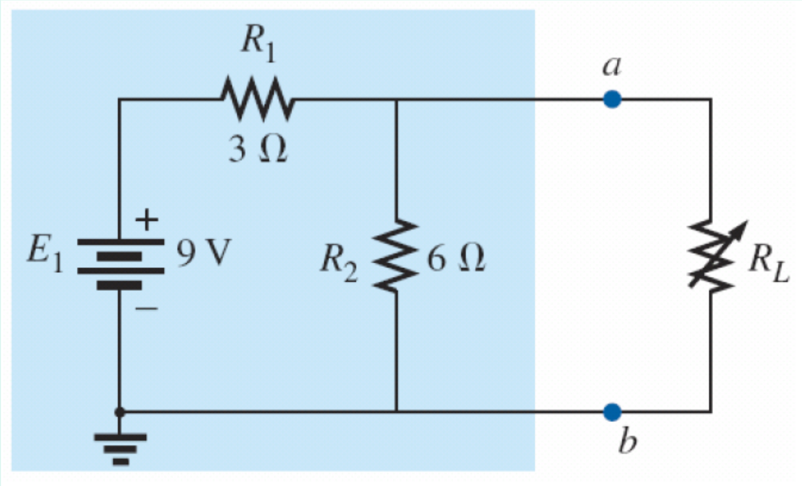




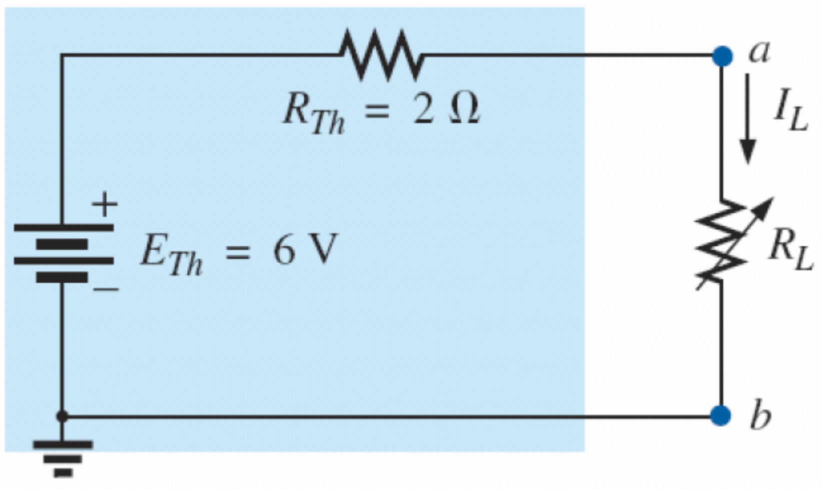
Useful for simplification of circuits

- Find the Thevenin equivalent between terminals a-b

$$R_{TH} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

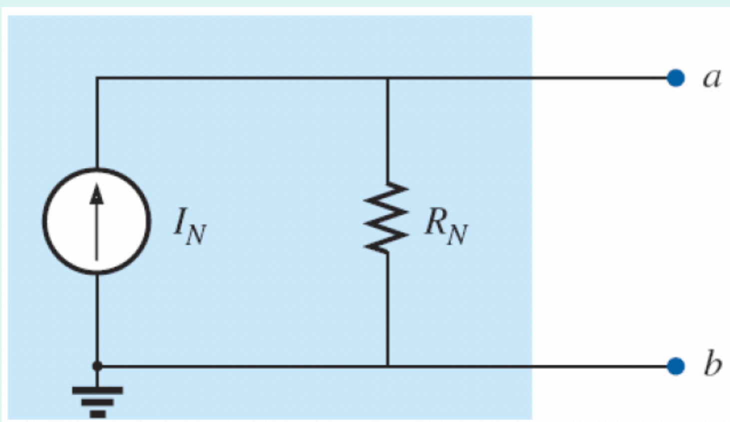


$$V_{TH} = E_1 \frac{R_2}{R_1 + R_2}$$



Norton's theorem

- Any linear 2-terminal network can be replaced by an equivalent circuit consisting of a current source and a parallel impedance

