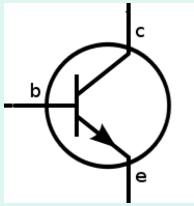
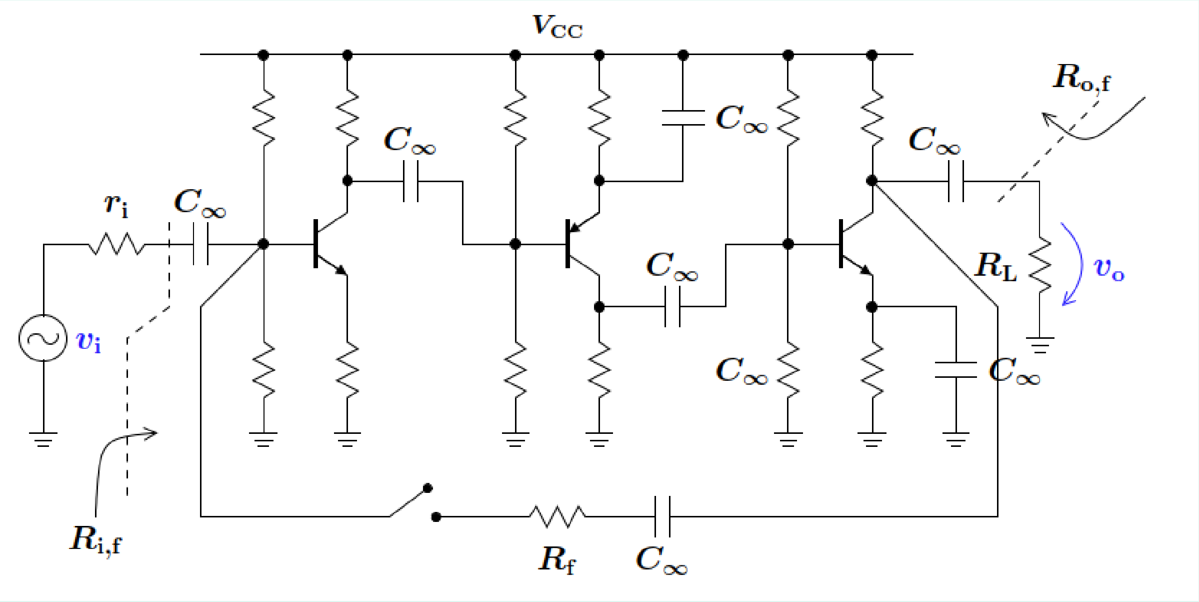
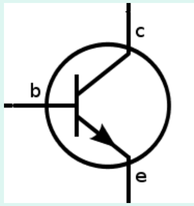


ELEC 301 - Bode plots

L07 - Sep 18

Instructor: Edmond Cretu

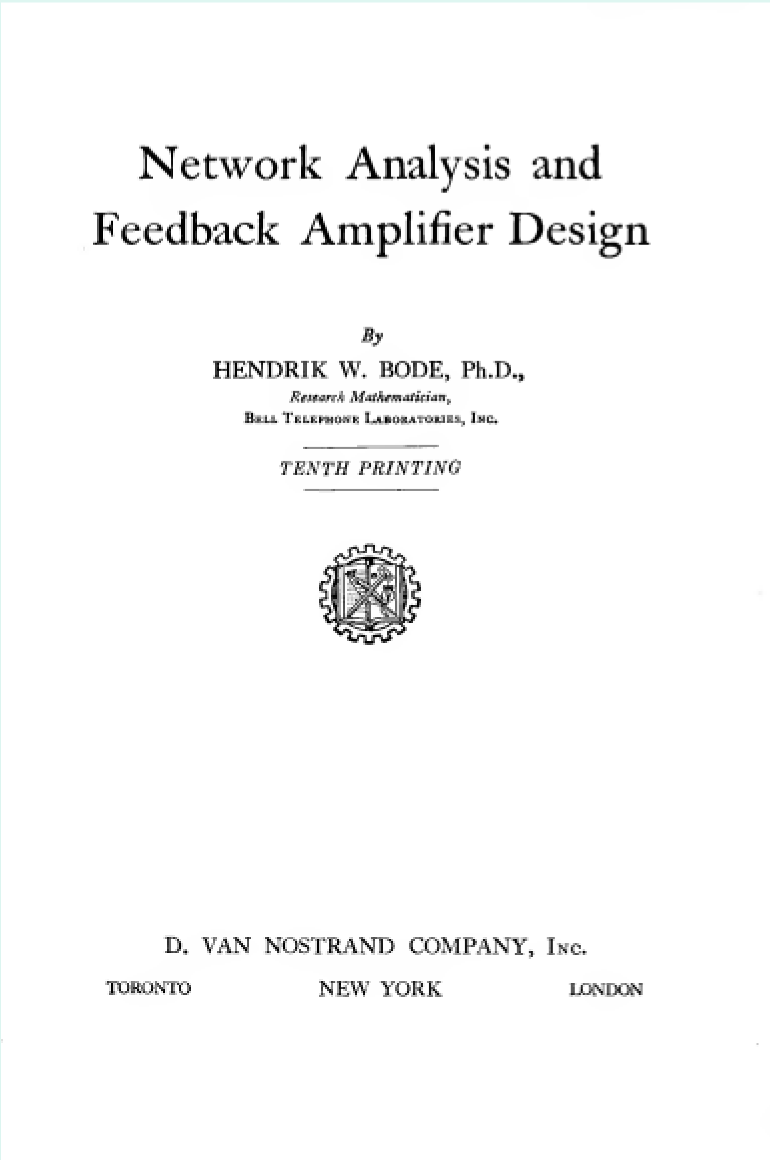




Last time

- Laplace transform, transfer functions
- The use of LT in solving ODEs
- The use of LT for mapping circuits from the time domain to s-domain
- Introduction in Bode plots

Bode - Network analysis and feedback amplifier design



Network Analysis and
Feedback Amplifier Design

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TENTH PRINTING

D. VAN NOSTRAND COMPANY, INC.
TORONTO NEW YORK LONDON

- Old reference (1945), available on the internet archives

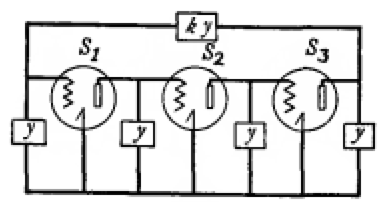


FIG. 8.20

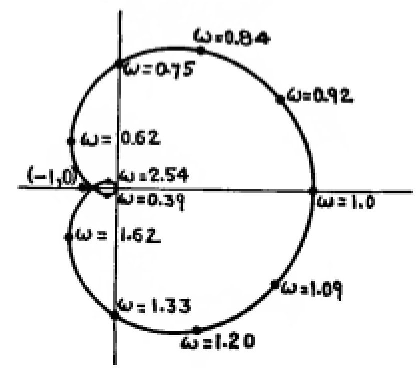
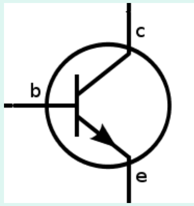


FIG. 8.21



Transfer function - example

- Amplifier with all poles and zeros in the negative half-plane, real and distinct

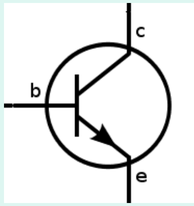
$$T(s) = K \frac{(s+\omega_{z1})(s+\omega_{z2})\ldots(s+\omega_{zn})}{(s+\omega_{p1})(s+\omega_{p2})\ldots(s+\omega_{pN})}$$

$$n < N$$

Frequency response - phasor representation

$$T(j\omega) = K \frac{(j\omega+\omega_{z1})(j\omega+\omega_{z2})\ldots(j\omega+\omega_{zn})}{(j\,\omega+\omega_{p1})(j\,\omega+\omega_{p2})\ldots(j\,\omega+\omega_{pN})}$$

$$T(j\omega) = K \frac{M_{z1}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{z1}}}\, M_{z2}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{z2}}} \ldots M_{zn}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{zn}}}}{M_{p1}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{p1}}}\, M_{p2}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{p2}}} \ldots M_{pN}(\omega)e^{j\tan^{-1}\frac{\omega}{\omega_{pN}}}}$$



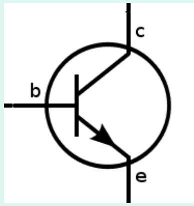
Magnitude and phase separation

Magnitude:

$$
\begin{aligned}
20\log|T(j\omega)| &= 20\log|K| \\
&+ 20\log\sqrt{\omega^2+\omega_{z1}^2} + 20\log\sqrt{\omega^2+\omega_{z2}^2} + \ldots + 20\log\sqrt{\omega^2+\omega_{zn}^2} \\
&- 20\log\sqrt{\omega^2+\omega_{p1}^2} - 20\log\sqrt{\omega^2+\omega_{p2}^2} - \ldots - 20\log\sqrt{\omega^2+\omega_{pN}^2}
\end{aligned}
$$

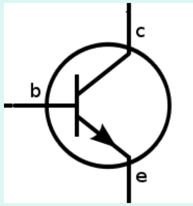
Phase: we must add 0 if $K>0$ and π if K is negative

$$
\begin{aligned}
\phi(\omega) &= \tan^{-1}\frac{\omega}{\omega_{z1}} + \tan^{-1}\frac{\omega}{\omega_{z2}} + \ldots + \tan^{-1}\frac{\omega}{\omega_{zn}} \\
&- \tan^{-1}\frac{\omega}{\omega_{p1}} - \tan^{-1}\frac{\omega}{\omega_{p2}} - \ldots - \tan^{-1}\frac{\omega}{\omega_{pN}}
\end{aligned}
$$



