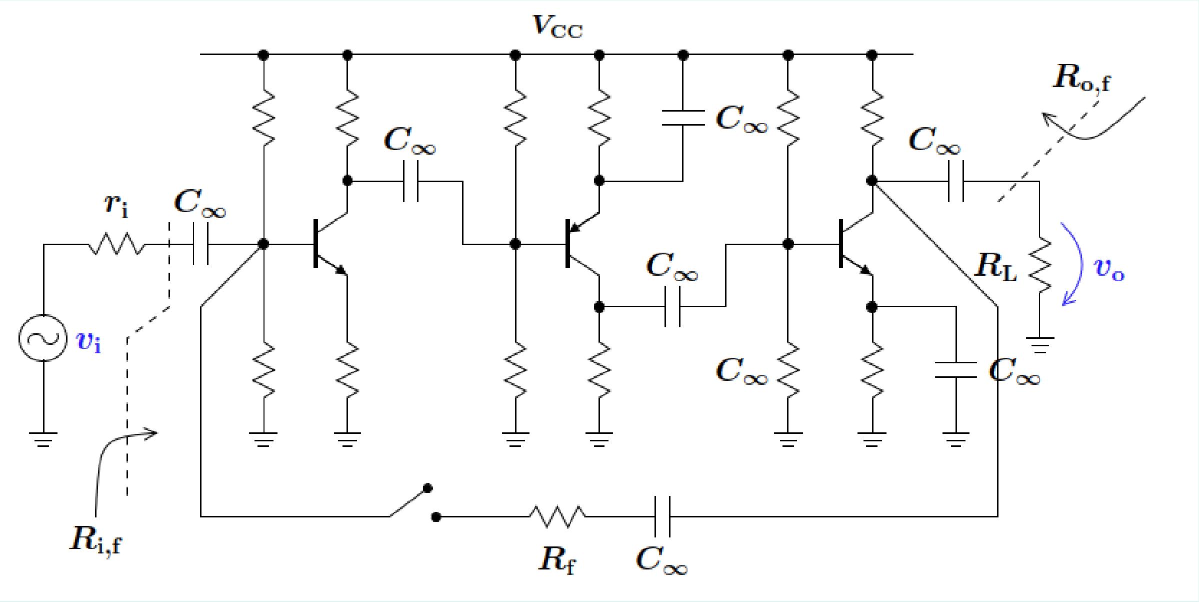
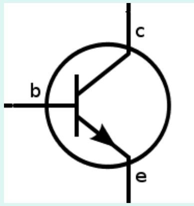
ELEC 301 - Band-pass circuits

L08 - Sep 22

Instructor: Edmond Cretu





Last time

- Bode plots - log magnitude and phase plots
- Examples
- Approximation errors

L08 Q01 - Bode plots for a 2nd order system

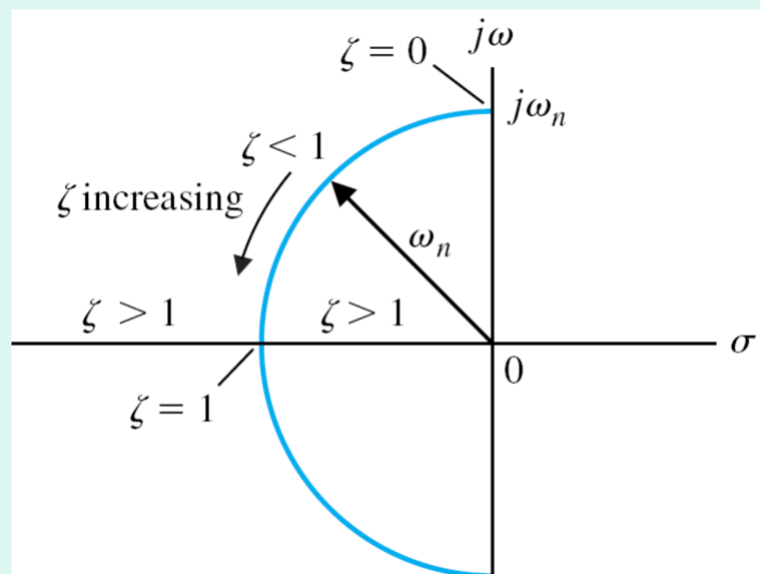
- Given a second order system, in which case the Bode technique provides a better approximation of the real frequency response?