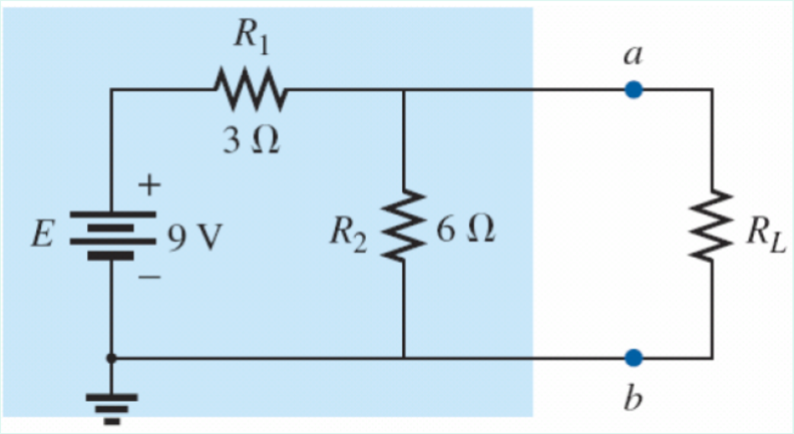


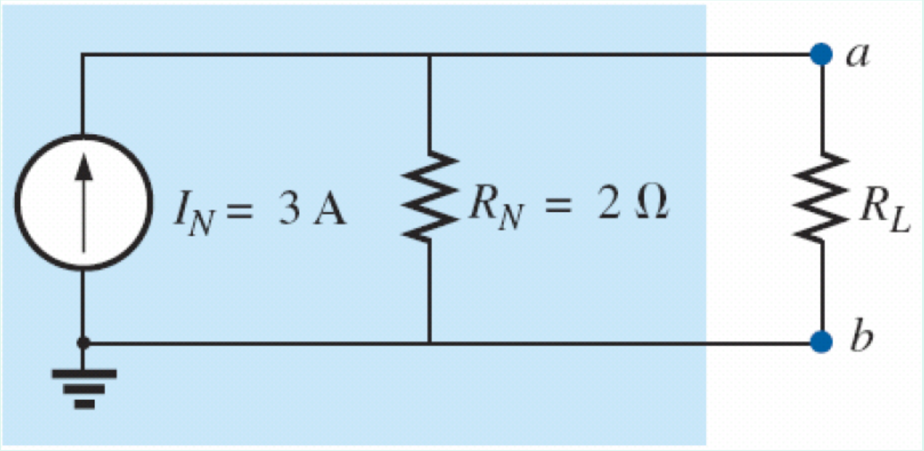
Norton equivalent circuit

1. $Z_N(s)$ is computed by setting all sources to zero (including all ICs) and then finding the resultant impedance between a-b ($Z_N(s)$=Zth(s))
2. Calculate $I_N(s)$ by returning all sources to their original position and then finding the short-circuit current between a-b

Exm

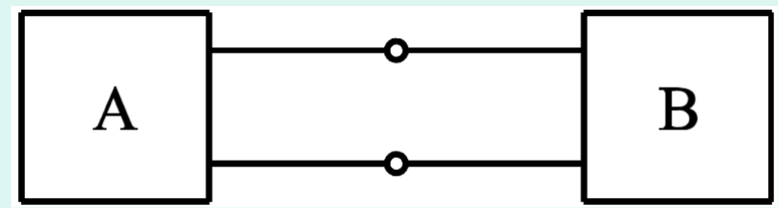
- Find the Norton equivalent circuit between a-b terminals