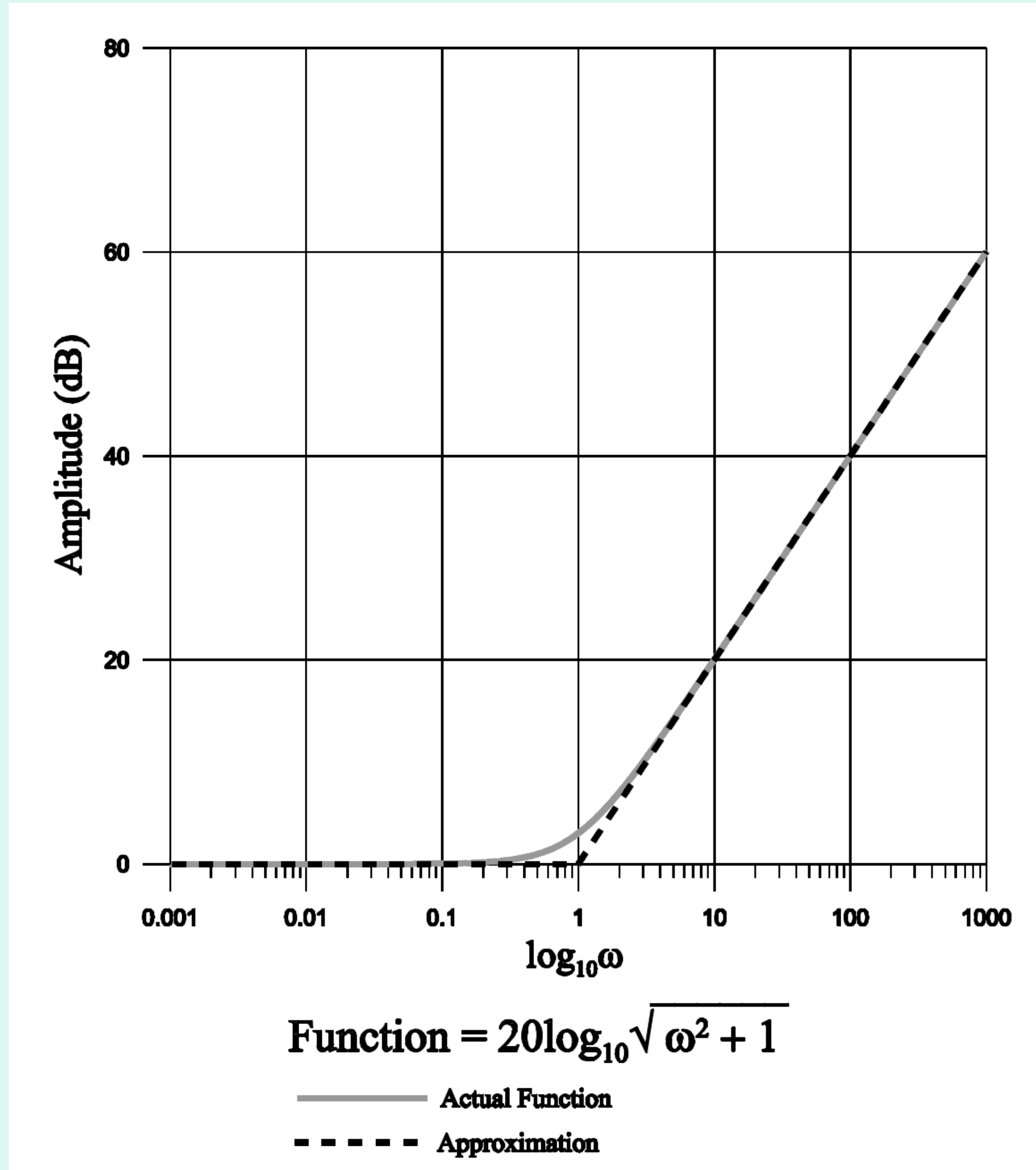
Remarks

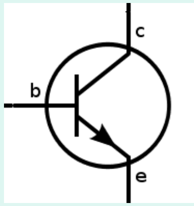
- The log operation separates the frequency response into additive primitive components
- While related, we can separate the visual representations for magnitude and phase
- We only need to identify the patterns of variations in the magnitude - phase representation for the primitive components (poles/zeros)



Simple zero z1=-1

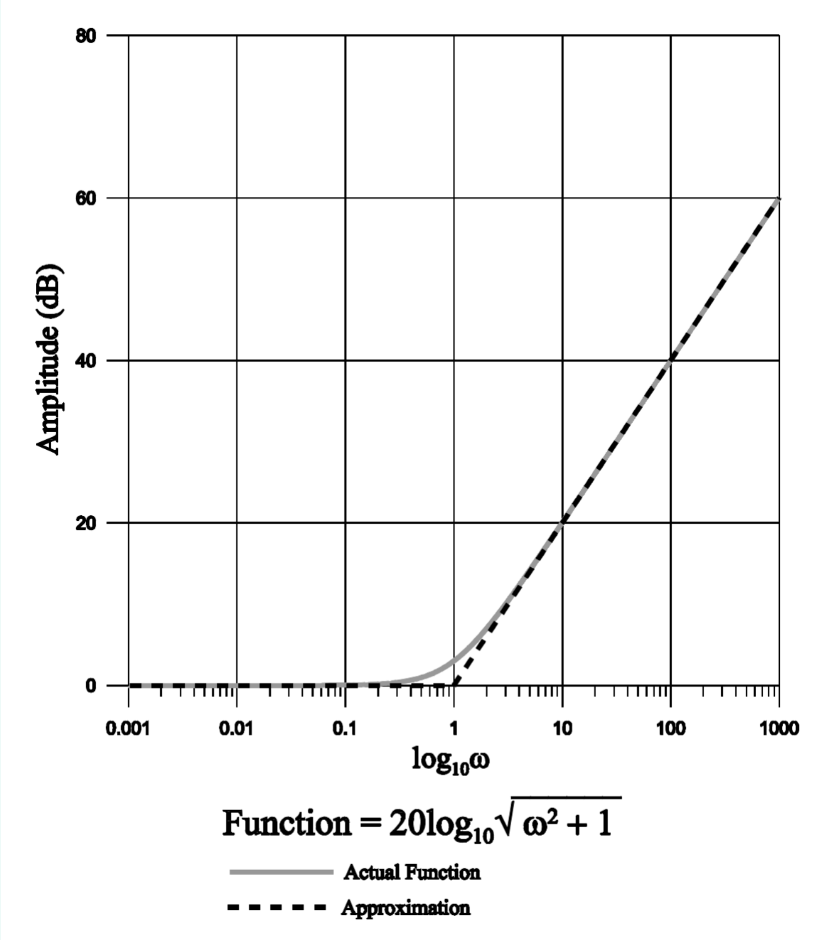
- Assume a simple zero in the transfer function





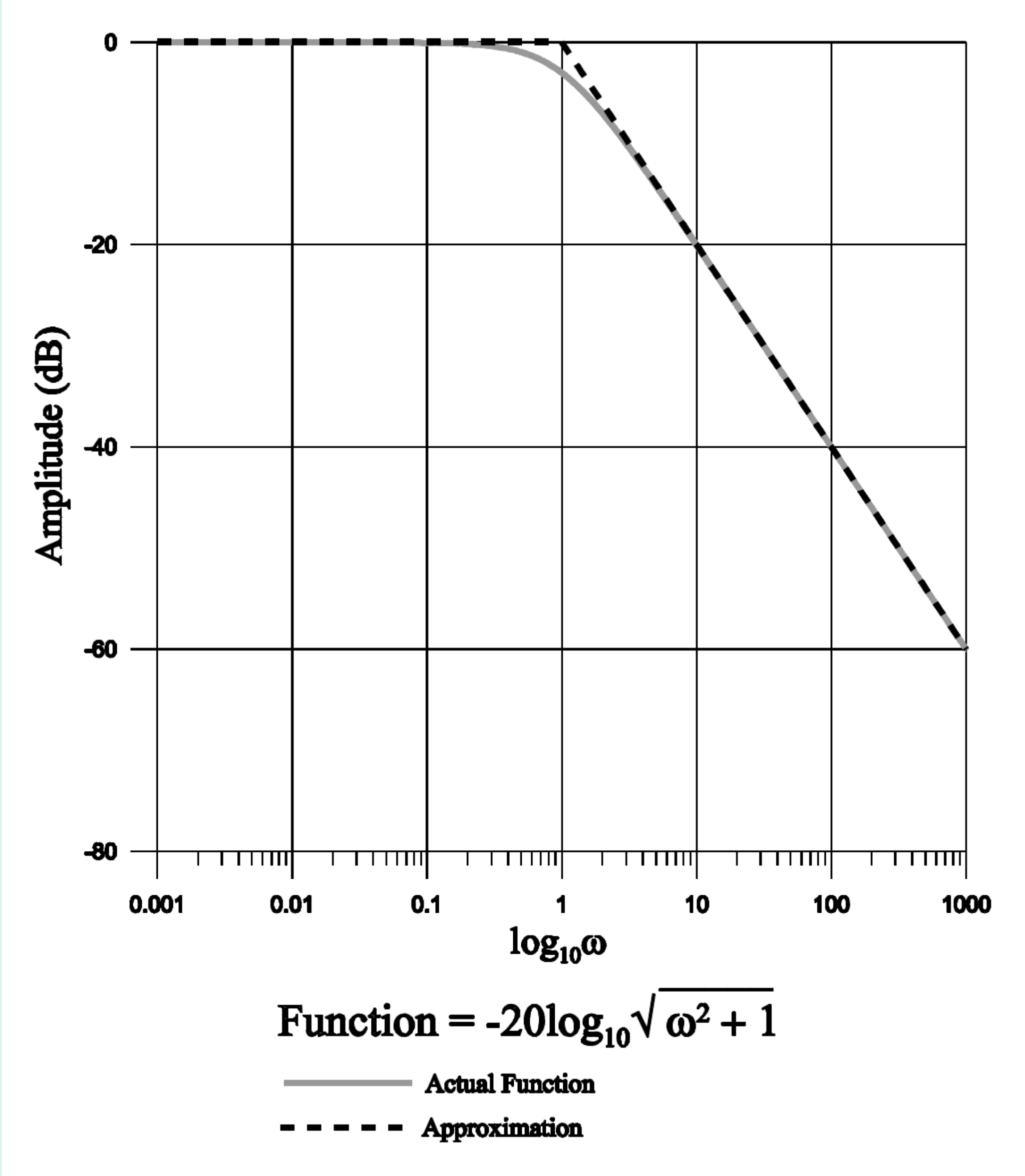
Effects of a single zero on magnitude

- Zero at ω_z
- Global effect felt for $\omega > \omega_z$
- Magnitude ($20\log|H(j\omega)|$) increase rate of +20dB/dec



