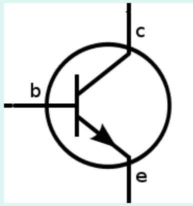
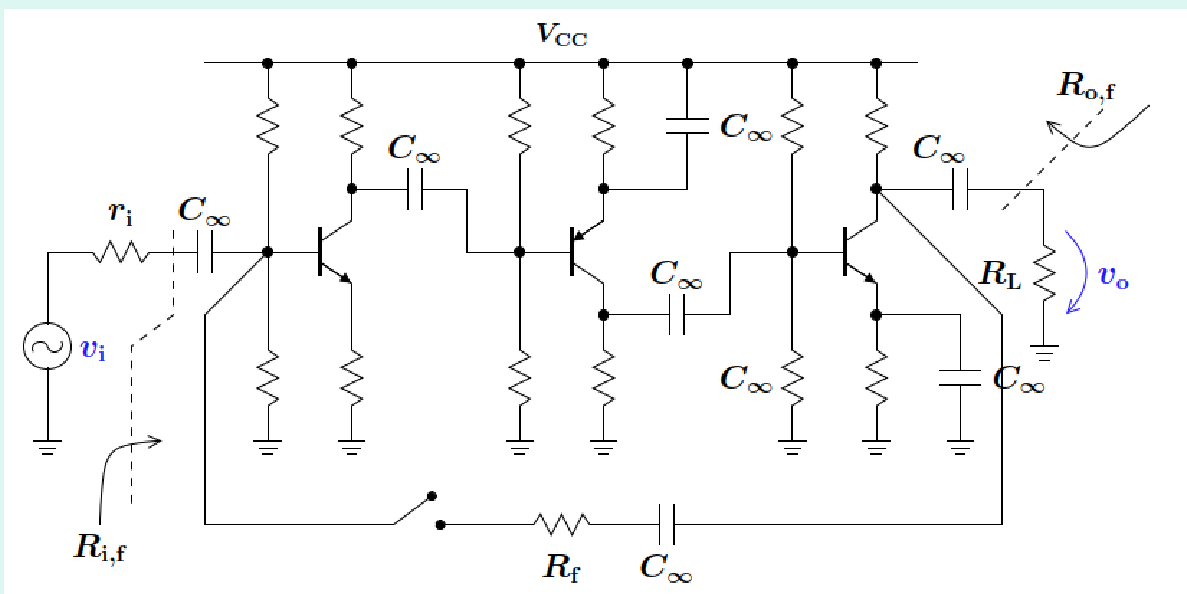
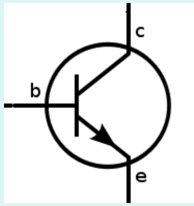


ELEC 301 - BJT small signal model

L10 - Sep 25

Instructor: Edmond Cretu



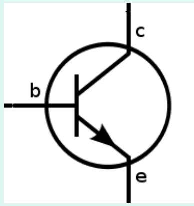


Last time

- The open-circuit (OCT) and short-circuit (SCT) time constants method
- SCT - used to approximate the LF cut-off frequency
- OCT - used to approximate the HF cut-off frequency

$$\omega_{L3dB} \approx \omega_{Lp1} + \omega_{Lp2} + .. + \omega_{LpN} = \frac{1}{\tau_{C_1}^{sc}} + \frac{1}{\tau_{C_2}^{sc}} + .. + \frac{1}{\tau_{C_3}^{sc}}$$

$$\frac{1}{\omega_{H3dB}} \approx \frac{1}{\omega_{Hp1}} + \frac{1}{\omega_{Hp2}} + .. \frac{1}{\omega_{HpM}} = \tau_{C_1}^{oc} + \tau_{C_2}^{oc} + .. + \tau_{C_M}^{oc}$$