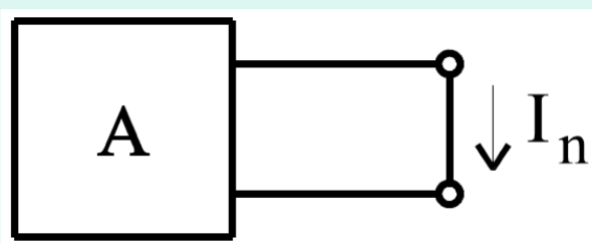


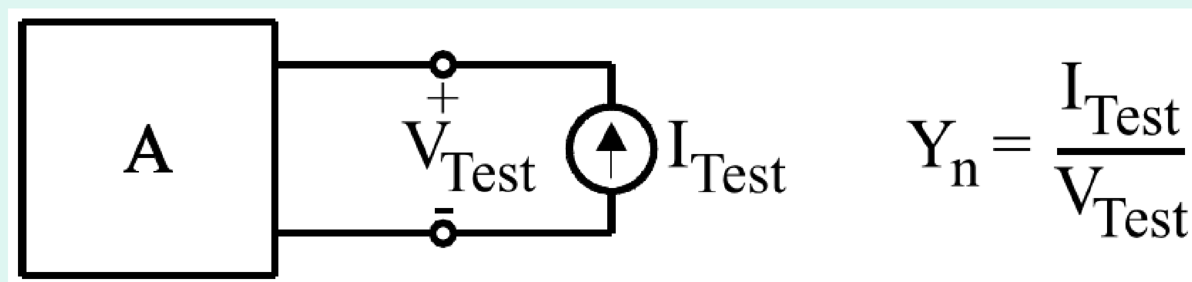


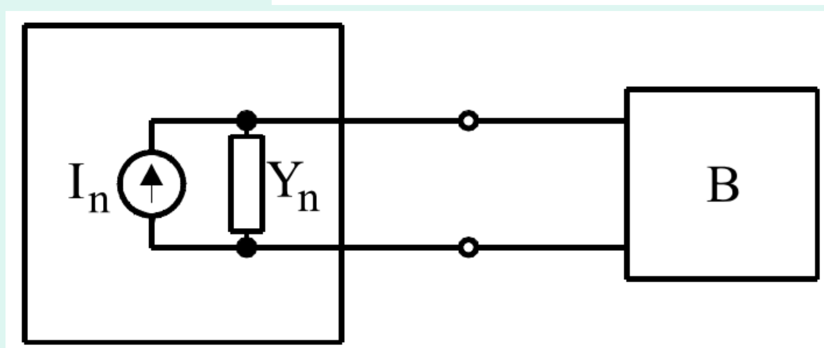
Norton's theorem - external view

- A portion of linear network may be represented at an electrical port by an equivalent circuit source I_n and an equivalent admittance Y_n