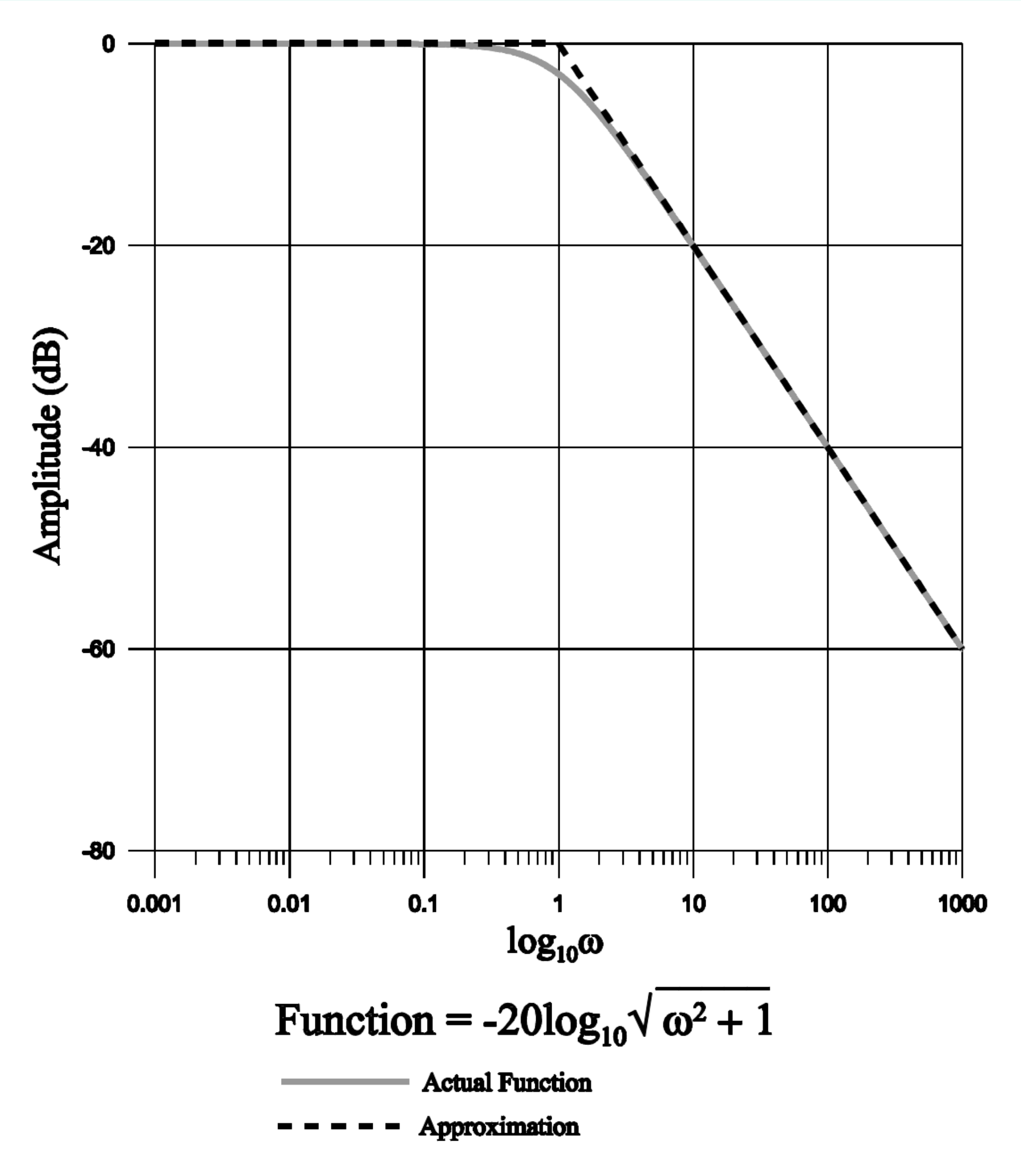
Simple pole p1=-1

- Contribution of a single pole to the magnitude



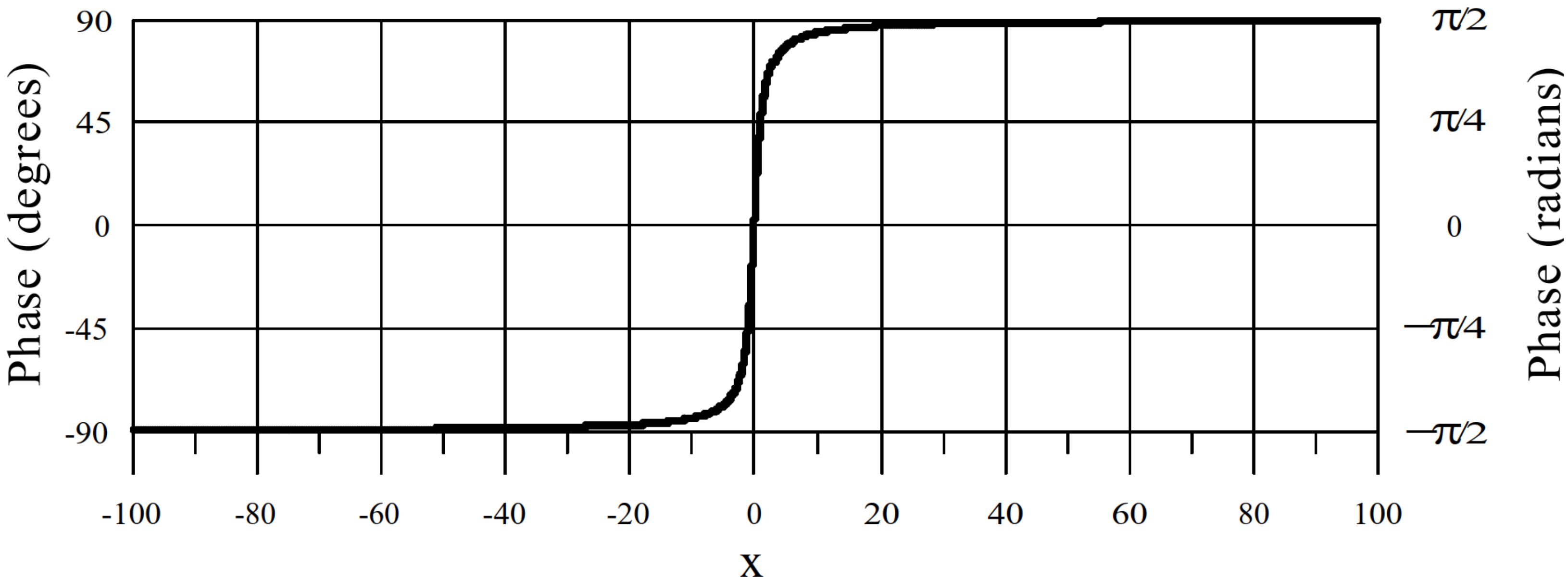
Effects of a single pole on magnitude

- pole at ω_p
- Global effect on magnitude for $\omega > \omega_p$
- Magnitude ($20\log|H(j\omega)|$) decrease rate of -20dB/dec



Phase contributions of single poles/zeros

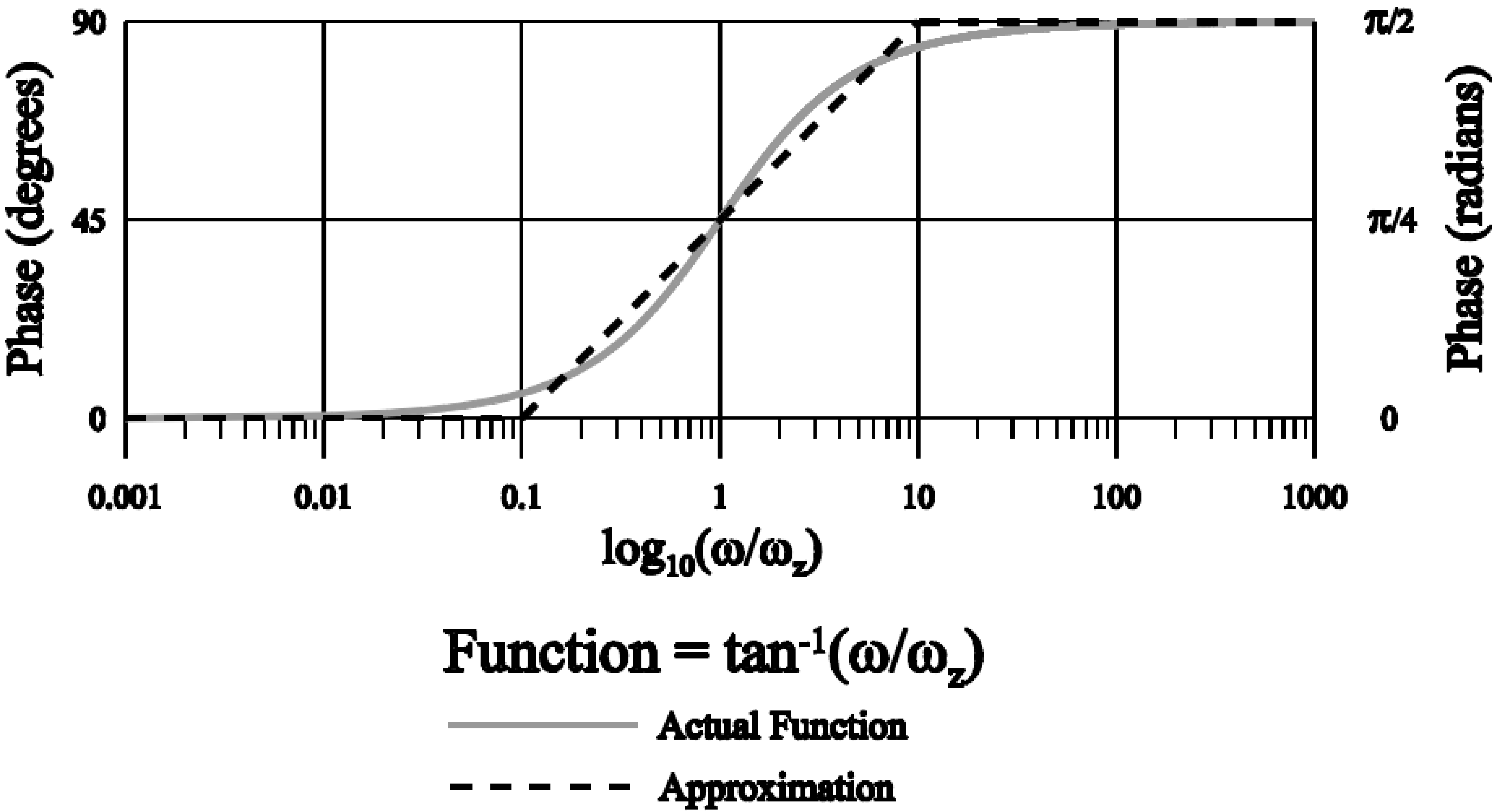
- A zero adds a phase term $+\arctan(\omega/\omega_z)$
- A pole adds a phase term $-\arctan(\omega/\omega_p)$