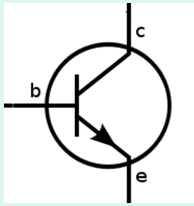
A. quality factor Q=1000

B. quality factor Q=10

C. Quality factor Q=0.5

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} X(s)$$



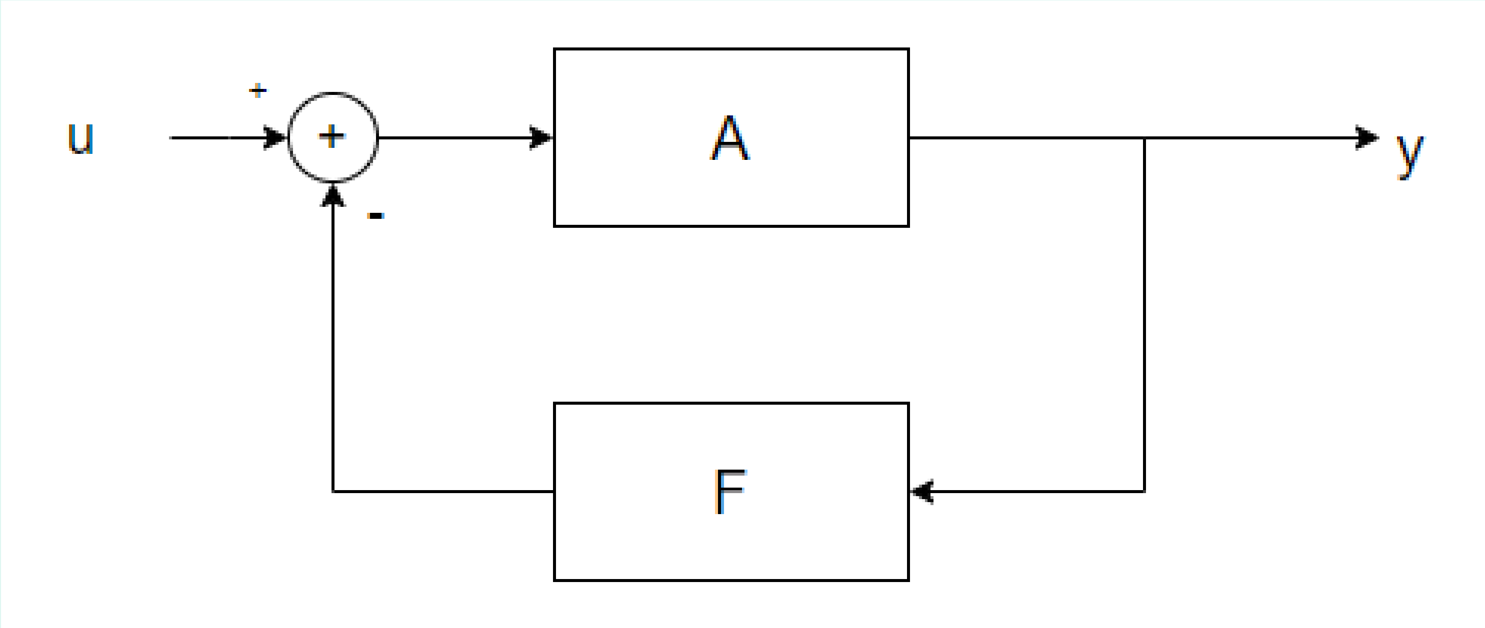


Conclusions

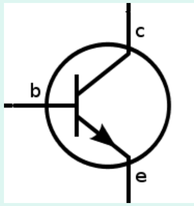
- Bode plots is a fast visual approximation technique for the frequency response of a linear system
- It provides a good approximation for systems with distinct individual poles on the real axis
- The typical cases for electronic amplifiers correspond to such a situation

Simple LP active amplifier

- Simple low-pass amplifier stage with passive feedback network



$$A(j\omega) = \frac{A_0}{1 + j\dfrac{\omega}{\omega_H}}, \quad F = F_0 = \frac{R_1}{R_1 + R_2} < 1$$



L08 Q02 LP amplifier

- Which statement is **false** for the previous configuration?

A. The closed loop gain decreases because of the negative feedback

B. The critical frequency of the closed loop transfer function decreases as result of the negative feedback action

C. The critical frequency of the closed loop transfer function increases, compared to the open loop case

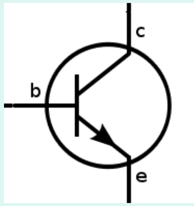
D. The closed loop system is less sensitive to errors in the value of A_0

Negative feedback action

- The critical frequency ω_H is increased by the loop gain

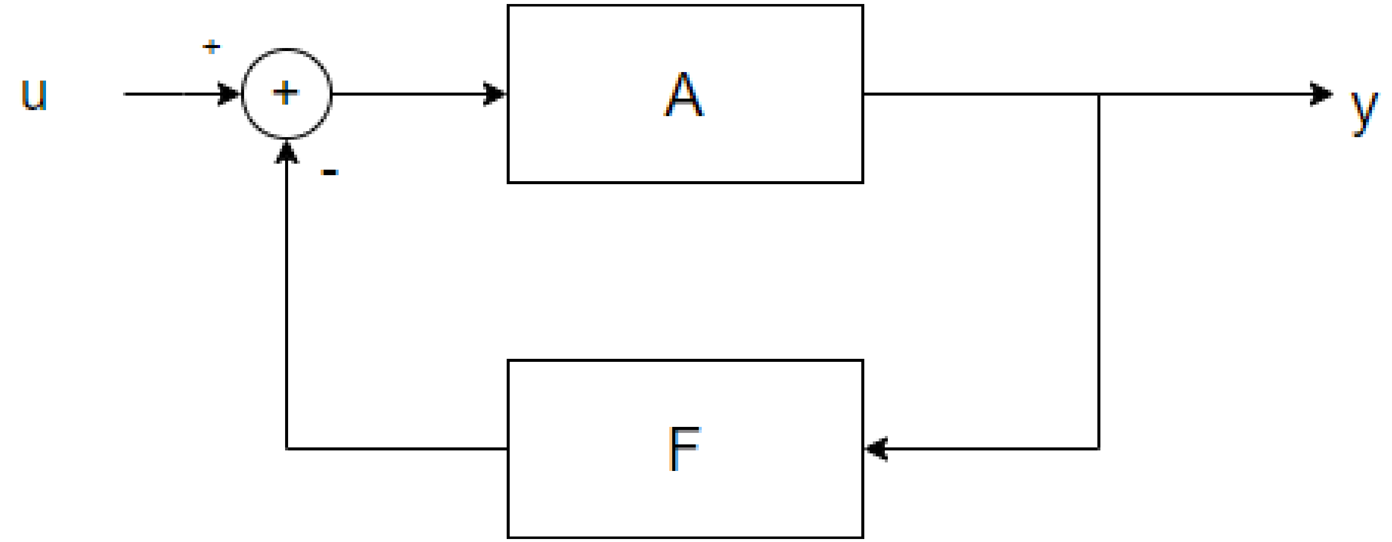
$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)F(j\omega)} = \frac{\dfrac{A_0}{1 + j\dfrac{\omega}{\omega_H}}}{1 + \dfrac{A_0}{1 + j\dfrac{\omega}{\omega_H}} F_0} = \frac{A_0}{1 + j\dfrac{\omega}{\omega_H} + A_0 F_0}$$