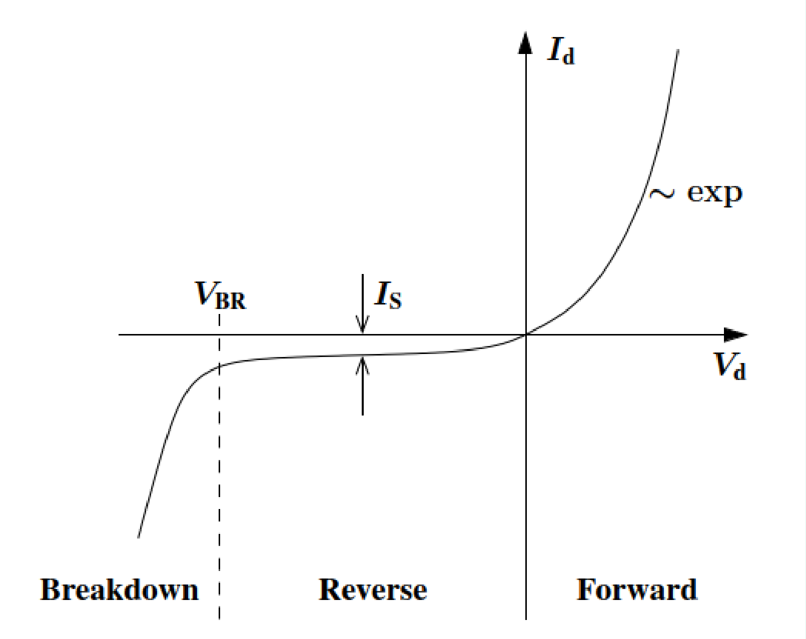
Today - Amplifiers with BJT

- Small-signal model for BJT ("hybrid-π" model)
- Notation conventions:
  - capital I's and V's with capital subscripts denote d.c. values
  - capital I's and V's with lowercase subscripts denote complex values (phasors)
  - lowercase i's and v's with capital subscripts denote instantaneous values, and
  - lowercase i's and v's with lowercase subscripts denote small signal values

$$i_C = I_C + i_c$$

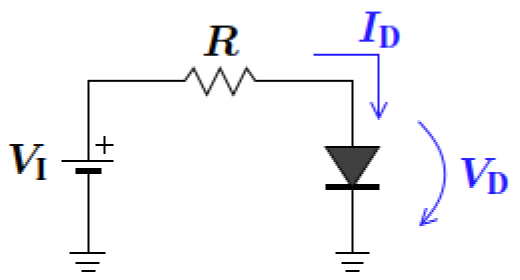
Diodes - recall

- Three operating regions:
  - Forward-bias region ($v_D>0$)
  - Reverse-bias region ($V_{BR}<v_D<0$)
  - Breakdown region ($v_D<V_{BR}$)
- Diode law (Shockley diode eqn.
  - I_S=reverse-bias saturation current
  - n = ideality factor (1..2)
  - V_T=kT/q=thermal voltage



$$i_D = I_s\left(e^{v_D/nV_T} - 1\right)$$

Biasing - quiescent point



$$\begin{cases} V_I = R \cdot I_D + V_D \\ I_D = I_S \exp \dfrac{V_D}{V_T} \end{cases} \rightarrow \begin{cases} \text{No closed-form solution} \\ \text{To be solved numerically} \end{cases}$$

- DC level biasing - (V_D, I_D) is called the quiescent point
- $V_D = V_T \ln(I_D/I_S) \approx 0.6..0.7V$ for a large range of currents

$$I_{D0} = \frac{V_I - 0.6\,\mathrm{V}}{R}$$

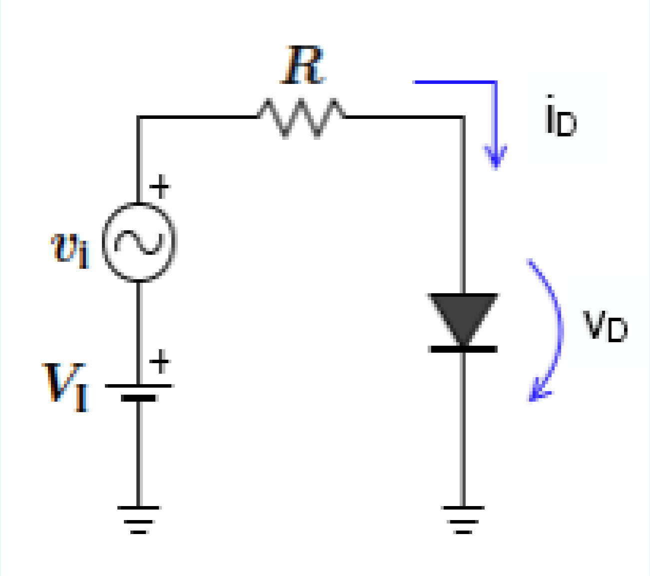
$$V_{D0} = 0.6\,\mathrm{V}$$

L10 Q01 - diode model

- Can we apply superposition to compute the total current through the circuit?