Function = $\tan^{-1}x$

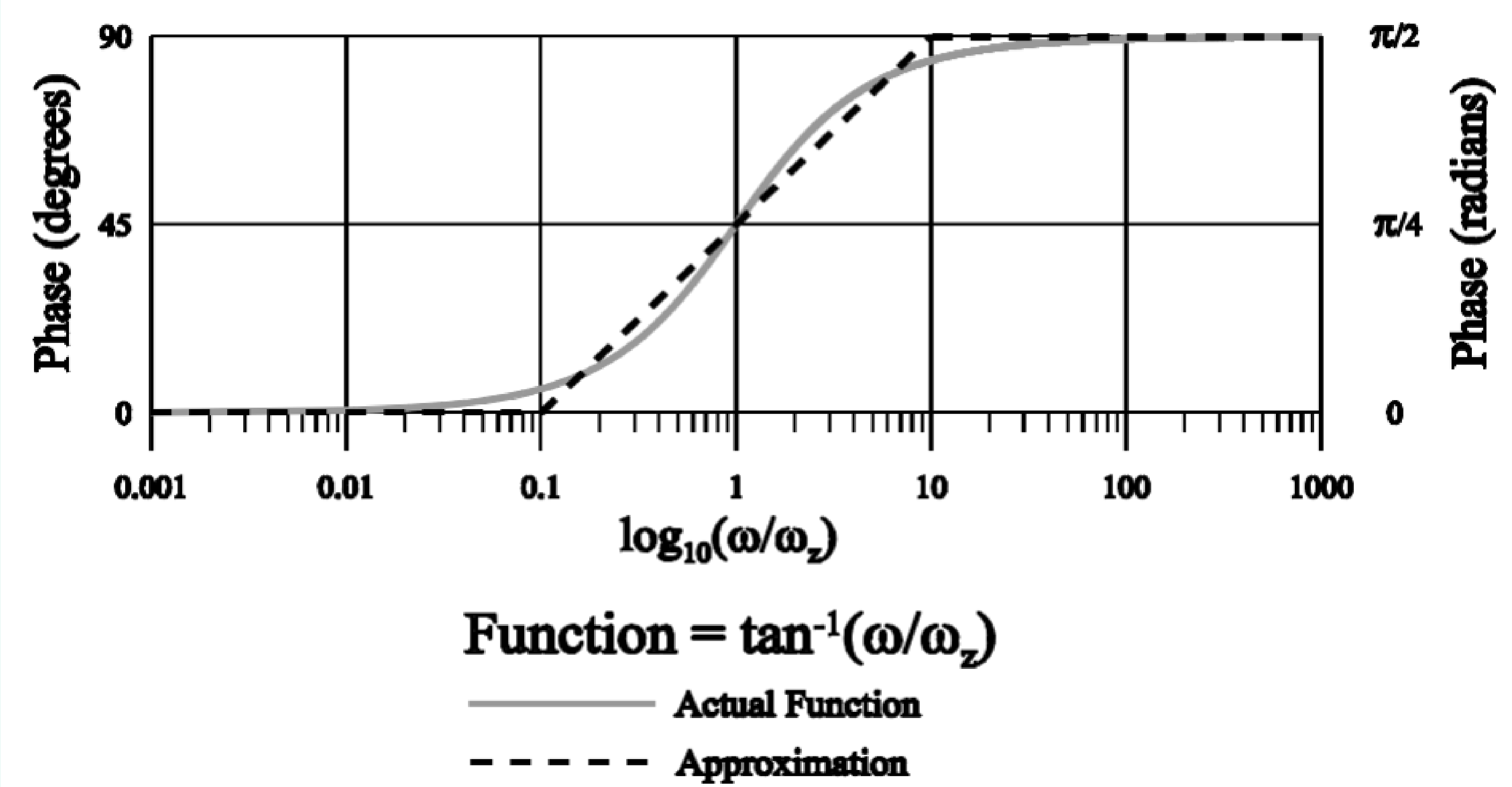
Phase contribution approximation

- Phase contribution of a zero ω_z



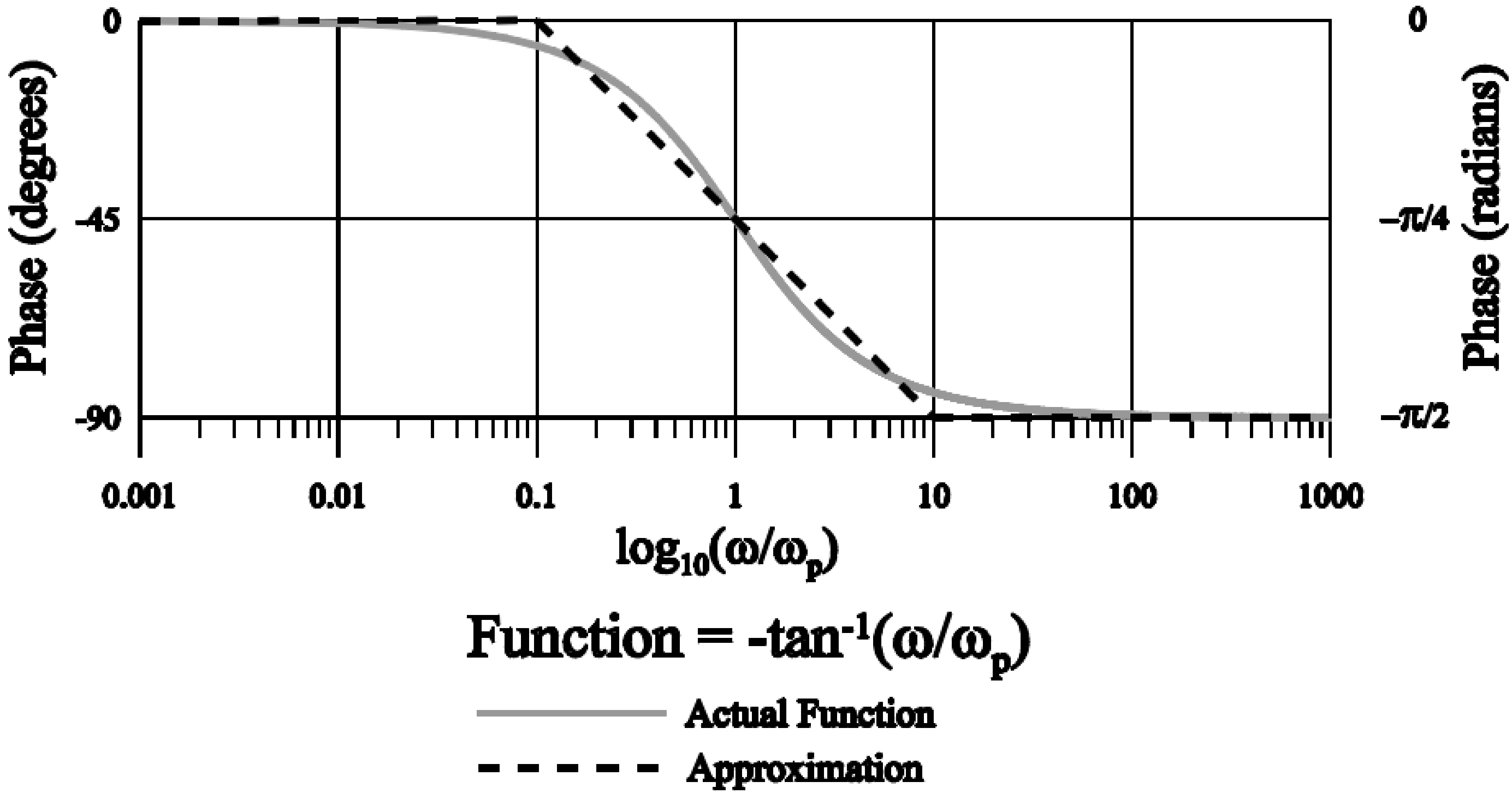
A zero contribution to phase

- Simple zero at ω_z
- Localized effect for $0.1\omega_z < \omega < 10\omega_z$
- Phase increase with +45deg/dec, total change +90deg



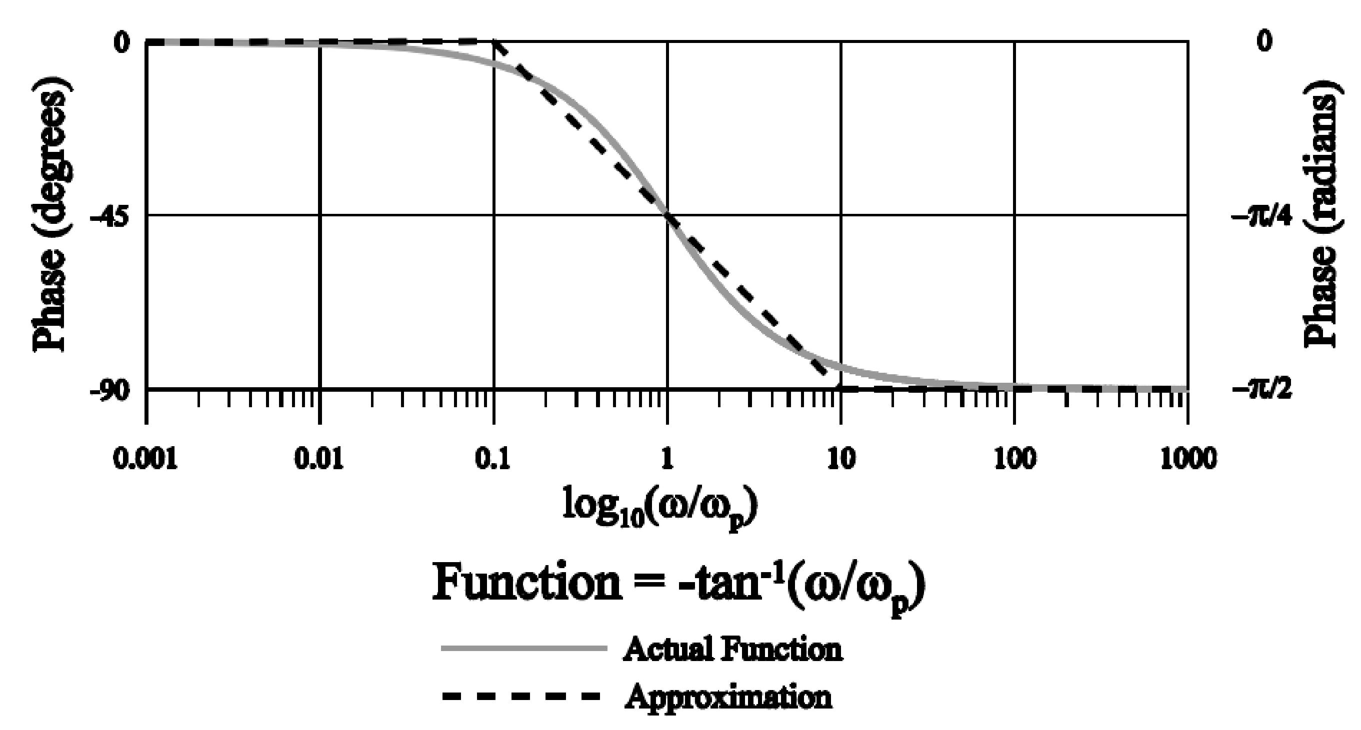
Phase contribution approximation - pole

- Simple pole at ω_p



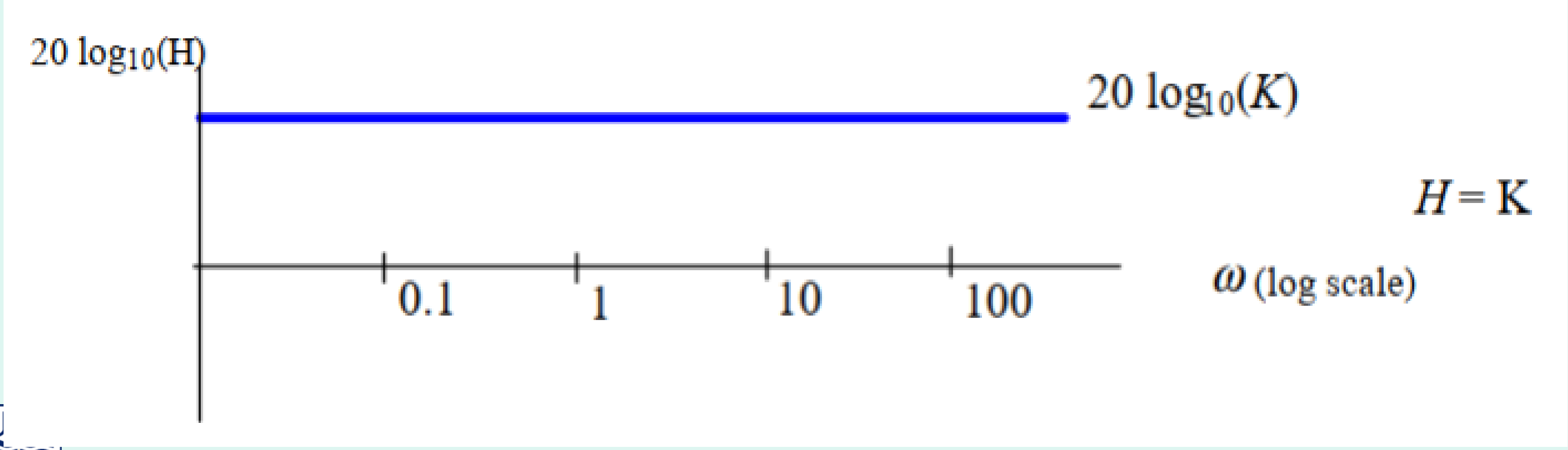
Simple pole contribution to phase

- Simple pole at ω_p
- Localized effect for $0.1\omega_p<\omega<10\omega_p$
- Phase decrease with -45deg/dec, total change -90deg



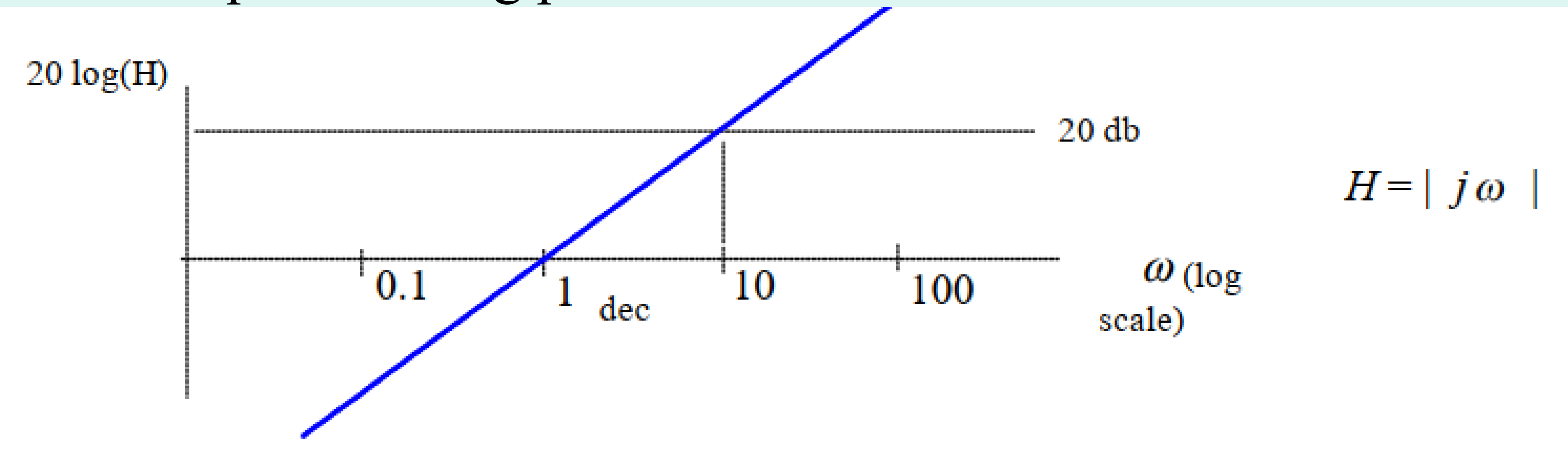
Constant term contribution

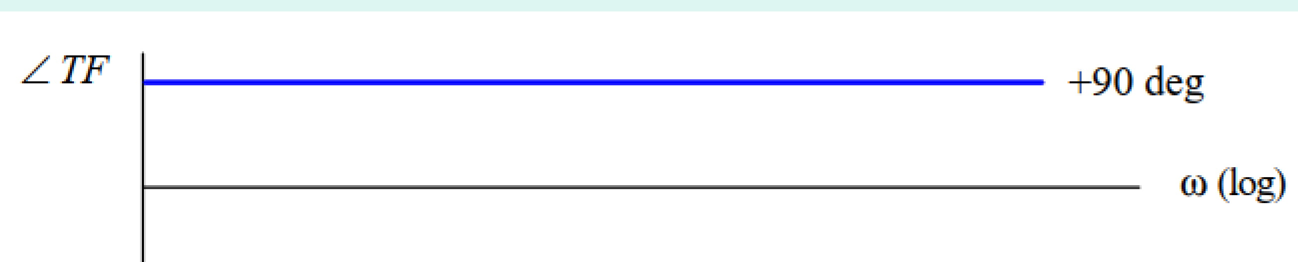
- Magnitude plot: a constant gain K contributes with a straight horizontal line of magnitude $20\log_{10}(K)$
- Phase plot - zero phase contribution (or 180deg contribution for K<0)