$$A_f(j\omega) = \frac{A_0}{1 + A_0 F_0} \frac{1}{1 + j\dfrac{\omega}{\omega_H(1 + A_0 F_0)}} \overset{A_0 F_0 >> 1}{\approx} \frac{1}{F_0} \frac{1}{1 + j\dfrac{\omega}{\omega_H(1 + A_0 F_0)}}$$

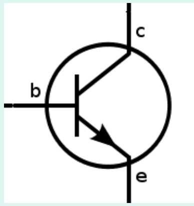


High-pass amplifier case

- Similar active network, but with a high-pass amplifier characteristics



$$A(j\omega) = \frac{A_0 j \dfrac{\omega}{\omega_L}}{1 + j\dfrac{\omega}{\omega_L}}, \quad F = F_0 = \frac{R_1}{R_1 + R_2} < 1$$



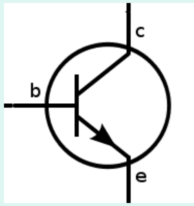
L08 Q03 HP amplifier with feedback

- What is the effect of the negative feedback on the critical frequency ω_L?

A. It is not changed

B. It is increased

C. It is decreased

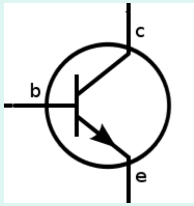


HP active amplifier with feedback

- The critical frequency is lowered by the loop gain

$$A(j\omega) = \frac{A_0 j \dfrac{\omega}{\omega_L}}{1 + j\dfrac{\omega}{\omega_L}}, \quad F = F_0 = \frac{R_1}{R_1 + R_2} < 1$$

$${}_c(j\omega) = \frac{A_0 j \dfrac{\omega}{\omega_L}}{A_0 F_0 j \dfrac{\omega}{\omega_L} + 1 + j\dfrac{\omega}{\omega_L}} = \frac{A_0}{1 + A_0 F_0} \frac{\dfrac{j\dfrac{\omega}{\omega_L}}{1 + A_0 F_0}}{1 + j\dfrac{\dfrac{\omega}{\omega_L}}{(1 + A_0 F_0)}}$$

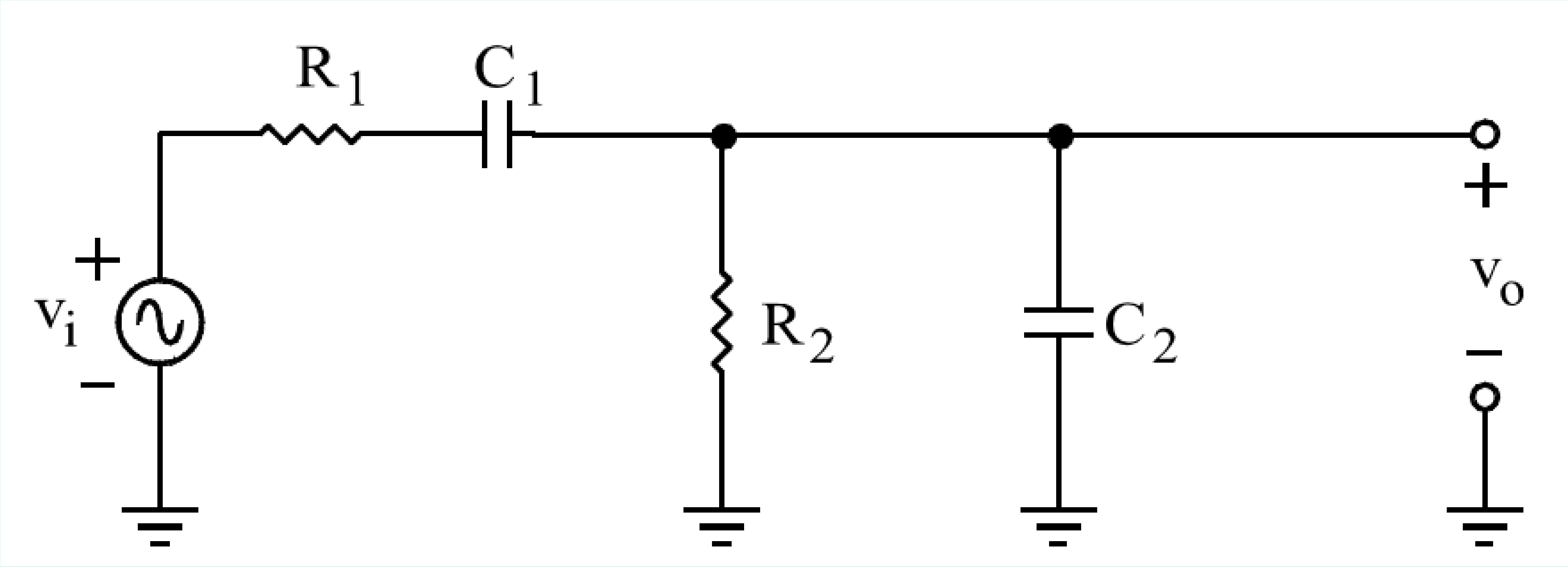


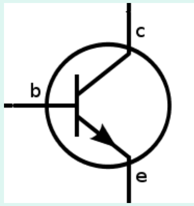
Open-circuit/short-circuit time constant method