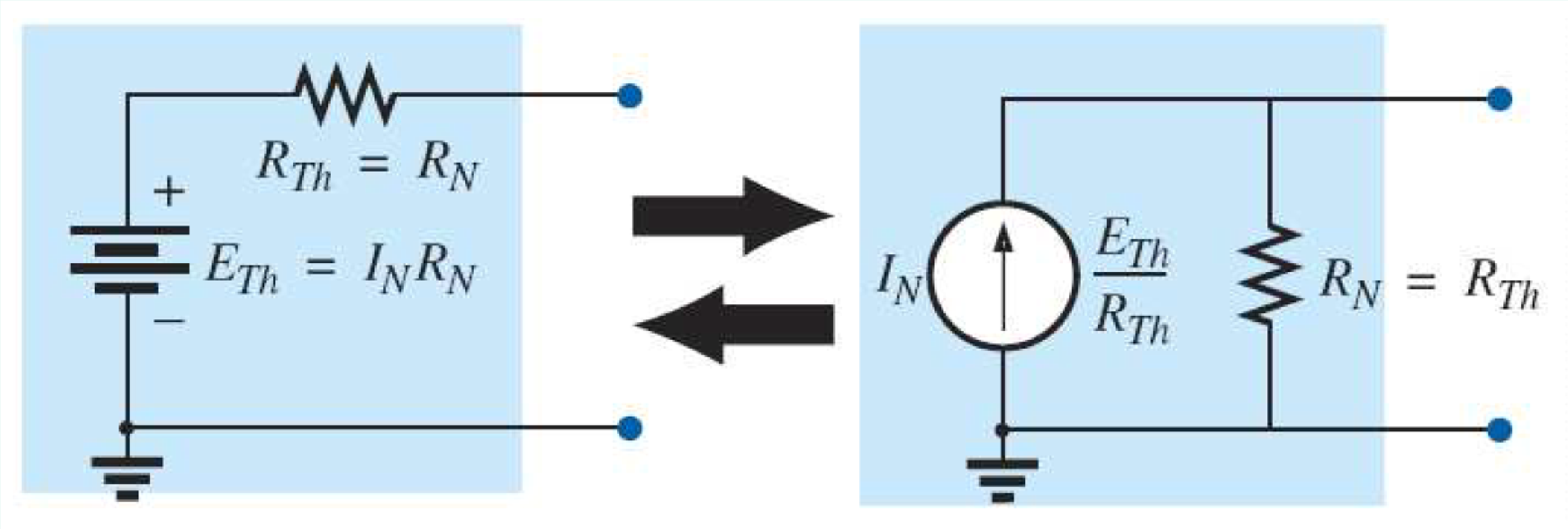


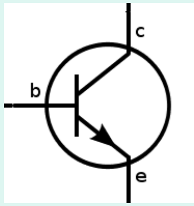






Converting between Thevening and Norton equivalent uniports



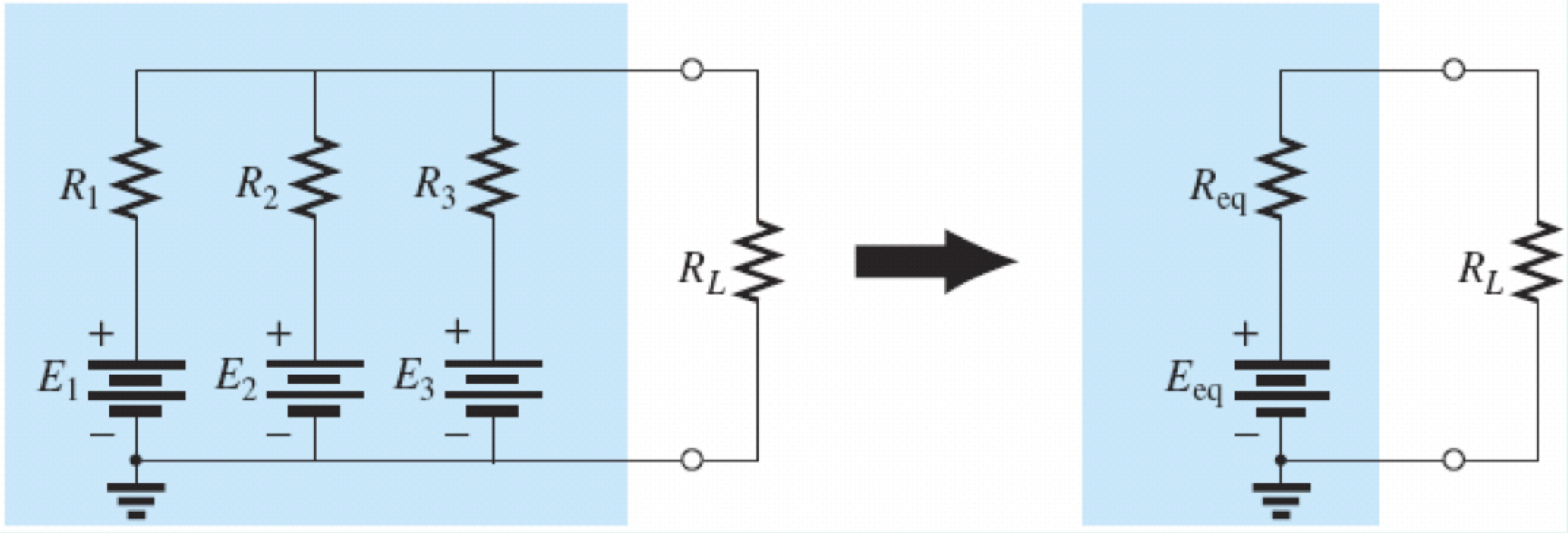


Question

- What to do if do not have access to the inner circuit (only to the port of A)? How do you determine the Thevenin/Norton equivalent?