Individual zero at the origin

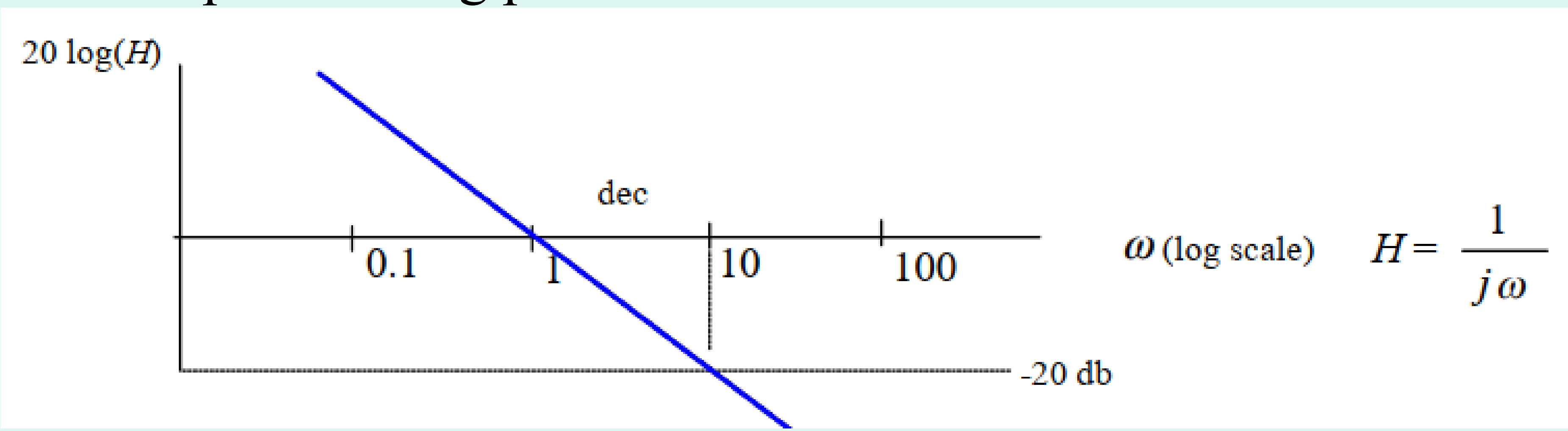
- $H(s)=s \Rightarrow H(j\omega)=j\omega$ (ideal derivator)
- Magnitude plot: positive slope line with +20db/dec, passing through $\omega=1$
- Phase plot: +90deg phase shift for each zero

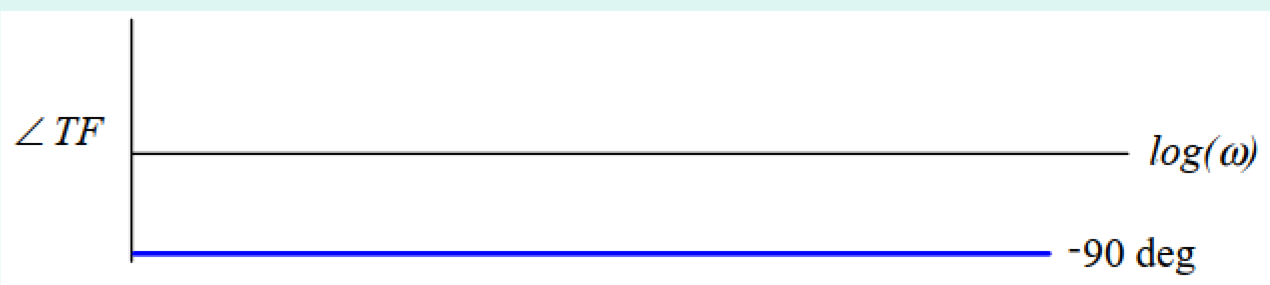




Simple pole at origin

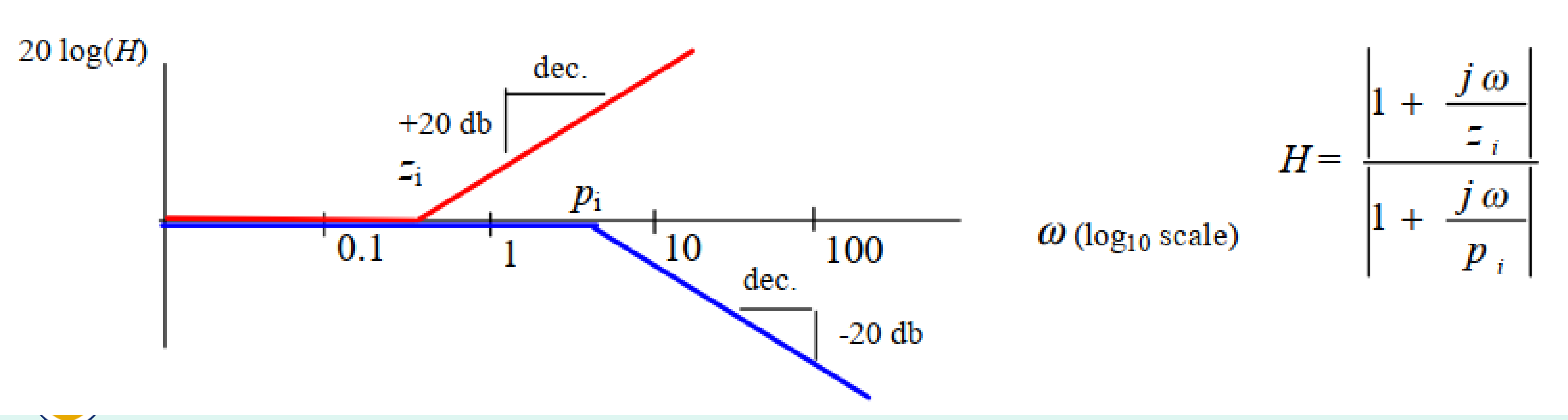
- H(s)=1/s => H(jω)=1/(jω) (ideal integrator)
- Magnitude plot: line passing through ω=1 with a drop of 20dB/dec
- Phase plot: -90deg phase shift





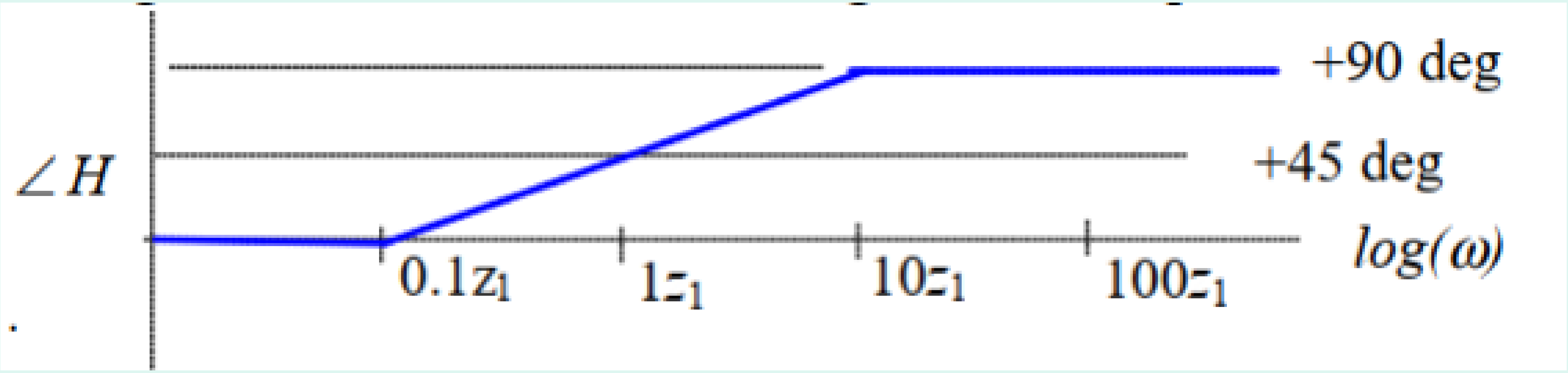
Simple zeros and poles not at origin

- Zero: $H(j\omega)=(1+j\omega/z_i)$
- Pole: $H(j\omega)=1/(1+j\omega/p_i)$
- Magnitude plot: no contribution below the critical frequency (break frequency). Above the critical frequency, they add a ramp unction of +20db/dec for a zero, and -20dB/dec for a pole
- Phase plot: a zero will introduce a +90 phase shift within two decades, and a pole a phase shift of -90deg within two decades



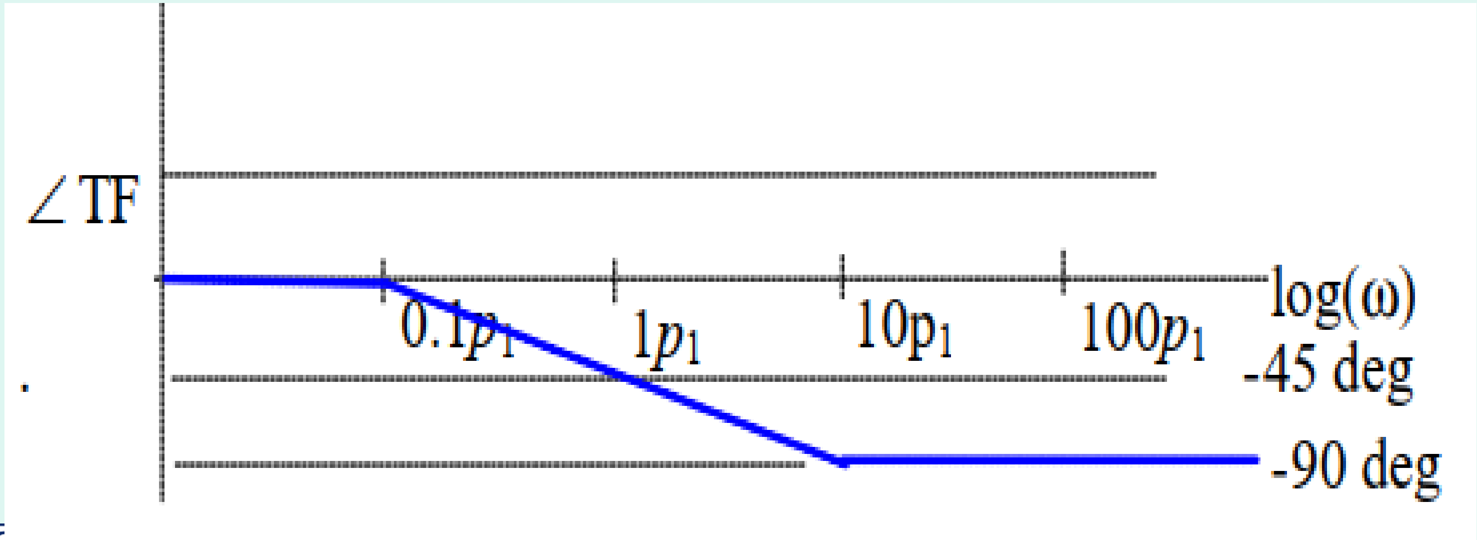
Phase plot - Zeros not at the origin

- terms of the form $(1+j\omega/z_1)$ - no phase shift for $\omega<0.1z_1$, +45deg shift at z_1 and +90deg shift for $\omega>10z_1$