- Most circuits we operate with are bandpass, with capacitors as the only reactive components
- We desire a fast method to approximate the critical frequencies ω_L, ω_H

Example - simple bandpass filter

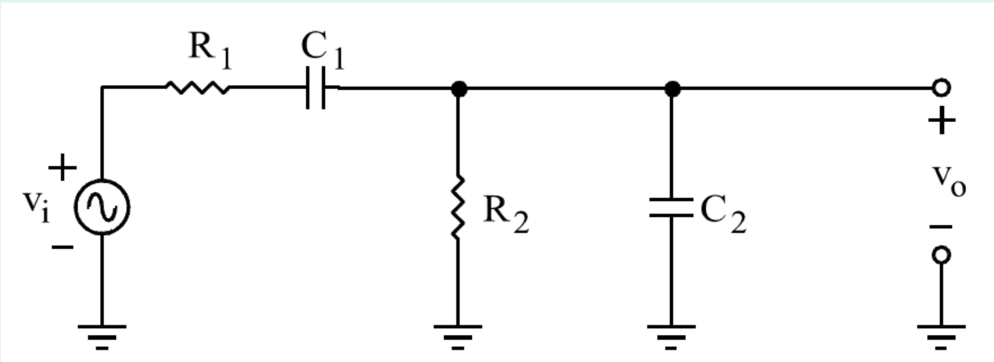
- C1 blocks the LF components, C2 attenuates the HF components
- To be a bandpass filter C1>>C2

