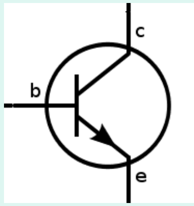
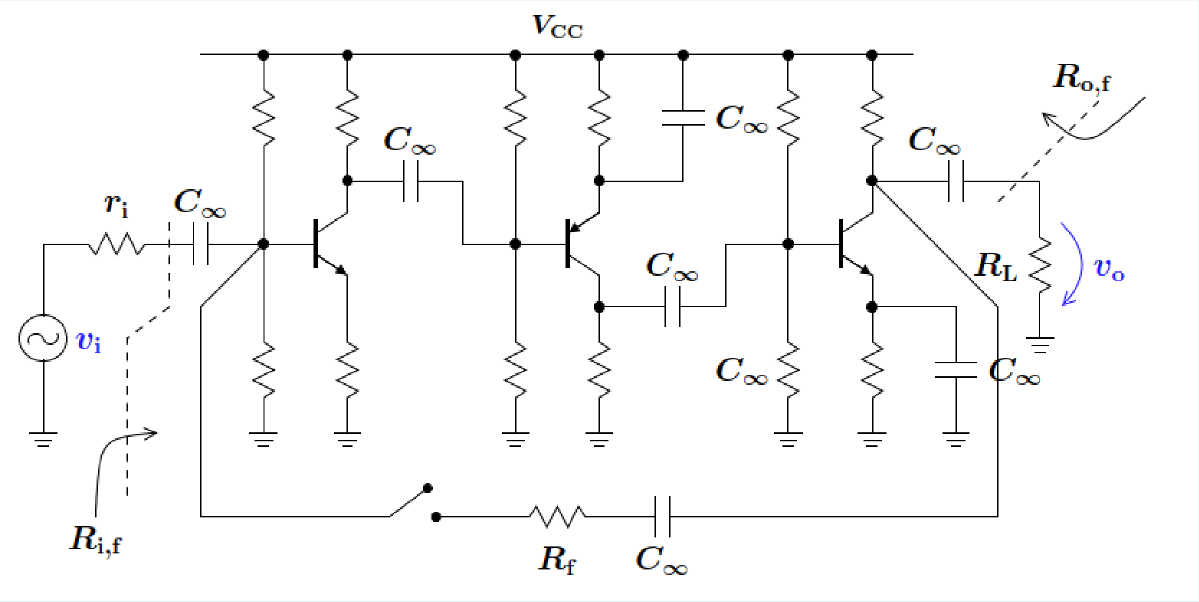
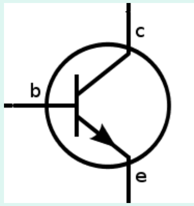


ELEC 301 - OC/SC time constants method

L09 - Sep 23

Instructor: Edmond Cretu





Last time

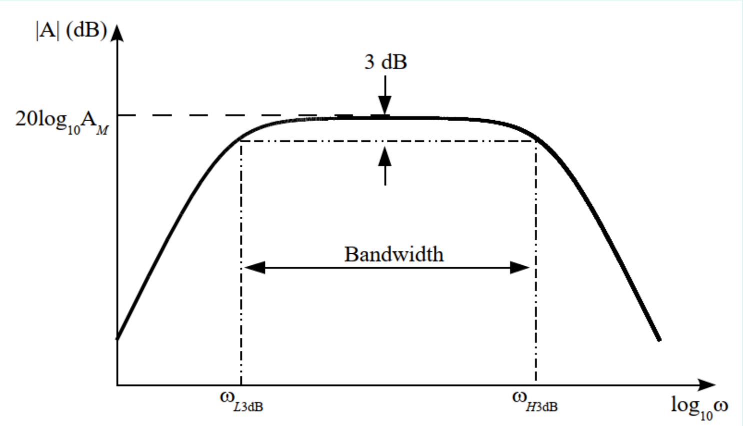
- Split the transfer function of a band-pass circuit into LF, MF, HF bands $T(s) = A_M F_L(s) F_H(s)$
- Effects of negative feedback on ω_L, ω_H