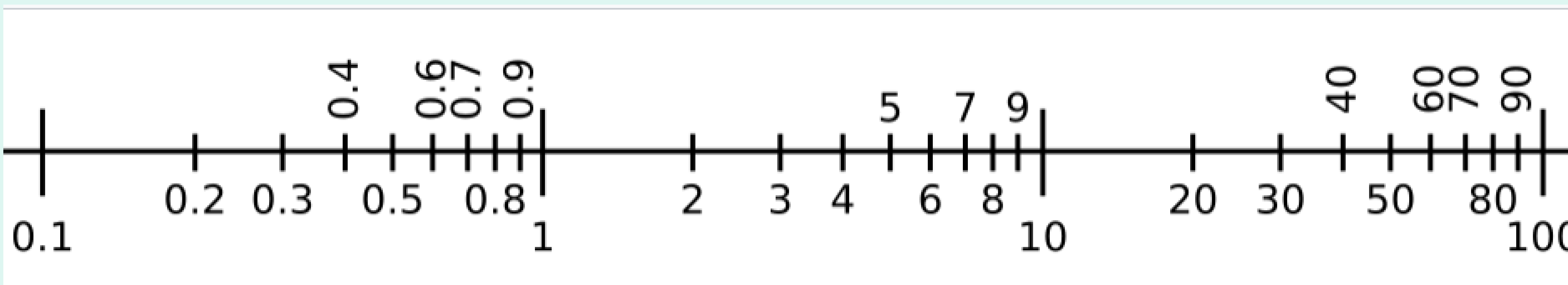
Phase plot - poles not at the origin

- Terms of the form $1/(1+j\omega/p_1)$
- No phase shift for $\omega<0.1p_1$, -45deg for $\omega=p_1$, and a -90deg shift for $\omega>10p_1$



Logarithmic scale

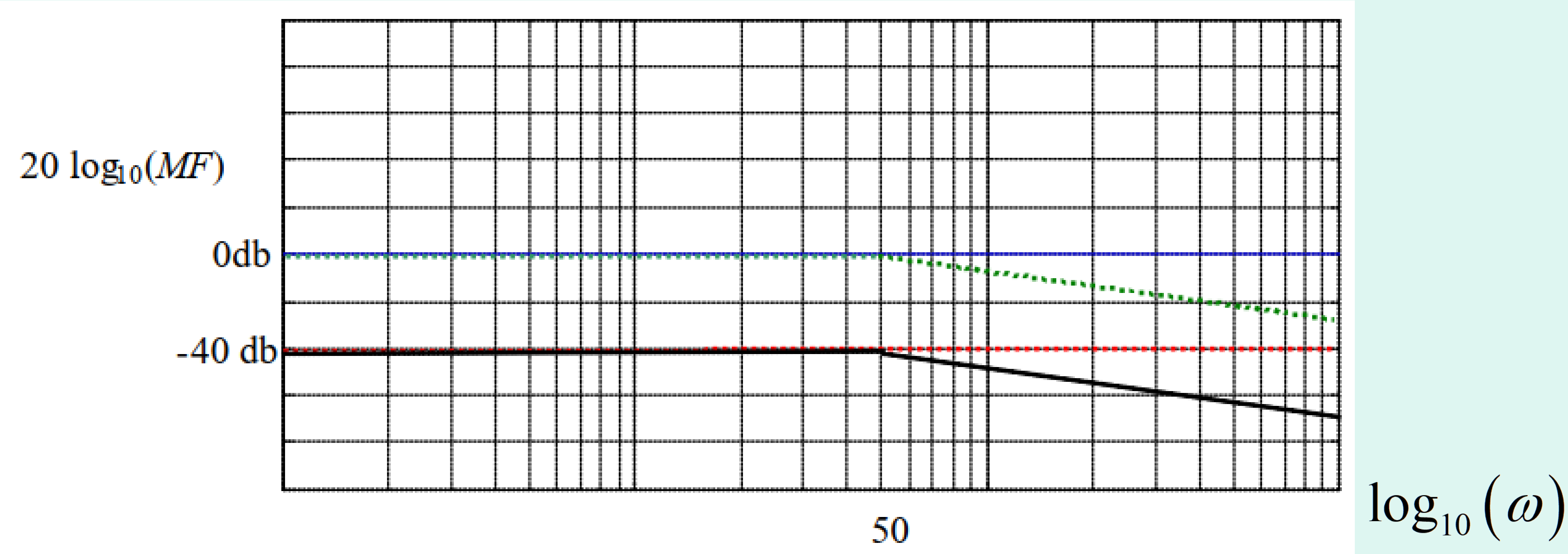
- When drawing by hand - 1,2,5,10 are almost equidistant: $\log(2) \approx 0.3$, $\log(3)=0.477$, $\log(5) \approx 0.7$



Exm 1- simple low-pass filter

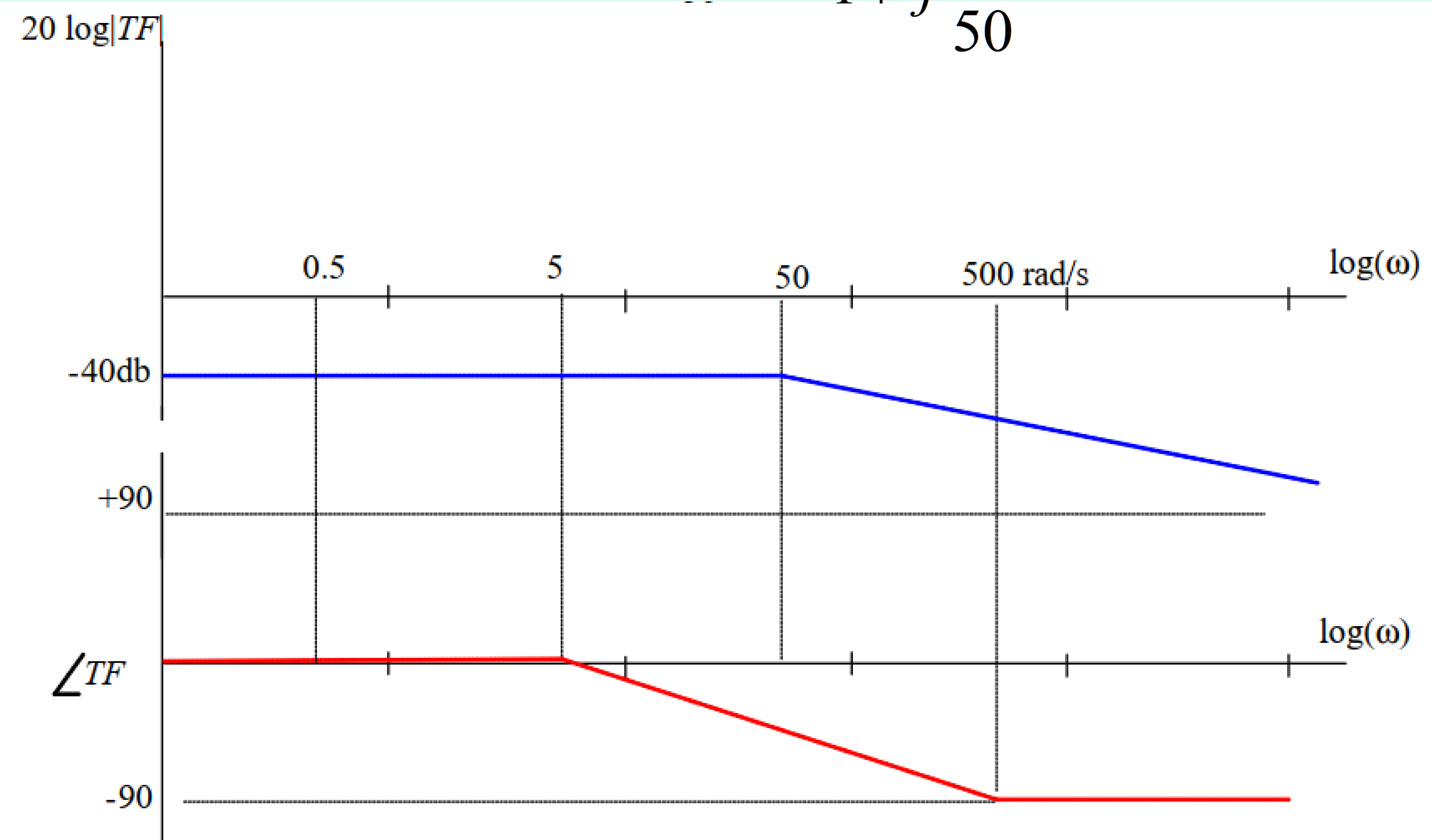
$$H(s)=\frac{1}{2s+100}=\frac{1}{100}\frac{1}{1+\frac{s}{50}}\Rightarrow H(j\omega)=\underbrace{\frac{1}{100}}_{H_{DC}}\frac{1}{1+j\frac{\omega}{50}}$$

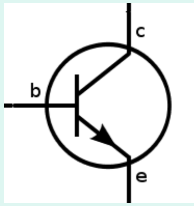
- Low-pass filter
- H_{DC}=0.01 => $20\log_{10}(0.01)$=-40dB
- Pole with critical frequency p_1=50rad/s



Bode - magnitude and phase plots

$$H(j\omega) = 0.01\frac{1}{1+j\dfrac{\omega}{50}}$$





Exm 2

- Second order system

$$H(s)=\frac{5\cdot 10^4 s}{s^2+505s+2500}=\frac{5\cdot 10^4 s}{(s+5)(s+500)}=\frac{5\cdot 10^4}{5\cdot 500}\frac{s}{\left(1+\frac{s}{5}\right)\left(1+\frac{s}{500}\right)}$$

$$H(j\omega)=20\frac{j\omega}{\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{500}\right)}$$