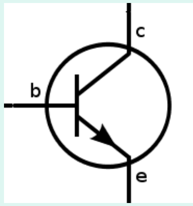
Circuit reduction techniques (2)

- **Millman's theorem** (for parallel branches) – any number of parallel branches can be reduced to one



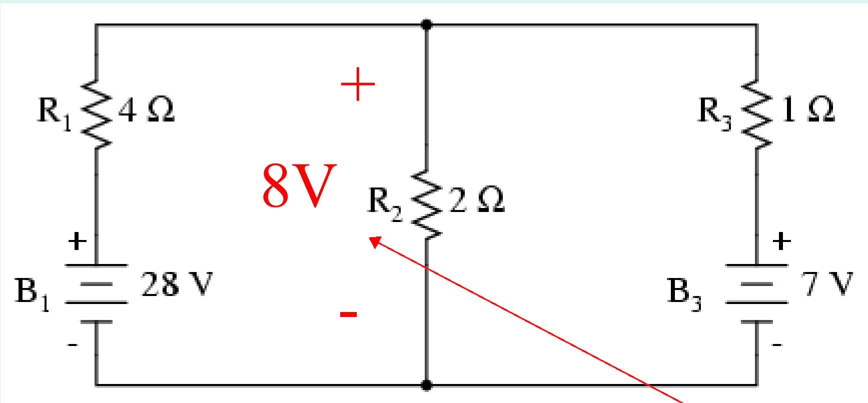
$$Y_{eq}(s) = Y_1 + Y_2 + Y_3 \quad \left(\frac{1}{Z_{eq}(s)} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right)$$

$$E_{eq}(s) = \frac{Y_1 E_1 + Y_2 E_2 + Y_3 E_3}{Y_1 + Y_2 + Y_3}$$



Exm

- Circuit computation through Millman's theorem



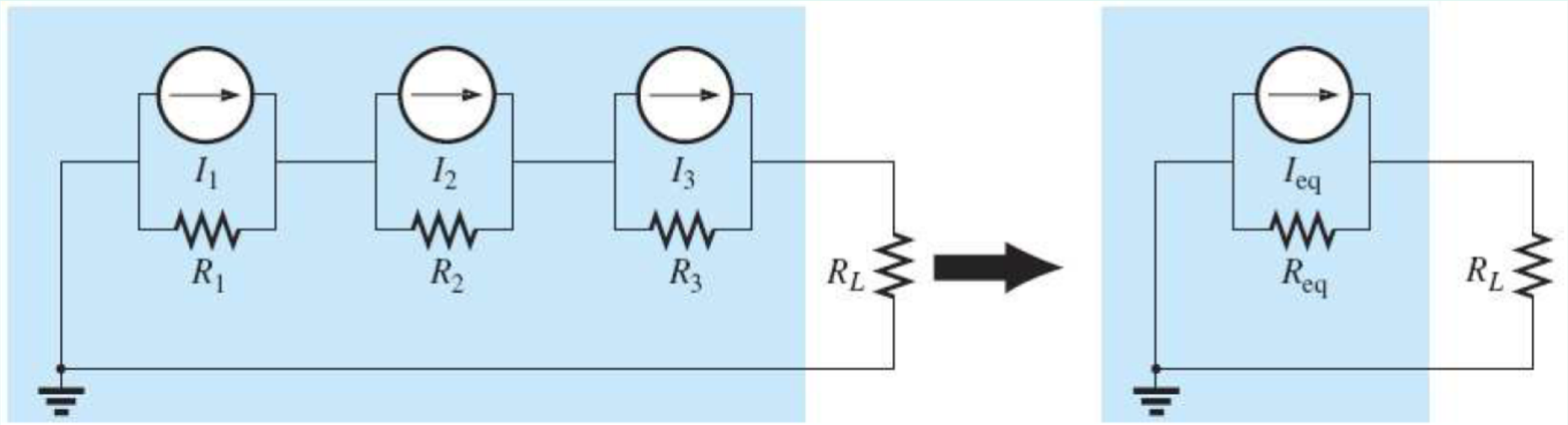
Millman's Theorem Equation

$$\frac{\frac{E_{B1}}{R_1}+\frac{E_{B2}}{R_2}+\frac{E_{B3}}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \text{Voltage across all branches}$$

$$\frac{\frac{28\text{ V}}{4\ \Omega}+\frac{0\text{ V}}{2\ \Omega}+\frac{7\text{ V}}{1\ \Omega}}{\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}} = 8\text{ V}$$