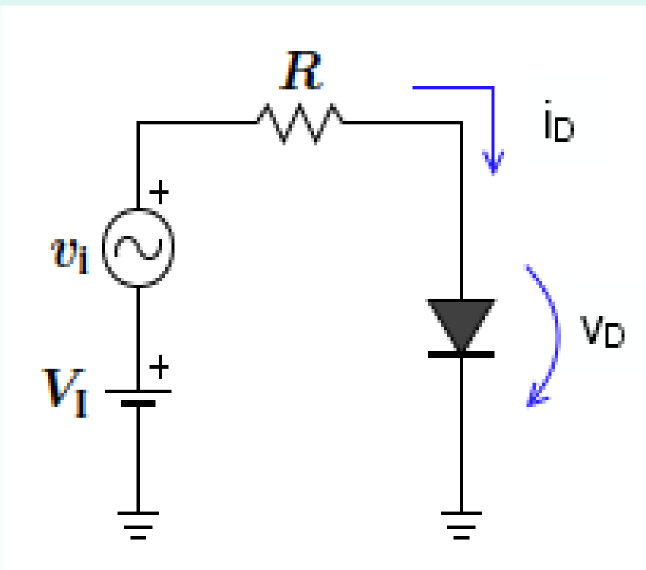
A. Yes

B. No



Diode - small signal model

- DC+ small AC voltages
- nonlinear circuit => no superposition!

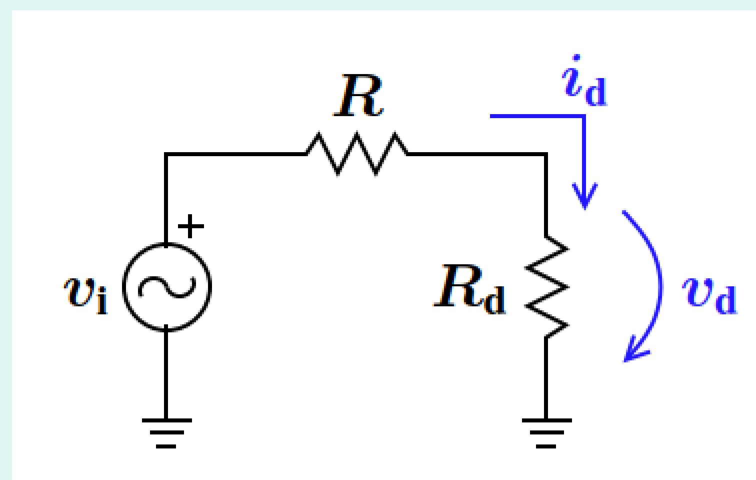


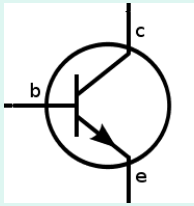
$$i_D = I_s\left(e^{v_D/V_T} - 1\right) \approx I_S e^{\frac{V_D + v_d}{V_T}} = \underbrace{I_S e^{\frac{V_D}{V_T}}}_{I_D} e^{\frac{v_d}{V_T}} = I_D e^{\frac{v_d}{V_T}} \overset{v_d << V_T}{\approx} I_D\left(1 + \frac{v_d}{V_T}\right)$$

$$I_D + i_d \simeq I_D\left(1 + \frac{v_d}{V_T}\right) \Rightarrow i_d \approx \frac{I_D}{V_T} v_d = \frac{v_d}{r_d}$$

$r_d = r_d(I_D)$=dynamic resistance of the diode

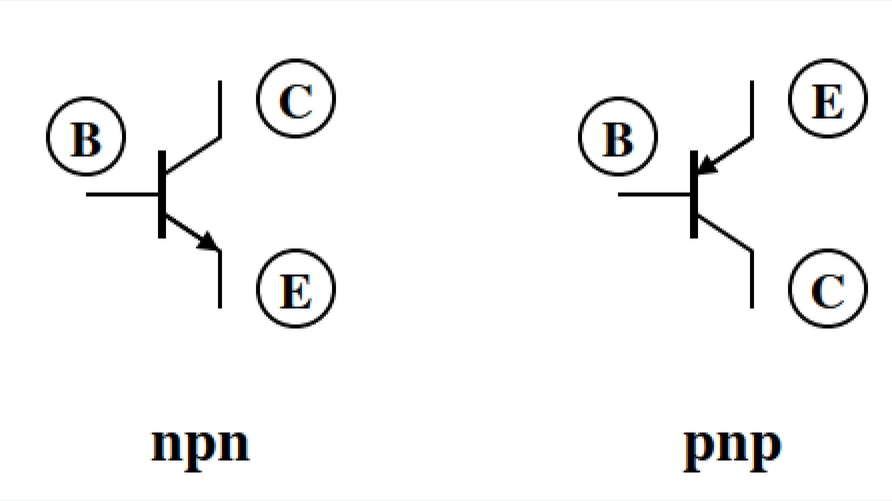
$$i_D = I_D + i_d \approx \frac{V_1 - 0.6V}{R} + \frac{v_i}{R + r_d}$$

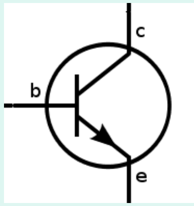




Bipolar junction transistors (BJTs)