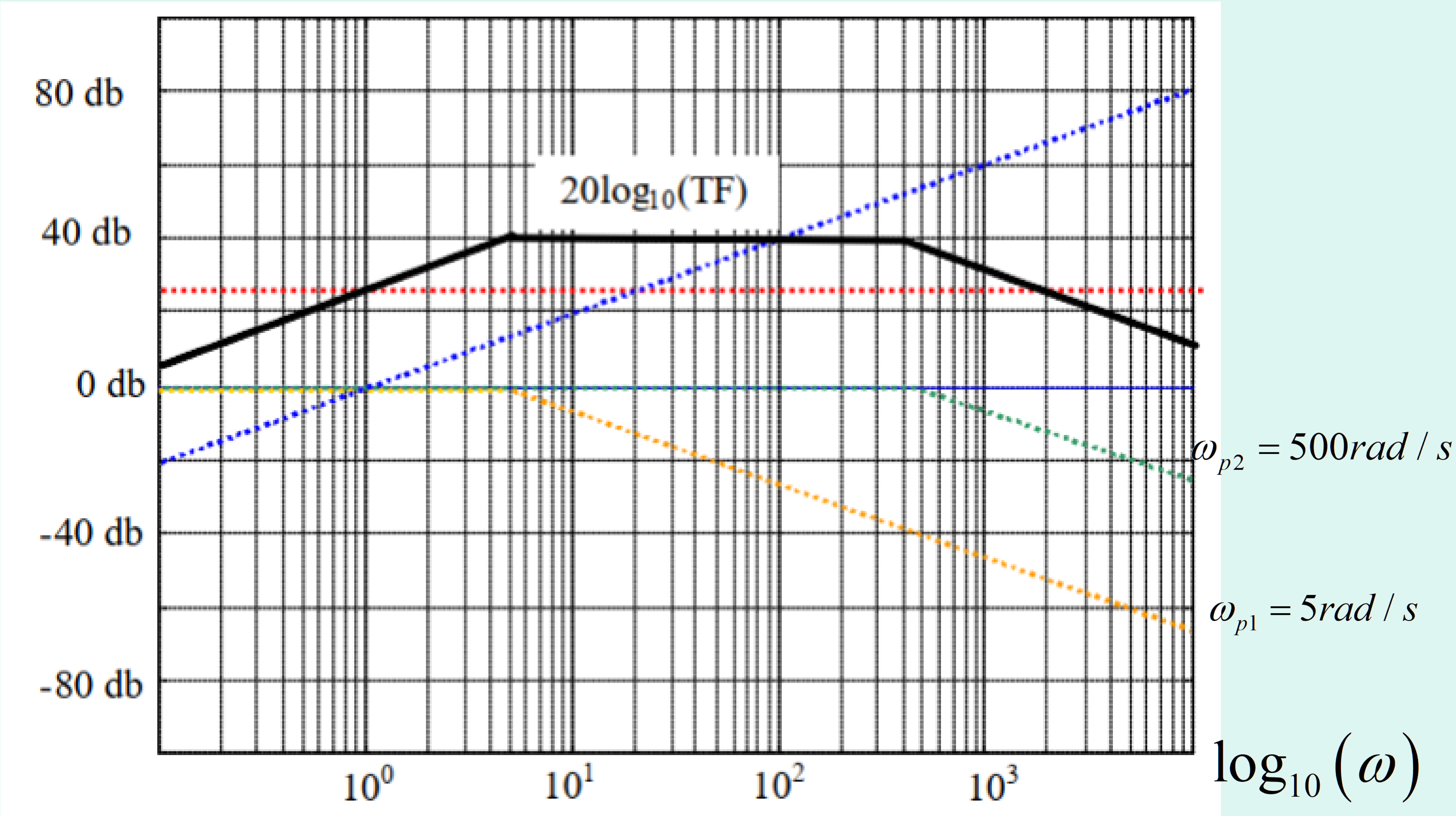
$$20\log\left(\left|H(j\omega)\right|\right)=\underbrace{20\log(20)}_{26.02}+20\log(\omega)-20\log\left(\sqrt{1+\left(\frac{\omega}{5}\right)^2}\right)-20\log\left(\sqrt{1+\left(\frac{\omega}{500}\right)^2}\right)$$

$$\varphi(\omega)=0+90^0-\tan^{-1}\left(\frac{\omega}{5}\right)-\tan^{-1}\left(\frac{\omega}{500}\right)$$

Steps

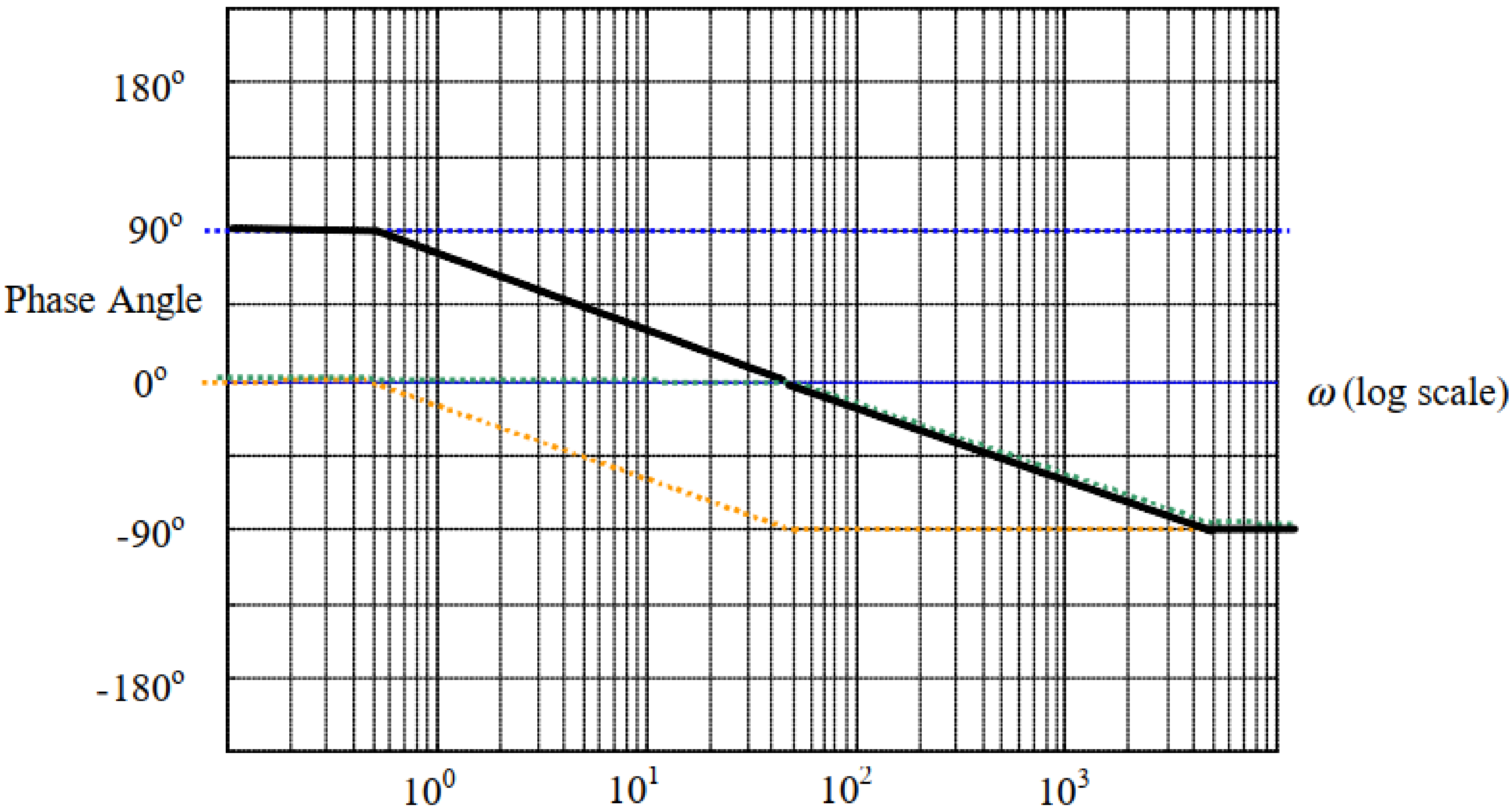
1. Draw the segments for each individual term on the graph
2. Start from the origin ($\omega << 1$)
3. Add the constant offset of the gain as starting line
4. Add the effects of the poles/zeros working from left to right along the $\log(\omega)$ axis

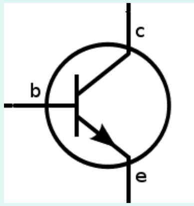
Exm2 - log-magnitude plot



Phase plot

$$H(j\omega) = 20\frac{j\omega}{\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{500}\right)}$$





Exm 3

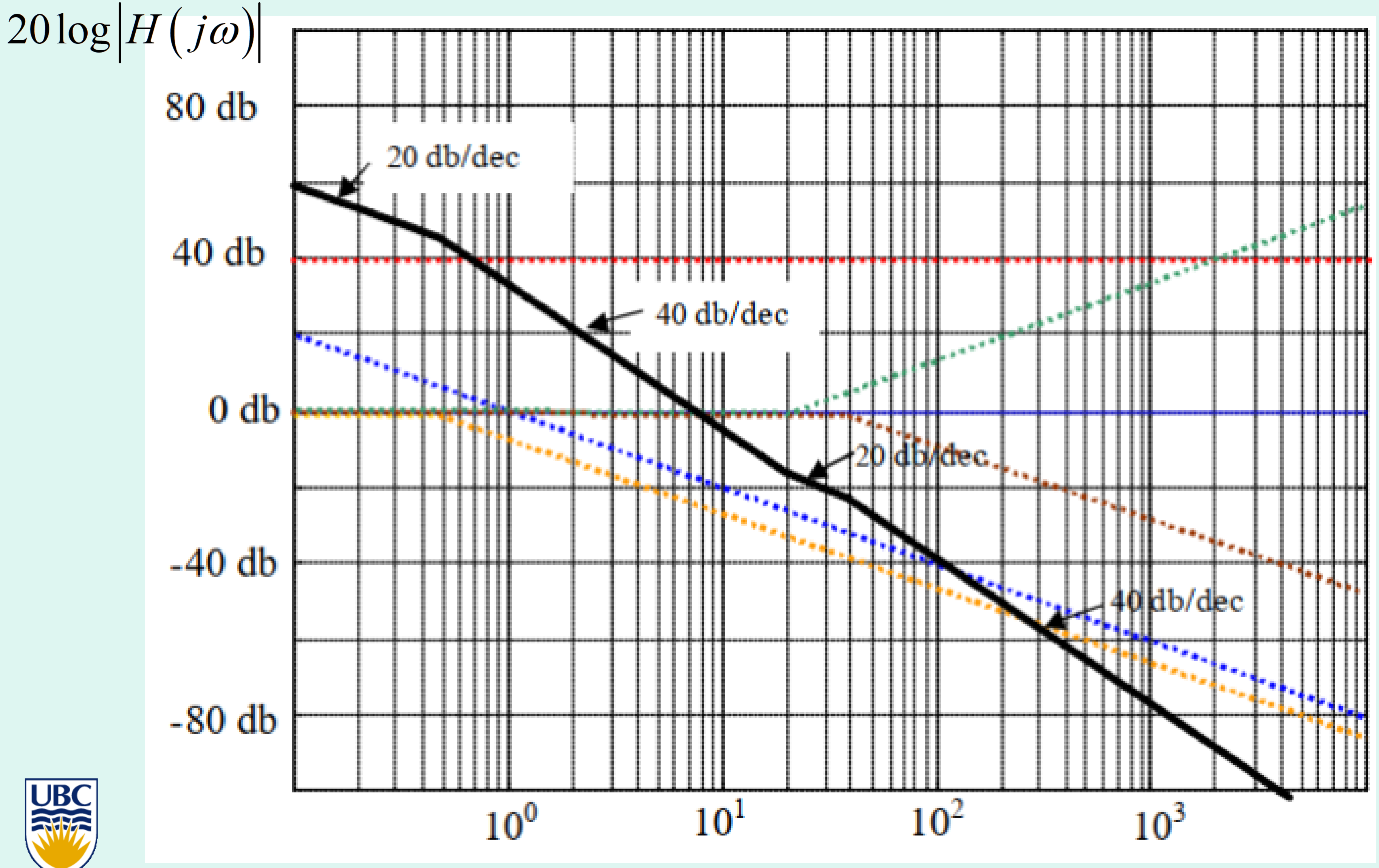
$$H(s) = \frac{200(s+20)}{s(2s+1)(s+40)} = \frac{200 \cdot 20}{40} \frac{1+\frac{s}{20}}{s\left(1+\frac{s}{0.5}\right)\left(1+\frac{s}{40}\right)}$$

$$H(j\omega) = 100 \frac{1+j\frac{\omega}{20}}{j\omega\left(1+j\frac{\omega}{0.5}\right)\left(1+j\frac{\omega}{40}\right)}$$

$$20\log\left|H(j\omega)\right| = 40dB + 20\log\left(\sqrt{1+\left(\frac{\omega}{20}\right)^2}\right) - 20\log(\omega) - 20\log\left(\sqrt{1+\left(\frac{\omega}{0.5}\right)^2}\right) - 20\log\left(\sqrt{1+\left(\frac{\omega}{40}\right)^2}\right)$$