Dual of Millman's theorem

- Reduces branches in series



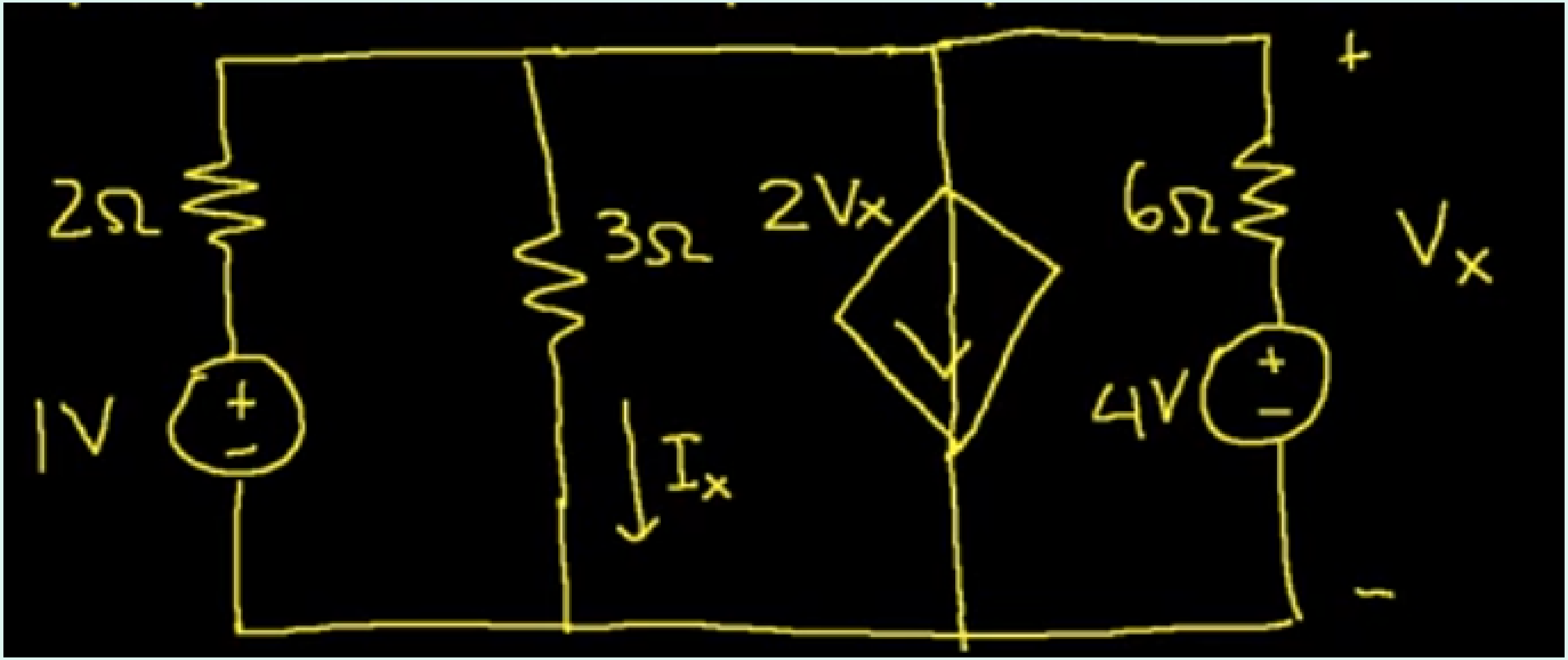
$$Z_{eq}(s) = Z_1 + Z_2 + Z_3$$

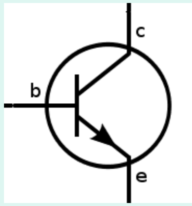
$$I_{eq}(s) = \frac{Z_1 I_1 + Z_2 I_2 + Z_3 I_3}{Z_1 + Z_2 + Z_3}$$