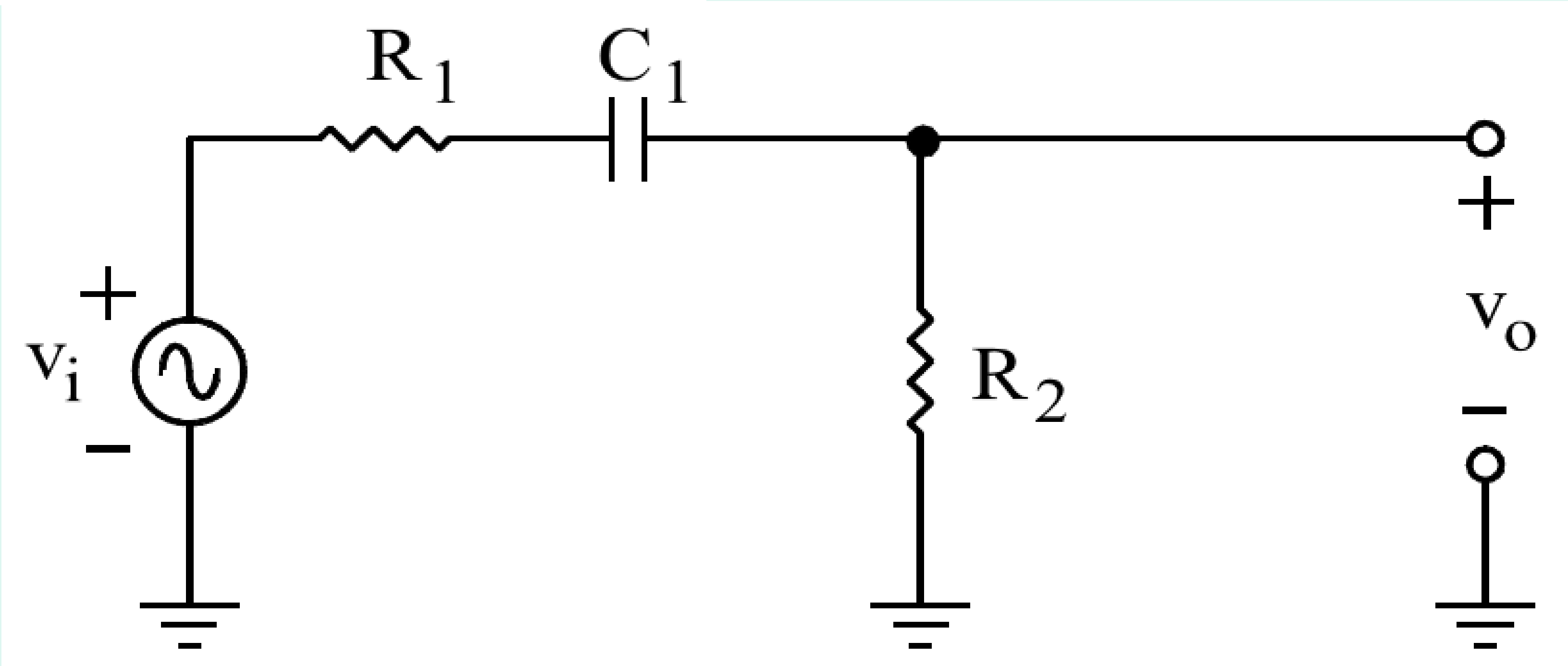




Circuit at LF

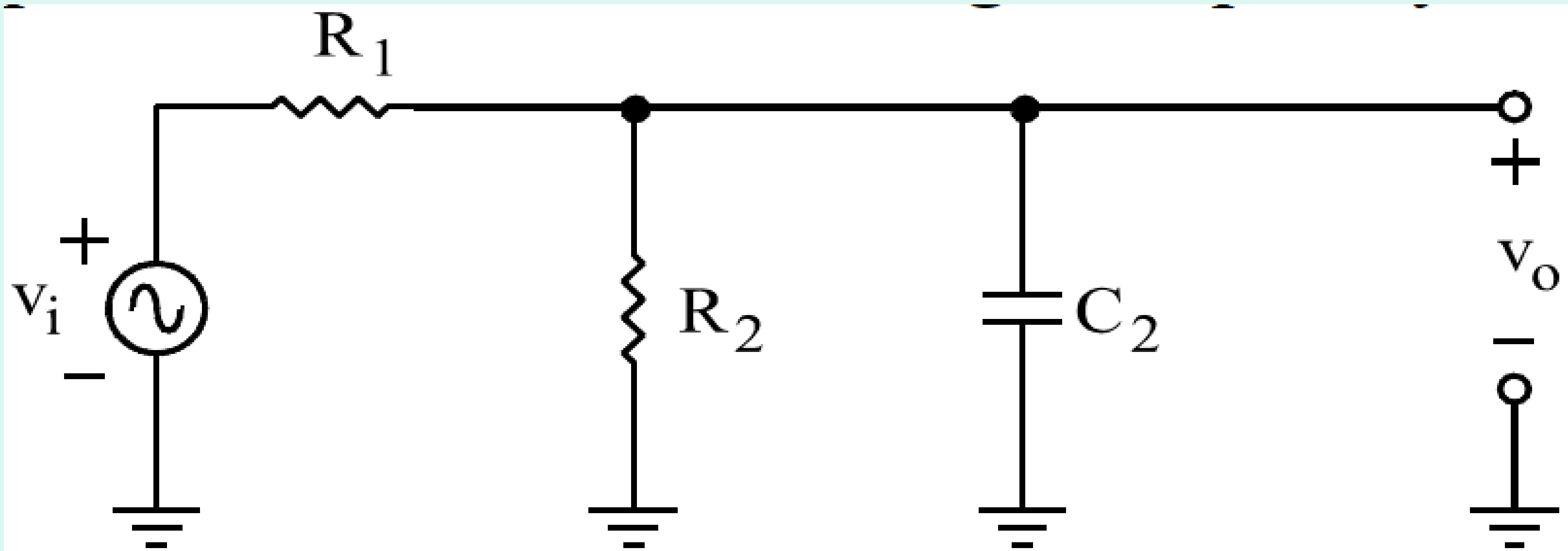
- Assume C1>>C2 - circuit behaves as high-pass filter for low frequency range





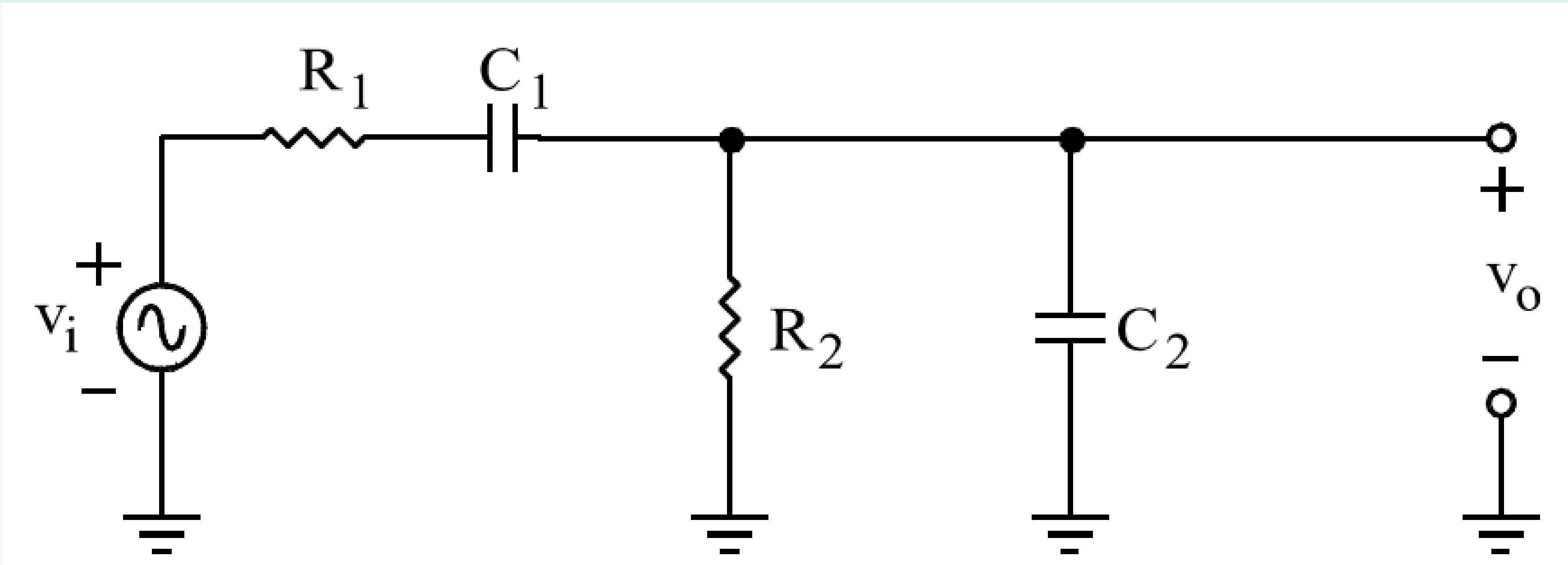
Circuit at HF

- Assume C1>>C2 - circuit behaves as low-pass filter for high frequency range



Transfer function

- Homework!