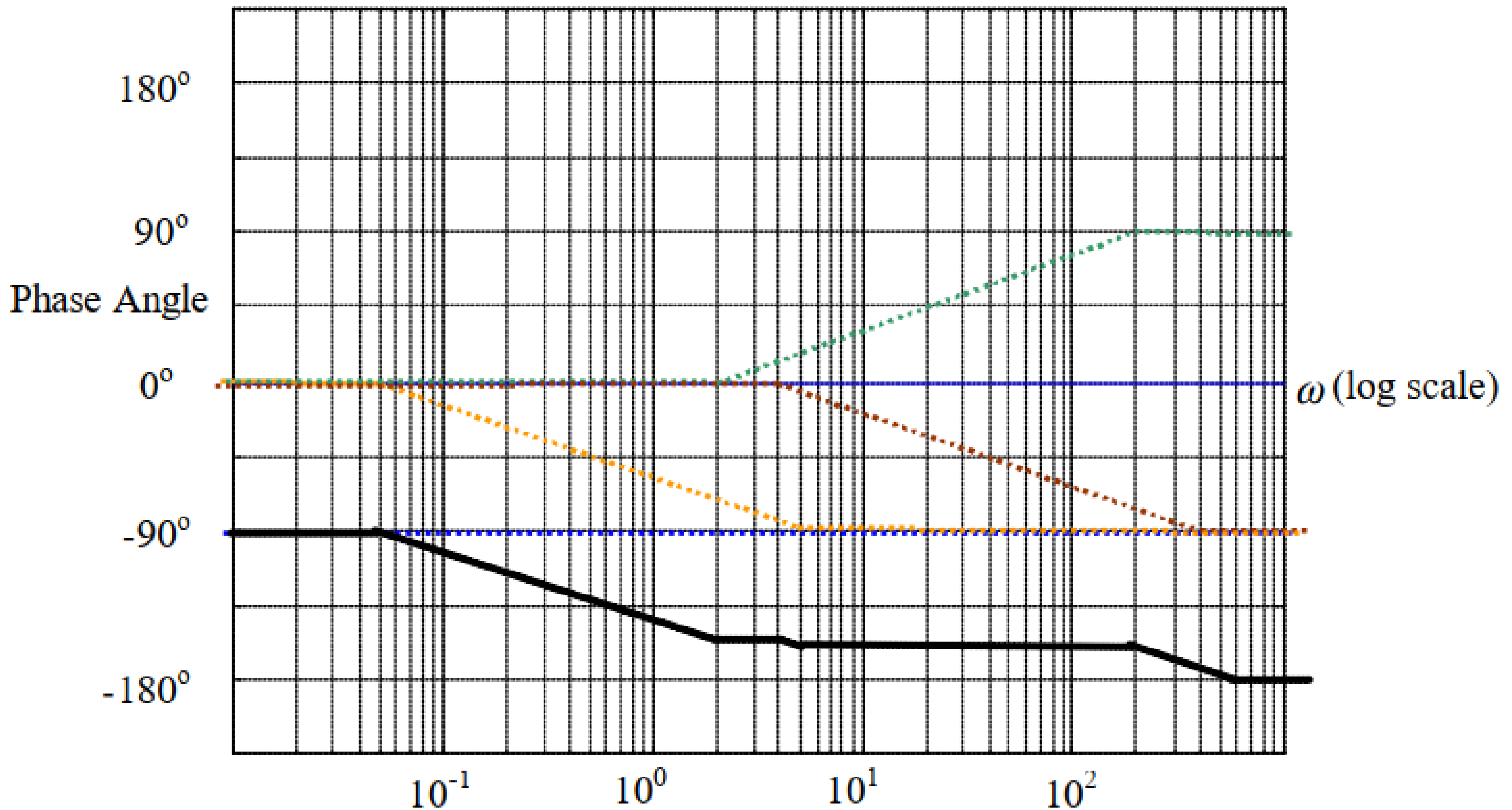
$$\varphi(\omega) = 0^0 + \tan^{-1}\left(\frac{\omega}{20}\right) - 90^0 - \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}\left(\frac{\omega}{40}\right)$$

Exm 3 - Bode magnitude plot



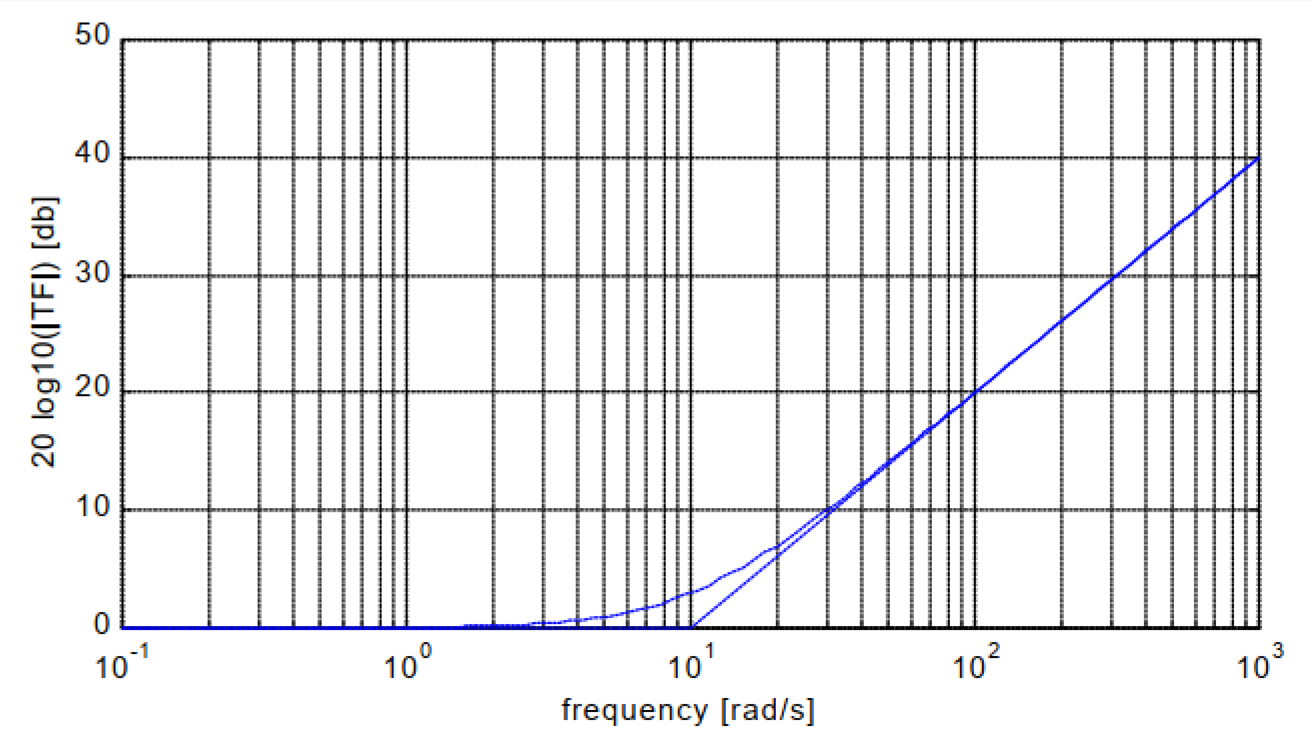
Exm 3 - Bode phase plot

$$H(j\omega) = 100\frac{1 + j\frac{\omega}{20}}{j\omega\left(1 + j\frac{\omega}{0.5}\right)\left(1 + j\frac{\omega}{40}\right)}$$



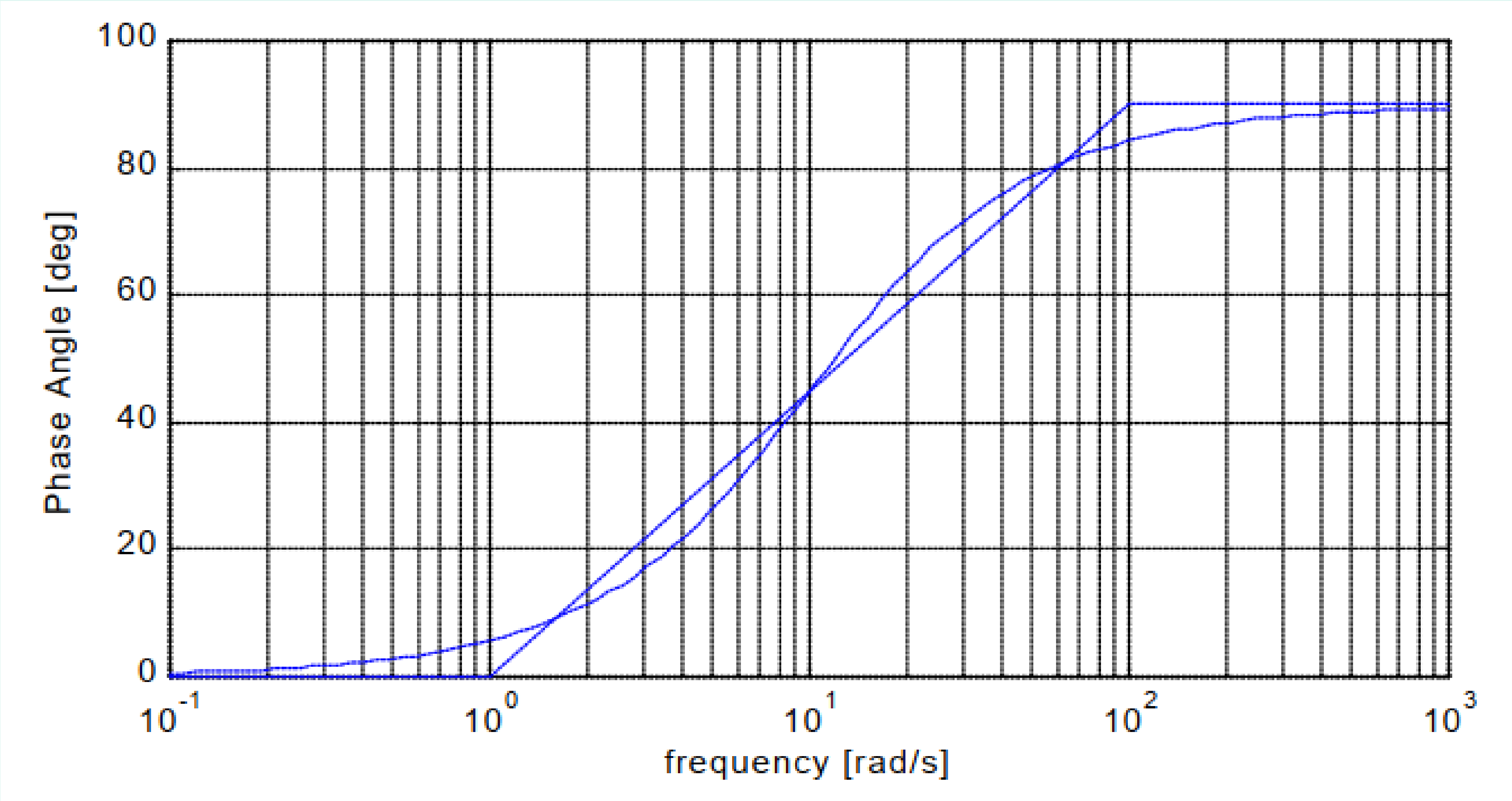
Approximation errors

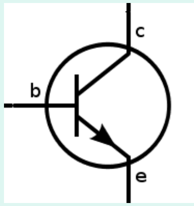
- Bode techniques are visual asymptotic approximations of the real magnitude and phase plots => there are approximation errors
- The largest errors for the magnitude plots - at the critical frequency (~3dB for simple zeros/poles)



Phase plot approximation errors

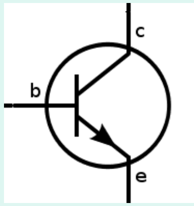
- The largest errors for the phase plots occur at $0.1\omega_{critical}$ and $10\omega_{critical}$ (~6deg for simple pole/zero)





Higher order poles/zeros

- The asymptotic trend: Nth order pole $1/(1+j\omega/p_1)^N$ will decrease the log-magnitude with -N*20dB/dec, and cause a phase shift of -N*90deg
- The max errors at the critical frequency will increase for higher order poles/zeros, but the asymptotic convergence remains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