More subtle case: superposition with linear dependent sources

- Steps:
  1. Identify sources: distinguish between independent and controlled sources in the circuit
  2. Activate one independent source (replace the independent V sources with SC and Independent I sources with OC). Do not deactivate any controlled sources => analyze the circuit to find the partial output(s)
  3. Repeat step 2 for all independent sources
  4. Sum the partial contribution to get the total response

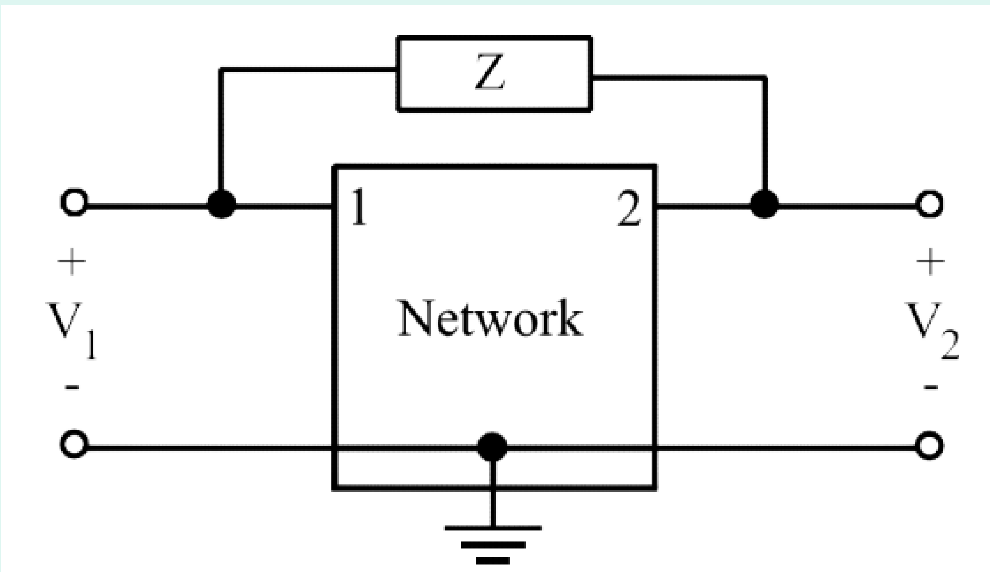
Homework example



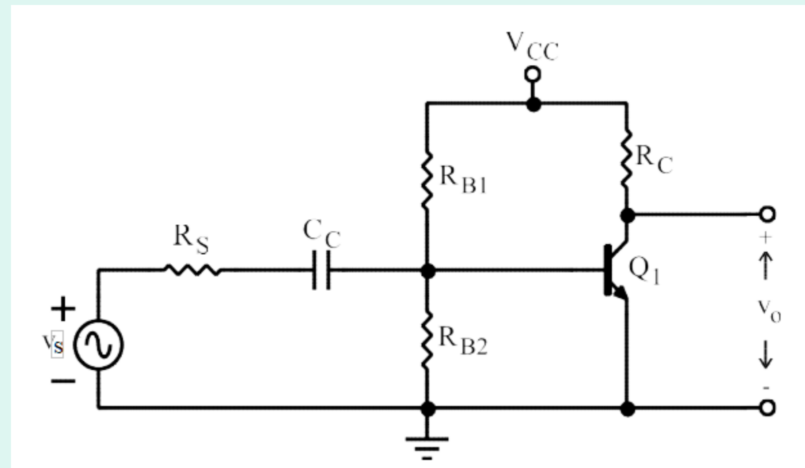