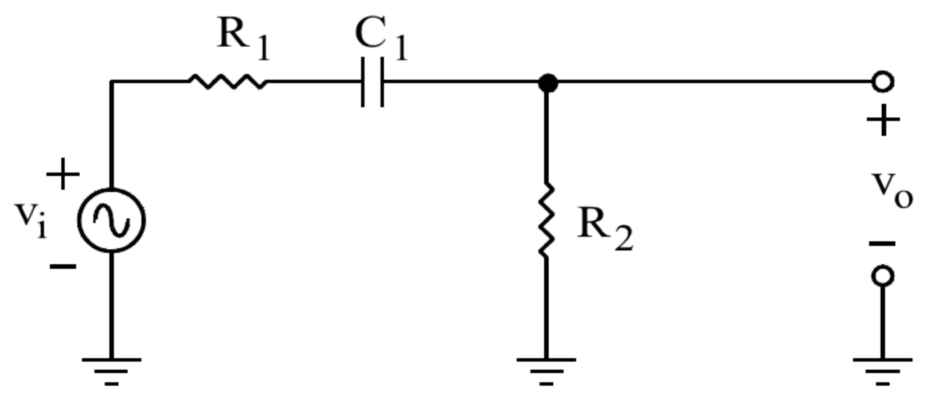


$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{s}{R_1 C_2}}{s^2 + \left[\dfrac{1}{R_2 C_2} + \dfrac{1}{R_1 C_2} + \dfrac{1}{R_1 C_1}\right] s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

Approximate LF pole location

- Recall the circuit approximations for the LF and HF cases

LF regime

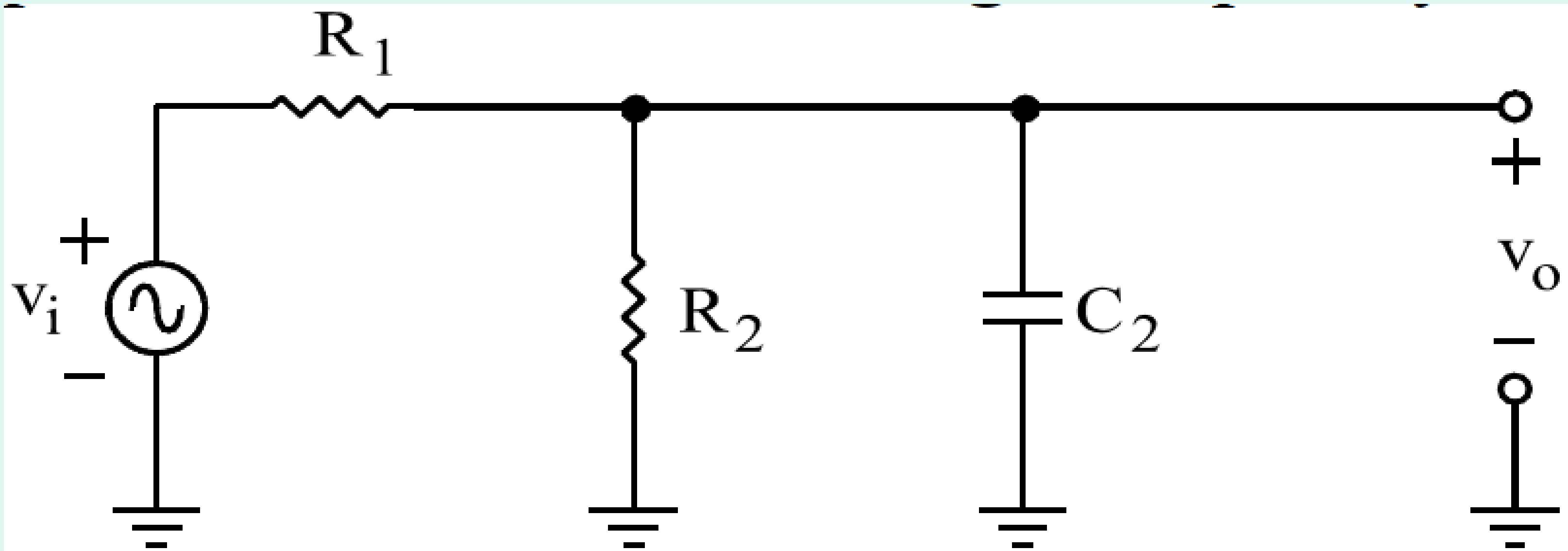


$$\left(\frac{V_o}{V_i}\right)_{LF} = \frac{R_2}{R_2 + R_1 + \dfrac{1}{sC_1}} = \frac{sR_2C_1}{1 + s\left(R_1 + R_2\right)C_1}$$

$$\omega_{p1} \approx \frac{1}{\left(R_1 + R_2\right)C_1} = \frac{1}{R_+ C_1}$$

Approximate HF pole location

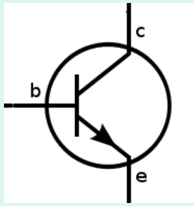
- The simplified circuit at HF:



$$\left(\frac{V_o}{V_i}\right)_{HF} \approx \frac{R_2 // \dfrac{1}{sC_2}}{R_1 + R_2 // \dfrac{1}{sC_2}} = \frac{\dfrac{R_2}{1+sR_2C_2}}{R_1 + \dfrac{R_2}{1+sR_2C_2}} = \frac{R_2}{R_1 + R_2 + sR_1R_2C_2}$$

$$\left(\frac{V_o}{V_i}\right)_{HF} \approx \frac{R_2}{R_1+R_2}\frac{1}{1+sC_2\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)^{-1}}$$

$$\omega_{p2} \approx \frac{1}{\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)^{-1} C_2} = \frac{1}{\left(R_1 // R_2\right) C_2}$$



Combined frequency response

- With the obtained poles approximations:

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{s}{R_1 C_2}}{s^2 + \left[\dfrac{1}{R_2 C_2} + \dfrac{1}{R_1 C_2} + \dfrac{1}{R_1 C_1}\right] s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{s}{R_1 C_2}}{\left(s + \dfrac{1}{R_+ C_1}\right)\left(s + \dfrac{1}{R_{||} C_2}\right)} = \frac{R_2}{R_1 + R_2} \frac{s}{\left(s + \dfrac{1}{R_+ C_1}\right)} \frac{\dfrac{1}{R_{||} C_2}}{\left(s + \dfrac{1}{R_{||} C_2}\right)}$$

$$= \frac{R_2}{R_1 + R_2} \frac{s}{(s + \omega_{p1})} \frac{\omega_{p2}}{(s + \omega_{p2})}$$

↗ Midband ↑ Low freq. ↖ High freq.