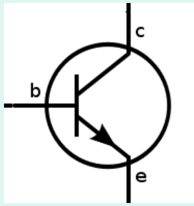
- Two types: npn, pnp
- Three terminals: emitter, base, collector



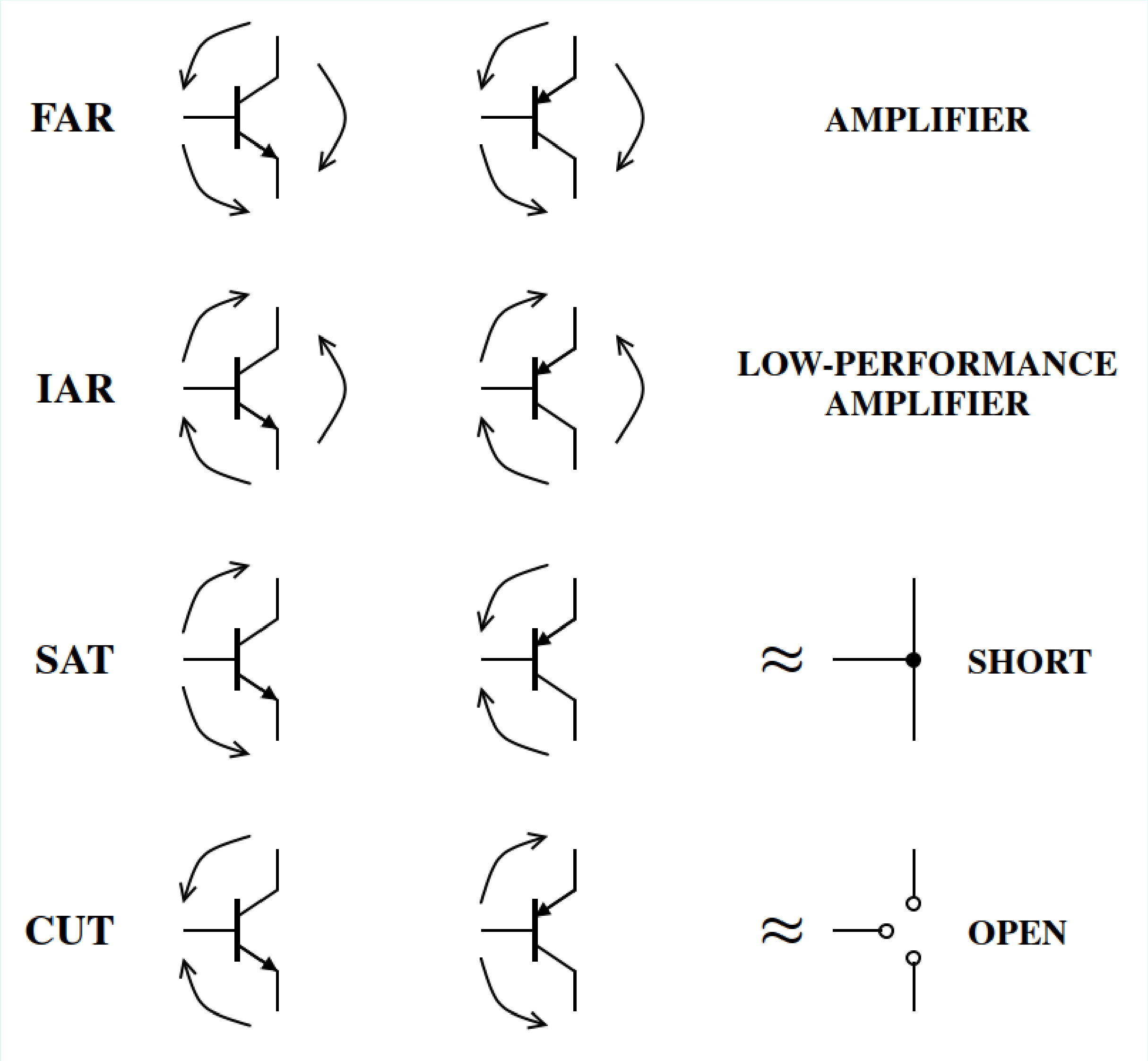


BJT operating modes

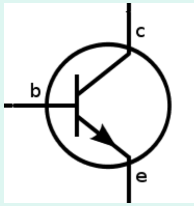
- Four transistor operating modes:
  - **Saturation** -- The transistor acts like a short circuit. Current freely flows from collector to emitter.
  - **Cut-off** -- The transistor acts like an open circuit. No current flows from collector to emitter.
  - **Active** -- The current from collector to emitter is proportional to the base current.
  - **Inverse-Active** -- Like active mode, the current is proportional to the base current, but it flows in reverse. Current flows from emitter to collector (not, exactly, the purpose transistors were designed for)