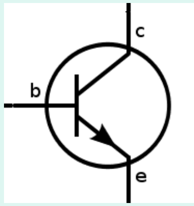
$$F_L(s) = \frac{(s+\omega_{Lz1})..(s+\omega_{LzN})}{(s+\omega_{Lp1})..(s+\omega_{LpN})}, \left|F_L(j\omega)\right| \xrightarrow{\omega\to\infty} 1$$

$$\begin{cases} \omega_{H,CL} = \omega_H\left(1+A_0F_0\right) \\ \omega_{L,CL} = \dfrac{\omega_L}{1+A_0F_0} \end{cases}$$

$$F_H(s) = \frac{\left(1+\dfrac{s}{\omega_{Hz1}}\right)..\left(1+\dfrac{s}{\omega_{Hzm}}\right)}{\left(1+\dfrac{s}{\omega_{Hp1}}\right)..\left(1+\dfrac{s}{\omega_{HpM}}\right)}, m < M$$

$$\left|F_H(j\omega)\right| \xrightarrow{\omega\to\infty} 0, \left|F_H(j\omega)\right| \xrightarrow{\omega\to 0} 1$$



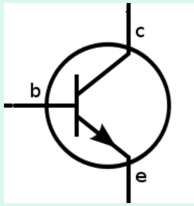


Bandwidth estimation

- Standard recipe:
  1. Derive T(s)
  2. Set s=jω
  3. Find |T(jω)|
  4. Set the magnitude lower by -3dB of the "midband" value
  5. Solve for ω_{L3dB}, ω_{H3dB}

$$\left|T\left(j\omega_{L3dB}\right)\right| = \frac{A_M}{\sqrt{2}}, \left|T\left(j\omega_{H3dB}\right)\right| = \frac{A_M}{\sqrt{2}}$$

- Difficulties:
  - difficult computation by hand
  - we desire design insight - which components set the BW limits => approximative tech., but with insight



Approximate (conservative) BW estimation

- LF case:

$$\frac{\omega_{L3dB}}{\omega_{Lz/p}} >> 1 \Rightarrow a_n\left(\frac{\omega_{L3dB}}{\omega_{Lz/p}}\right)^n + a_{n-1}\left(\frac{\omega_{L3dB}}{\omega_{Lz/p}}\right)^{n-1} + .. + a_1\left(\frac{\omega_{L3dB}}{\omega_{Lz/p}}\right) + a_0 \approx a_n\left(\frac{\omega_{L3dB}}{\omega_{Lz/p}}\right)^n + a_{n-1}\left(\frac{\omega_{L3dB}}{\omega_{Lz/p}}\right)^{n-1}$$

$$\omega_{L3dB}^2 \approx \sum_{i=1}^{N}\omega_{Lp_i}^2 - 2\sum_{j=1}^{N}\omega_{Lzj}^2$$

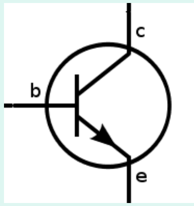
- HF case:

$$\frac{\omega_{H3dB}}{\omega_{Hz/p}} << 1 \Rightarrow a_n\left(\frac{\omega_{H3dB}}{\omega_{Hz/p}}\right)^n + a_{n-1}\left(\frac{\omega_{H3dB}}{\omega_{Hz/p}}\right)^{n-1} + .. + a_1\left(\frac{\omega_{H3dB}}{\omega_{Hz/p}}\right) + a_0 \approx a_1\left(\frac{\omega_{H3dB}}{\omega_{Hz/p}}\right) + a_0$$

$$\frac{1}{\omega_{H3dB}^2} \approx \sum_{i=1}^{M}\frac{1}{\omega_{Hp_i}^2} - 2\sum_{j=1}^{m}\frac{1}{\omega_{Hz_j}^2} \qquad \tau_{H3dB}^2 = \sum_{i=1}^{M}\tau_{Hp_i}^2 - 2\sum_{j=1}^{m}\tau_{Hz_j}^2$$

Method of OC and SC time constants

- OC=open-circuit, SC=short-circuit
- How to estimate the cut-off frequencies when the locations of poles and zeros are not known
- The approximation is reasonably accurate provided that the next nearest poles, on either side, are at least two octaves (4x) away, or better a decade (10x) away
- Design aspects: OCT and SCT, unlike typical circuit simulations, can identify which elements are responsible for BW limitations



SC time ct. - the cut-off frequency ω_{L3dB}

- Focus on $F_L(s)$

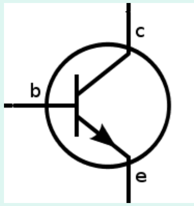
$$F_L(s)=\frac{(s+\omega_{Lz1})\cdots(s+\omega_{Lzn})}{(s+\omega_{Lp1})\cdots(s+\omega_{LpN})}$$

$$F_L(s)=\frac{s^n+a_1 s^{n-1}+\cdots+a_n}{s^N+b_1 s^{N-1}+\cdots+b_N}$$

$$b_1=\omega_{Lp1}+\omega_{Lp2}+\cdots+\omega_{LpN}=\frac{1}{\tau_{C_1}^{sc}}+\frac{1}{\tau_{C_2}^{sc}}+\cdots+\frac{1}{\tau_{C_N}^{sc}}=\sum_{i=1}^{N}\frac{1}{C_i R_{is}}$$