Generalization

- We can in general, provided that the poles groups are apart, separate an amplifier frequency response into the three bands components:

$$T(s) = A_M F_L(s) F_H(s)$$

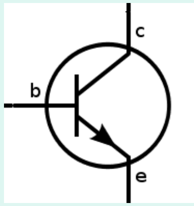
A_M ↗ Midband

$F_L(s)$ ↑ Lowf (high-pass)

$F_H(s)$ ↖ Highf (low-pass)

$$F_L(s) = \frac{(s+\omega_{Lz1})\cdots(s+\omega_{Lzn})}{(s+\omega_{Lp1})\cdots(s+\omega_{LpN})}$$

Here n=N to ensure that $|F_L(\infty)|=1$



Generalization (2)

- For the HF component:

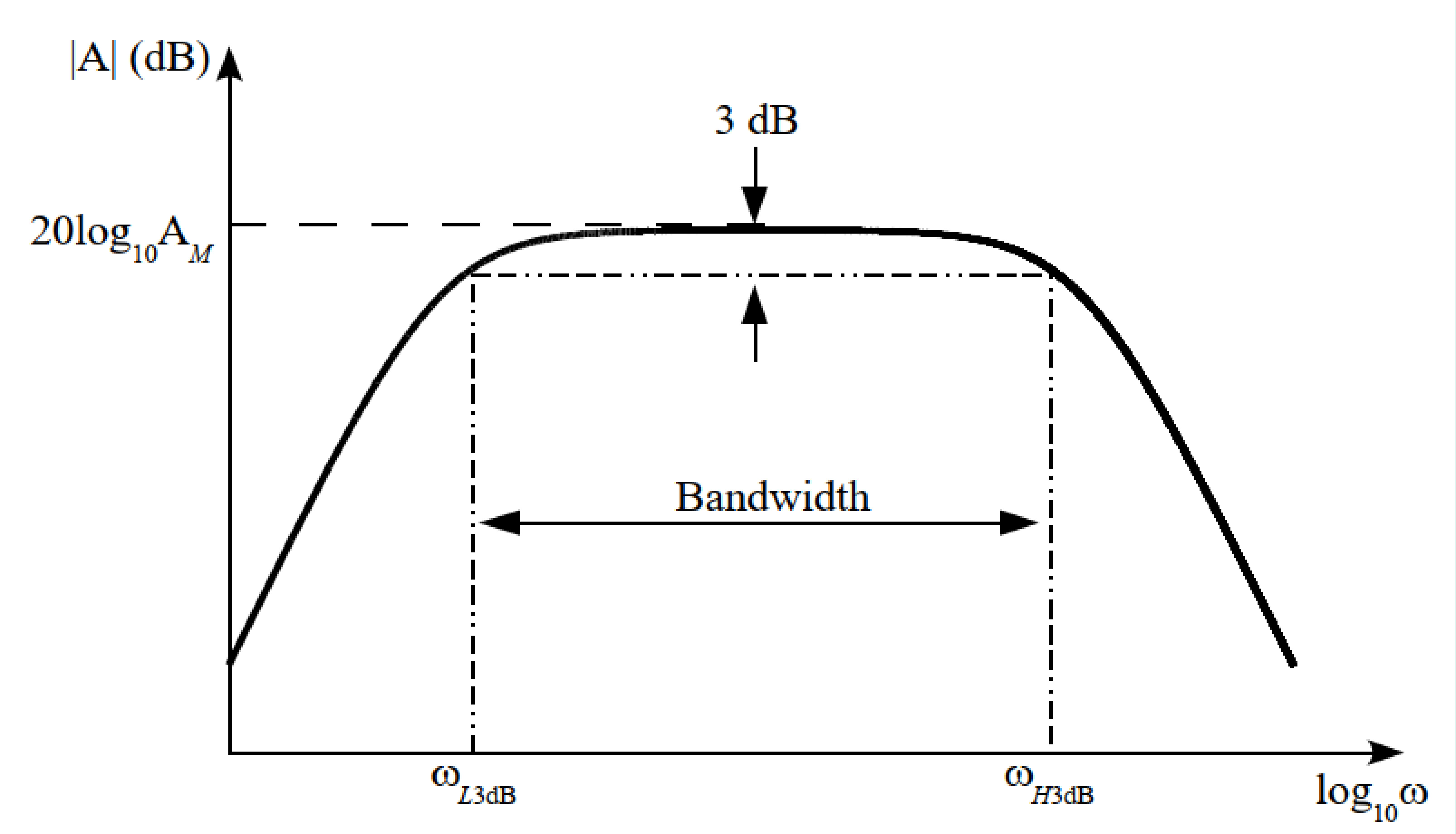
$$F_H(s) = \frac{\omega_{Hp1} \cdots \omega_{HpM}}{\omega_{Hz1} \cdots \omega_{Hzm}} \frac{(s+\omega_{Hz1}) \cdots (s+\omega_{Hzm})}{(s+\omega_{Hp1}) \cdots (s+\omega_{HpM})}$$