BJT operating modes



- FAR = forward active region
- IAR = inverse active region (poor β)
- SAT = saturation
- CUT = cut-off



BJT in active operating mode

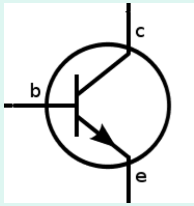
- EB junction must be forward biased (V_{BE}~0.6V for a npn transistor)
- CB junction must be reversed biased (V_{CB}>0V for npn)
- DC model:

$$i_C = I_C + i_c$$

$$I_C = \alpha I_E = \beta I_B$$

$$I_C = I_S e^{V_{BE}/V_T}$$

  - α=common base current gain factor
  - β=common emitter current gain factor
  - V_T=thermal voltage (average thermal energy available to charge carriers)=kT/q~25mV at 25deg
  - I_S= reverse saturation current



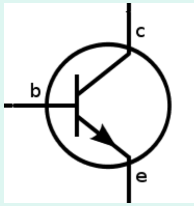
BJT DC model

$$I_E = I_B + I_C$$

$$I_C = \alpha I_E = \beta I_B$$

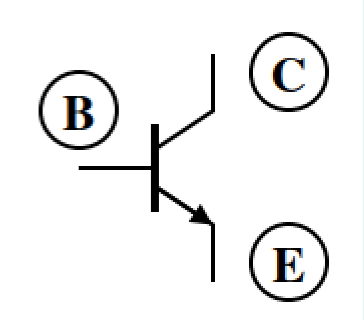
$$I_C = I_S e^{V_{BE}/V_T}$$

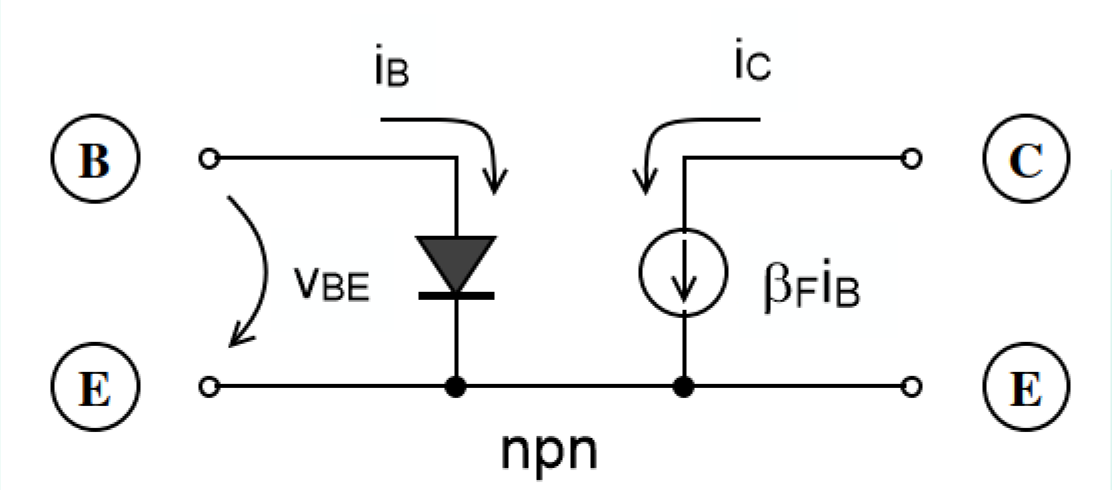
$$\alpha = \frac{\beta}{\beta + 1} \Leftrightarrow \beta = \frac{\alpha}{1 - \alpha}$$