Here τ^{sc} are the "short-circuit" time constants- each R_{is} is the resistance seen by the i-th low frequency capacitor, Ci, with all other LF capacitors replaced by short-circuits

- All HF capacitors are treated as open circuits



Finding the cut-off frequency ω_{L3dB} (2)

- If one of the pole frequencies (or one of the $1/\tau^{sc}$ terms) is greater than any of the other pole (or other $1/\tau^{sc}$) or zero frequencies by a factor of 4x (10x is better), then the Fl(s) can be approximated for frequencies above those of other poles and zeros, as:

$$F_L(s) = \frac{s^n + a_1 s^{n-1} + \cdots + a_n}{s^N + b_1 s^{N-1} + \cdots + b_N} \approx \frac{s^n}{s^N + b_1 s^{N-1}} = \frac{s}{s + b_1}$$

$$\boxed{\omega_{L3dB} \approx b_1 = \sum_{i=1}^{N} \frac{1}{C_i R_{is}}}$$

OC time ct. - the cut-off frequency ω_{3dBH}

- Focus on $F_H(s)$

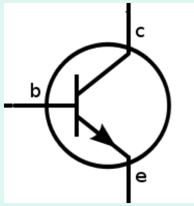
$$F_H(s)=\frac{\left(1+\frac{s}{\omega_{Hz1}}\right)\cdots\left(1+\frac{s}{\omega_{Hzm}}\right)}{\left(1+\frac{s}{\omega_{Hp1}}\right)\cdots\left(1+\frac{s}{\omega_{HpM}}\right)}$$

$$F_H(s)=\frac{1+c_1 s+\cdots+c_m s^m}{1+d_1 s+\cdots+d_M s^M}$$

$$d_1=\frac{1}{\omega_{Hp1}}+\frac{1}{\omega_{Hp2}}+\cdots+\frac{1}{\omega_{HpM}}=\tau_{C_1}^{oc}+\tau_{C_2}^{oc}+\cdots+\tau_{C_M}^{oc}=\sum_{i=1}^{M} C_i R_{io}$$

Here, τ^{oc} are **"open-circuit" time constants** - each R_{io} is the resistance seen by the i-th HF capacitor, with all other HF capacitors replaced by open-circuits

- All of the LF capacitors are treated as short-circuits



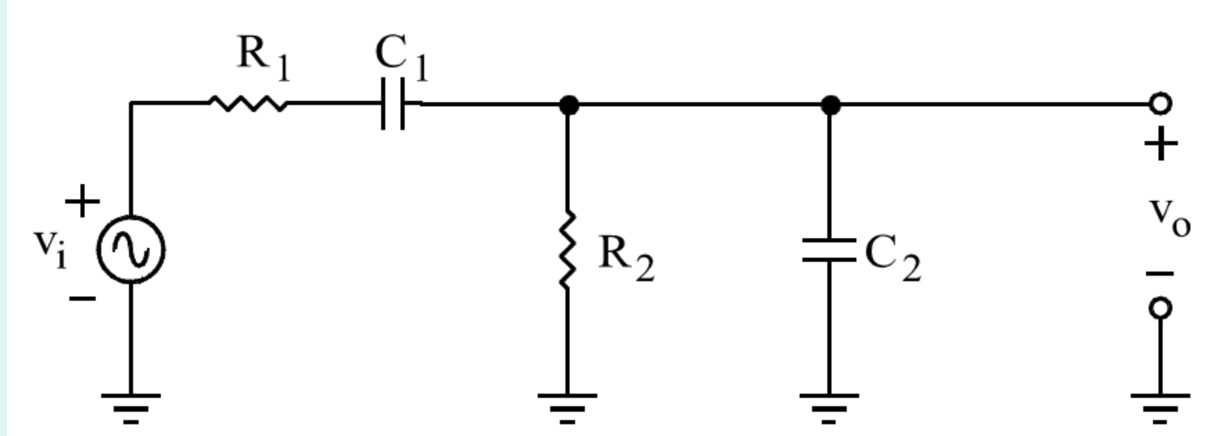
Finding the cut-off frequency ω_{3dBH} (2)