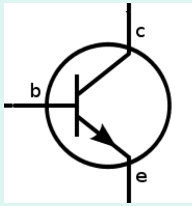
$$= \frac{\left(1+\frac{s}{\omega_{Hz1}}\right) \cdots \left(1+\frac{s}{\omega_{Hzm}}\right)}{\left(1+\frac{s}{\omega_{Hp1}}\right) \cdots \left(1+\frac{s}{\omega_{HpM}}\right)}$$

Here $M>m$, to ensure that $|F_H(\infty)|=0$

Generic amplifier frequency response

- Bandwidth $BW=\omega_{H3dB}-\omega_{L3dB}$





Finding the cut-off frequency ω_{L3dB}

- We assume we know the transfer functions (locations of poles and zeros)

$$|F_L(j\omega_{L3dB})|^2 = \frac{(\omega_{L3dB}^2+\omega_{Lz1}^2)(\omega_{L3dB}^2+\omega_{Lz2}^2)\cdots(\omega_{L3dB}^2+\omega_{Lzn}^2)}{(\omega_{L3dB}^2+\omega_{Lp1}^2)(\omega_{L3dB}^2+\omega_{Lp2}^2)\cdots(\omega_{L3dB}^2+\omega_{LpN}^2)} = \frac{1}{2}$$

$$2(\omega_{L3dB}^2+\omega_{Lz1}^2)\cdots(\omega_{L3dB}^2+\omega_{Lzn}^2) = (\omega_{L3dB}^2+\omega_{Lp1}^2)\cdots(\omega_{L3dB}^2+\omega_{LpN}^2)$$

$$2\left(\omega_{L3dB}^{2n}+(\omega_{Lz1}^2+\omega_{Lz2}^2+\cdots+\omega_{Lzn}^2)\omega_{L3dB}^{2(n-1)}+\cdots+(\omega_{Lz1}^2\omega_{Lz2}^2\cdots\omega_{Lzn}^2)\right)$$
$$=\left(\omega_{L3dB}^{2N}+(\omega_{Lp1}^2+\omega_{Lp2}^2+\cdots+\omega_{LpN}^2)\omega_{L3dB}^{2(N-1)}+\cdots+(\omega_{Lp1}^2\omega_{Lp2}^2\cdots\omega_{LpN}^2)\right)$$

Finding the cut-off frequency ω_{L3dB} (2)

- Since ω_{L3dB} is larger than any pole or zero, we can approximate the eqn. considering only the highest powers
- Rem: many of the zeros may be located at 0

$$2\,\omega_{L3dB}^{2N} + 2\left(\omega_{Lz1}^{2} + \cdots + \omega_{Lzn}^{2}\right)\omega_{L3dB}^{2(N-1)} \approx \omega_{L3dB}^{2N} + \left(\omega_{Lp1}^{2} + \cdots + \omega_{LpN}^{2}\right)\omega_{L3dB}^{2(N-1)}$$

$$\boxed{\omega_{L3dB} \approx \sqrt{\omega_{Lp1}^{2} + \cdots + \omega_{LpN}^{2} - 2\,\omega_{Lz1}^{2} - \cdots - 2\,\omega_{Lzn}^{2}}}$$

Finding the cut-off frequency ω_{3dBH}

- At ω_{3dBH}, the normalized $F_H(s)$ should be reduced by sqrt(2)

$$|F_H(j\omega_{H3dB})|^2 = \frac{\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hz1}^2}\right)\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hz2}^2}\right)\cdots\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hzm}^2}\right)}{\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hp1}^2}\right)\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hp2}^2}\right)\cdots\left(1+\frac{\omega_{H3dB}^2}{\omega_{HpM}^2}\right)} = \frac{1}{2}$$

$$2\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hz1}^2}\right)\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hz2}^2}\right)\cdots\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hzm}^2}\right) = \left(1+\frac{\omega_{H3dB}^2}{\omega_{Hp1}^2}\right)\left(1+\frac{\omega_{H3dB}^2}{\omega_{Hp2}^2}\right)\cdots\left(1+\frac{\omega_{H3dB}^2}{\omega_{HpM}^2}\right)$$

Finding the cut-off frequency ω_{3dBH} (2)

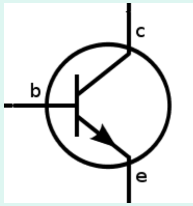
- Since M>m and ω_{3dBH} is smaller than any of the pole or zero frequencies, we can approximate the equality considering only the (dominant) low powers in the polynomials

$$2+\left(\frac{2}{\omega_{Hz1}^{2}}+\cdots+\frac{2}{\omega_{Hzm}^{2}}\right)\omega_{H3dB}^{2}\approx 1+\left(\frac{1}{\omega_{Hp1}^{2}}+\cdots+\frac{1}{\omega_{HpM}^{2}}\right)\omega_{H3dB}^{2}$$

$$\frac{1}{\omega_{H3dB}^{2}}\approx\frac{1}{\omega_{Hp1}^{2}}+\cdots+\frac{1}{\omega_{HpM}^{2}}-\frac{2}{\omega_{Hz1}^{2}}-\cdots-\frac{2}{\omega_{Hzm}^{2}}$$

$$\tau_{H3dB}\approx\sqrt{\tau_{Hp1}^{2}+\cdots+\tau_{HpM}^{2}-2\tau_{Hz1}^{2}-\cdots-2\tau_{Hzm}^{2}}$$

$$\omega_{H3dB}=\frac{1}{\tau_{H3dB}}$$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