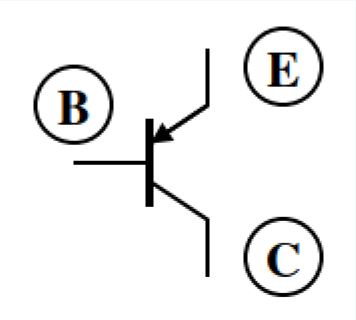
Large-signal model for BJT

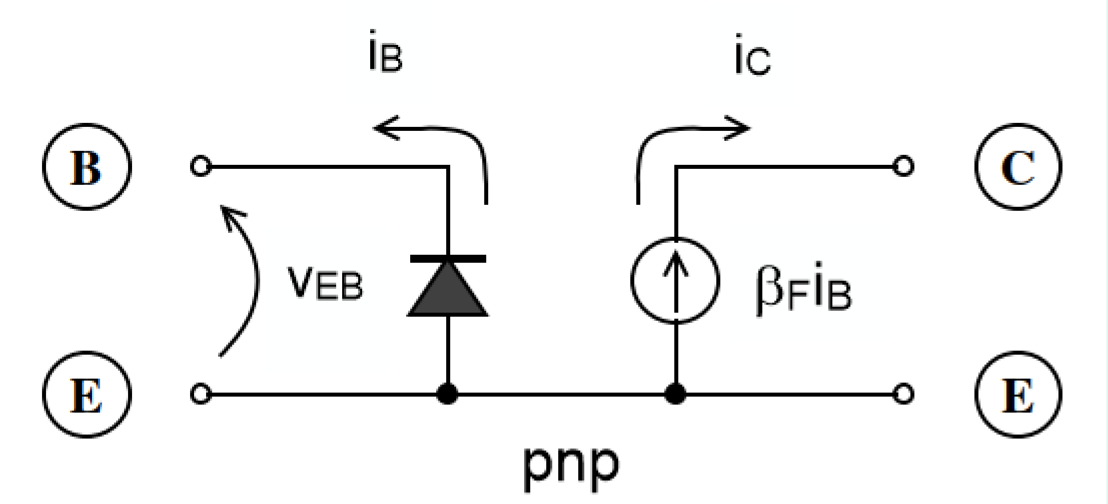
- REM: coupled junctions



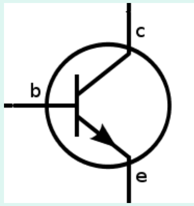


$$i_C = I_S e^{\frac{v_{BE}}{V_T}}, i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}}$$



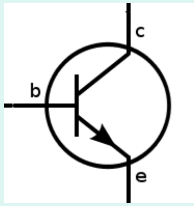


$$i_C = I_S e^{\frac{v_{EB}}{V_T}}, i_B = \frac{I_S}{\beta} e^{\frac{v_{EB}}{V_T}}$$



BJT parameters

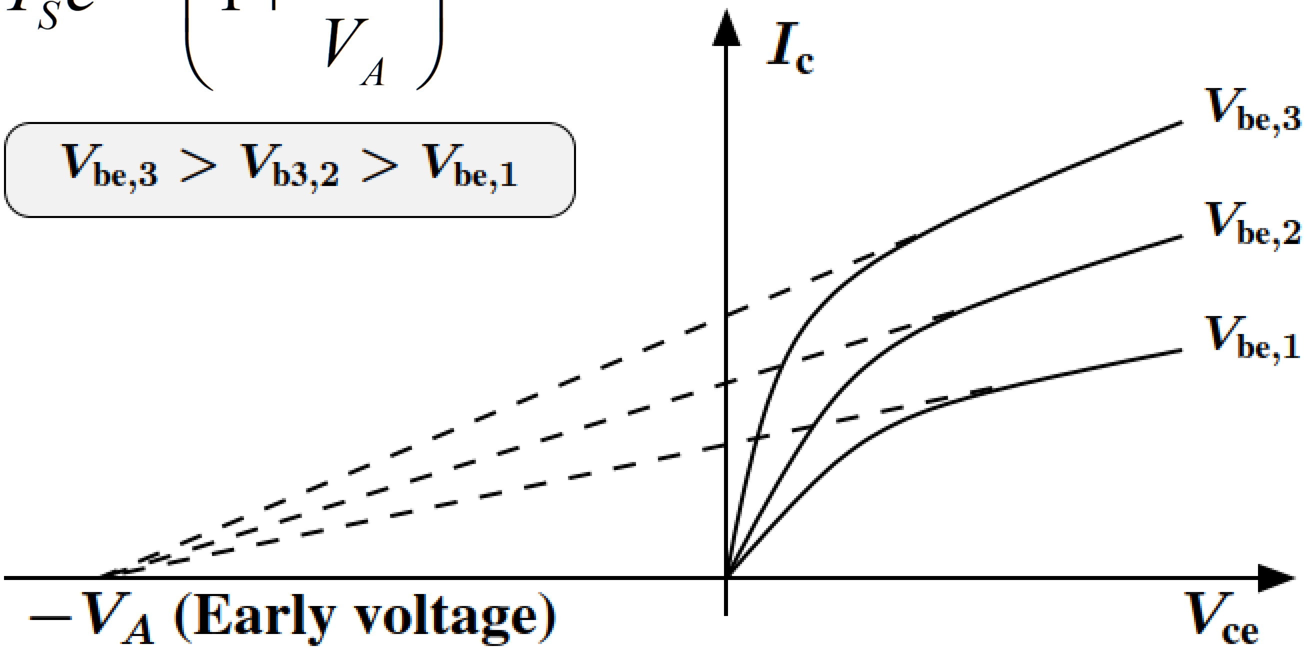
- Saturation current: $I_S=10^{-16}..10^{-14}A$
- Thermal voltage: $V_T=kt/q \simeq 25mV$ at 25^0C
- Forward current gain $\beta_F=\beta$ (assumed constant):
  - $\beta_{F,\ npn}= 50..500$ (technology optimized for npn)
  - $\beta_{F,pnp}= 10..100$