- If one of the pole frequencies (or $1/\tau^{oc}$ terms) is smaller than any of the other pole (or other $1/\tau^{oc}$) or zeros frequencies (by 4x, or better by 10x), then, for frequencies below those of the other poles and zeros, we can approximate $F_H(s)$ as:

$$F_H(s) = \frac{1 + c_1 s + \cdots + c_m s^m}{1 + d_1 s + \cdots + d_M s^M} \approx \frac{1}{1 + d_1 s}$$

$$\omega_{H\,3dB} \approx \frac{1}{d_1} = \frac{1}{\sum_{i=1}^{M} C_i R_{io}}$$

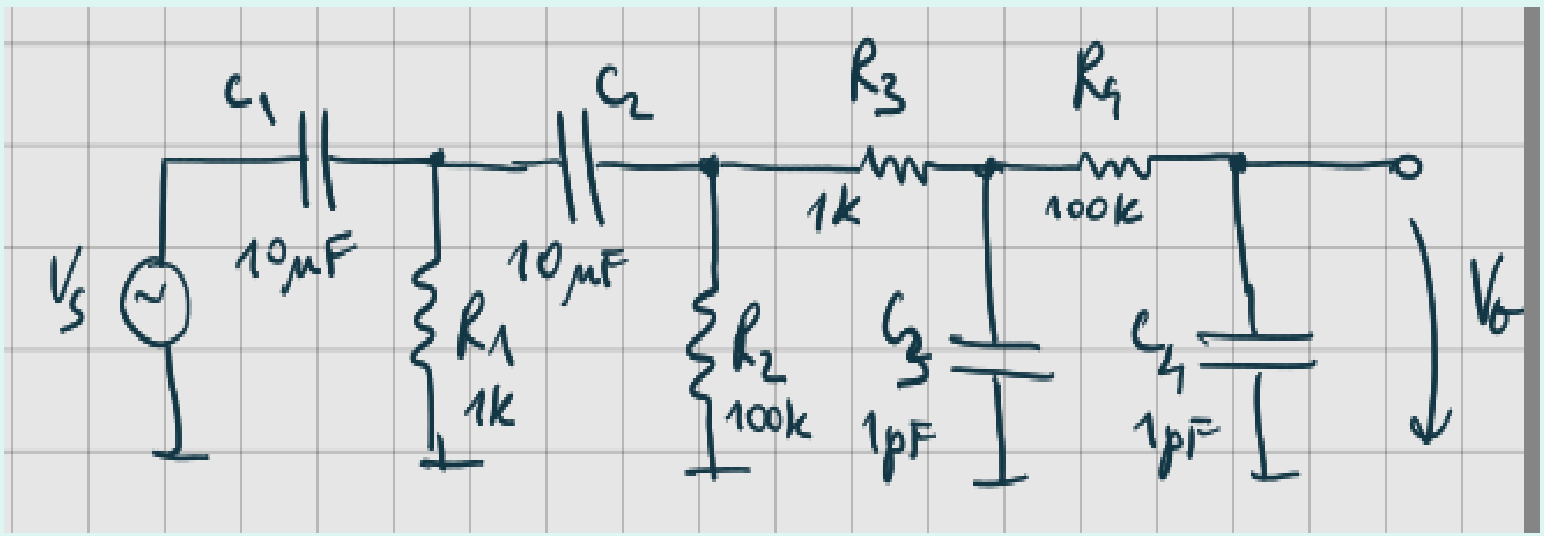
Remarks

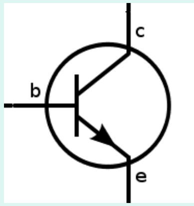


- For the initial circuit:
  - $\tau_{p1}=C_1R_+$ is the time ct. for C_1 with C_2 open-circuited
    $\tau_{p2}=C_2R_{//}$ is the time ct. for C_2 with C_1 short-circuited
- The OC/SC time constants method can be extended to provide further estimations for all poles of a transfer function, provided that the next nearest poles, on either side, are at least two octaves away

Example

- Use SC and OC time constants method to estimate the cut-off frequencies of the following circuit:





L09 Q01 - LF behavior

- Which capacitor limits the LF behavior?

A. C1

B. C2

C. C3

D. C4

L09 Q02 - HF behavior

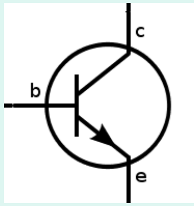
- Which capacitor limits the HF behavior?

A. C1

B. C2

C. C3

D. C4


c
b
e