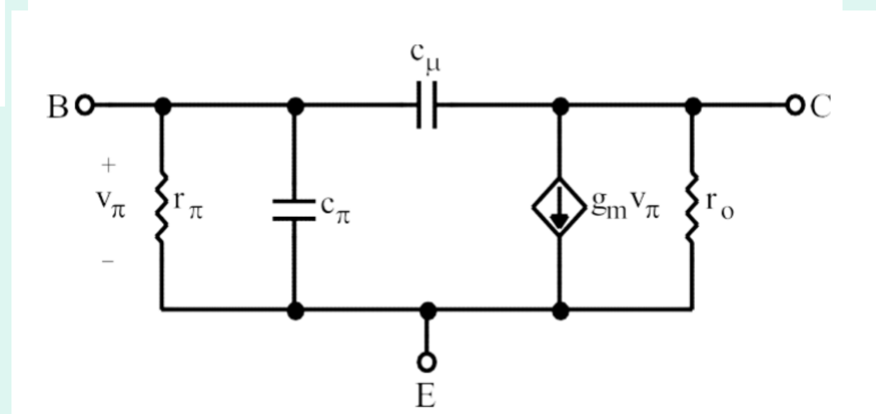
Miller’s theorem

- Helps in analysis by eliminating the feedback

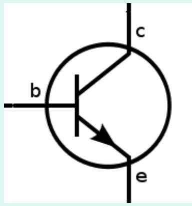


$$V_2 = kV_1$$



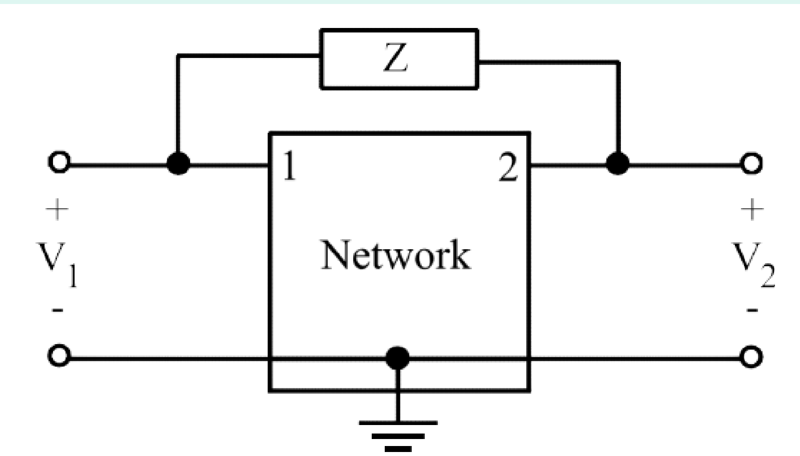


hybrid-π small signal model



Miller’s theorem (2)

- Replace the Z feedback with two impedances Z1 and Z2



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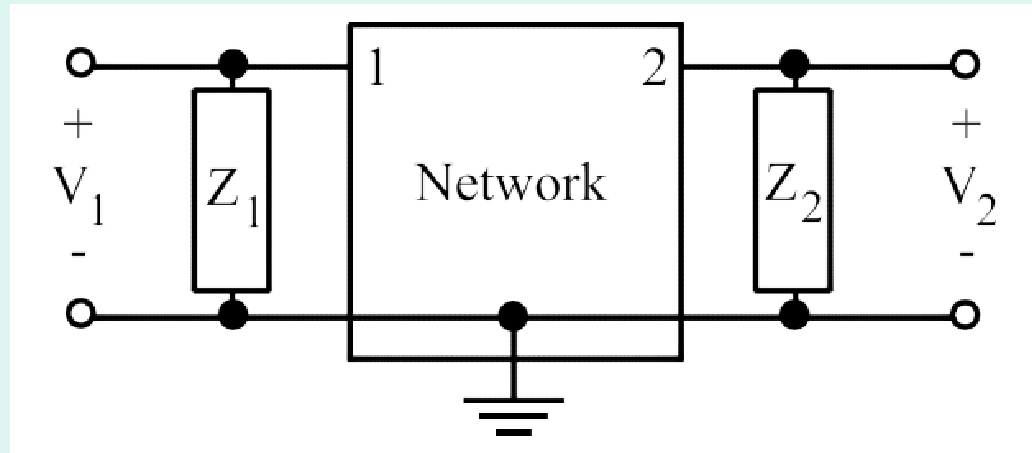
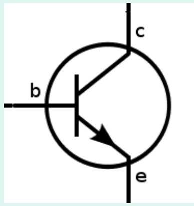


$$V_2 = kV_1$$

$$V_2 = kV_1$$

$$Z_1 = Z\frac{1}{1-k}$$

$$Z_2 = Z\frac{k}{k-1}$$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