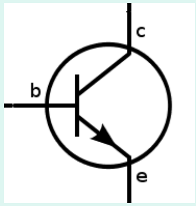
BJT current-voltage relations

- Early effect (discovered by James Early): $v_{CE} \nearrow \Rightarrow i_C \nearrow$
- Early voltage V_A - models the Early effect
- $V_{A,typ}$=50..100V (larger is better)

$$i_C = i_C\left(v_{BE}, v_{CE}\right) = I_S e^{\frac{v_{BE}}{V_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$



UBC



Small variations around the DC bias values

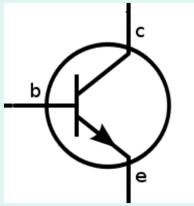
- Consider a small variation v_{be} around the DC bias value V_{BE}

$$v_{BE} = V_{BE} + v_{be} \Rightarrow i_C = I_S e^{v_{BE}/V_T} = I_S e^{V_{BE}/V_T} e^{v_{be}/V_T} = I_C e^{v_{be}/V_T}$$

$$i_C = I_C + i_c \overset{v_{be} << V_T}{\approx} I_C \left(1 + \frac{v_{be}}{V_T} \right) = I_C + \frac{I_C}{V_T} v_{be}$$

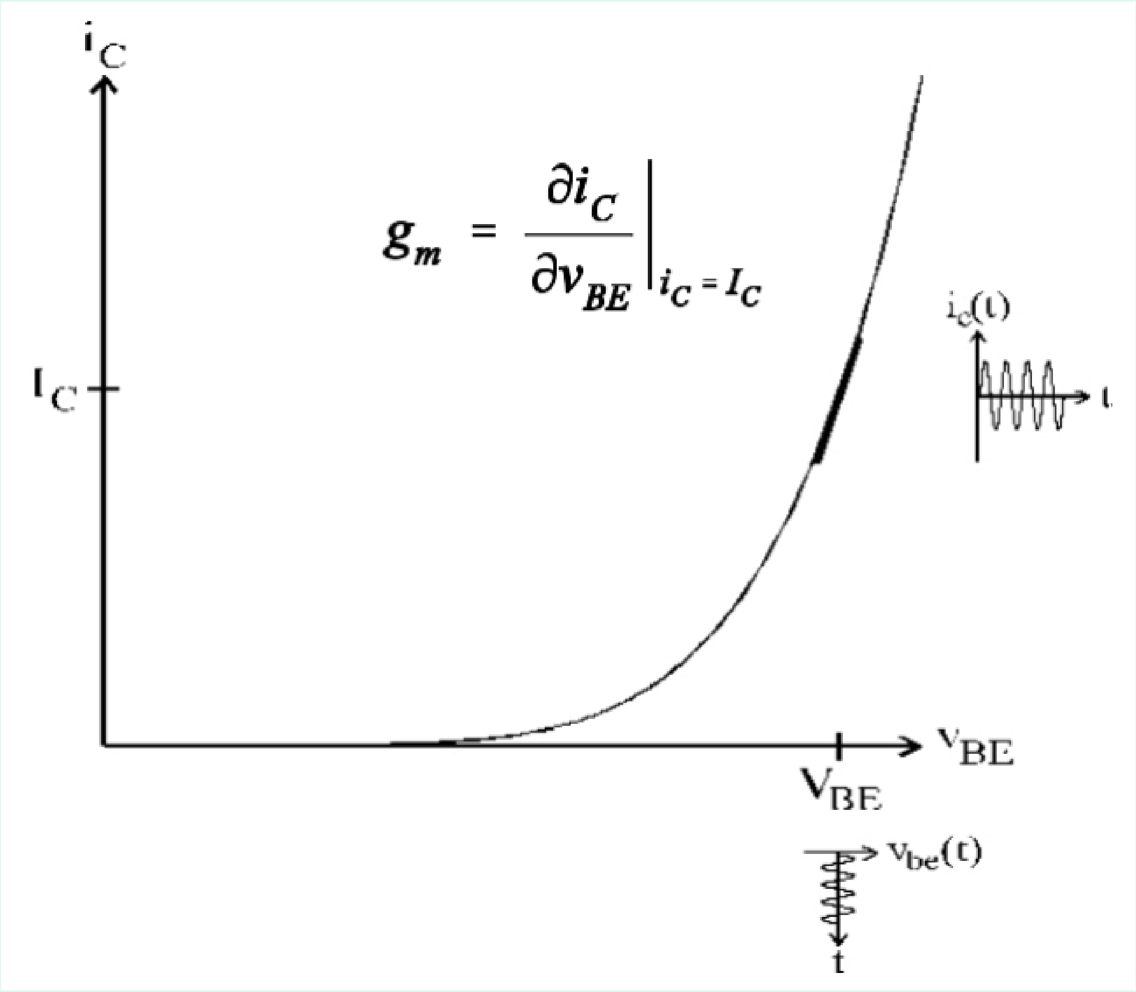
$$i_c = \underbrace{\frac{I_C}{V_T}}_{g_m} v_{be}$$

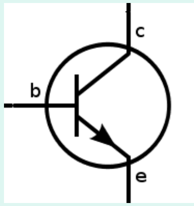
g_m = small signal transconductance



Transconductance g_m

- g_m= the slope of the $i_C(v_{BE})$ curve



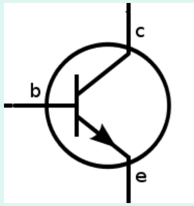


BE input port

$$i_B = I_B + i_b = \frac{i_C}{\beta} = \frac{I_C + i_c}{\beta} = \frac{I_C}{\beta} + \frac{g_m}{\beta} v_{be} \Rightarrow i_b = \frac{g_m}{\beta} v_{be}$$

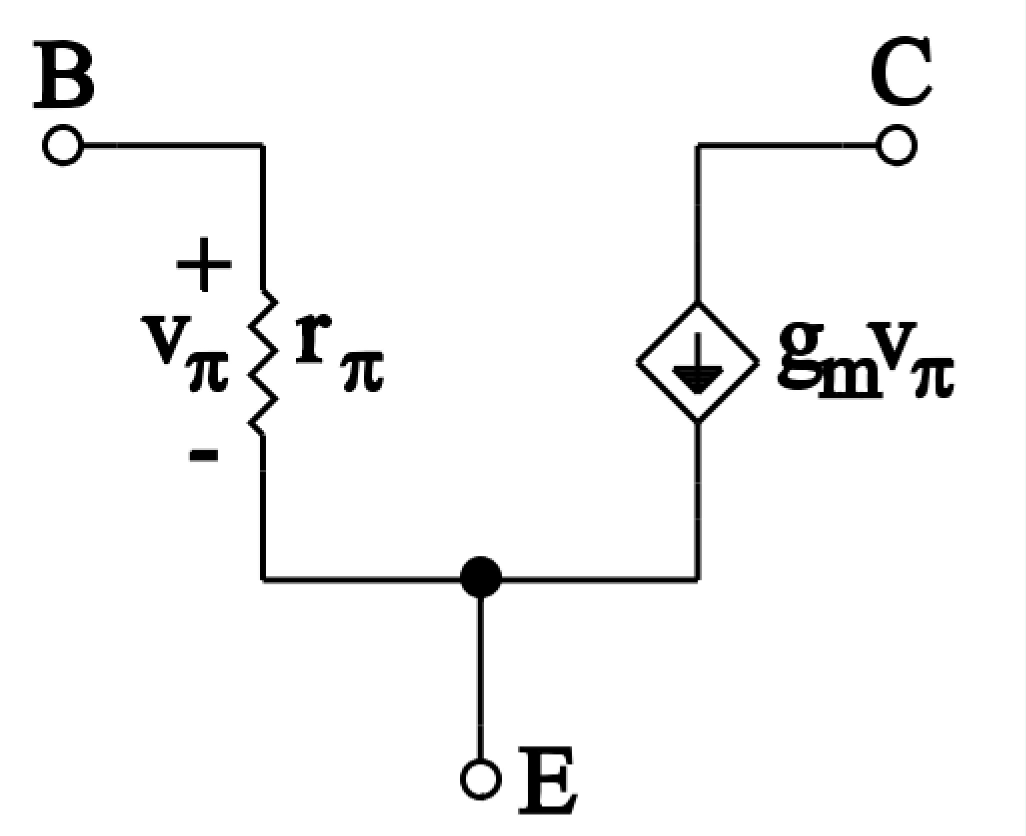
- Small-signal input resistance:

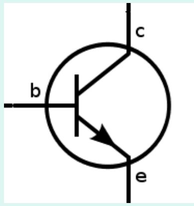
$$r_\pi = \frac{v_{be}}{i_{be}} = \frac{\beta}{g_m} = \beta \frac{V_T}{I_C} = \frac{V_T}{I_B}$$



Basic Hybrid-π model for BJT

- Small signal model without considering capacitive effects, nor the Early effect

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