ELEC 301 - Feedback in circuits

L21 - Oct 28

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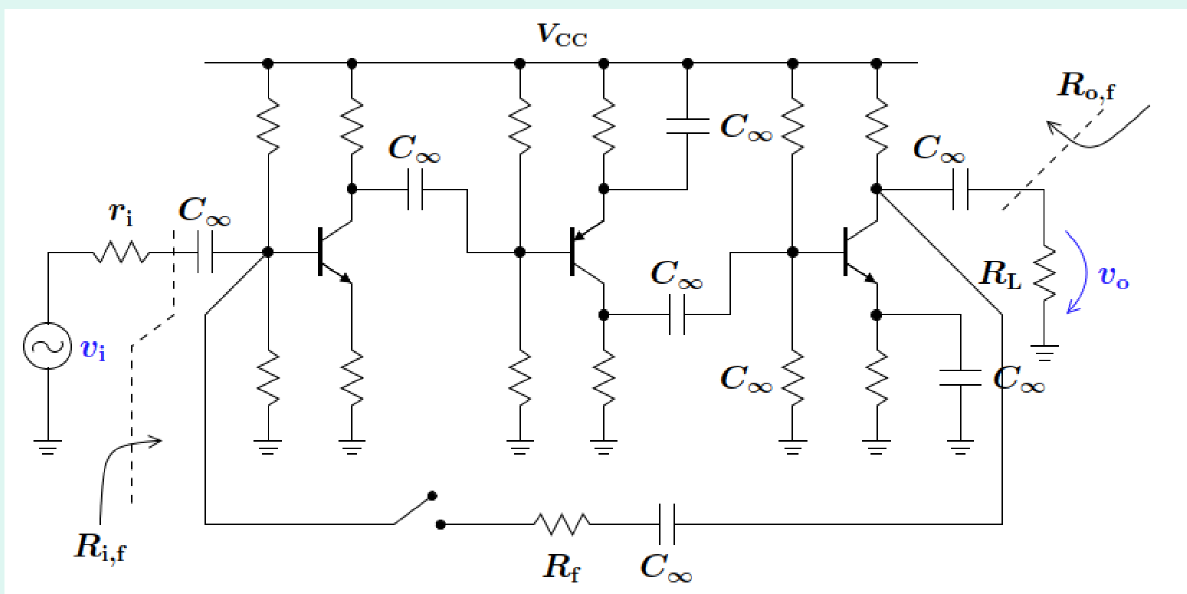
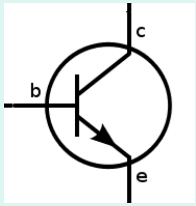




Last time: feedback networks

- Feedback theory:

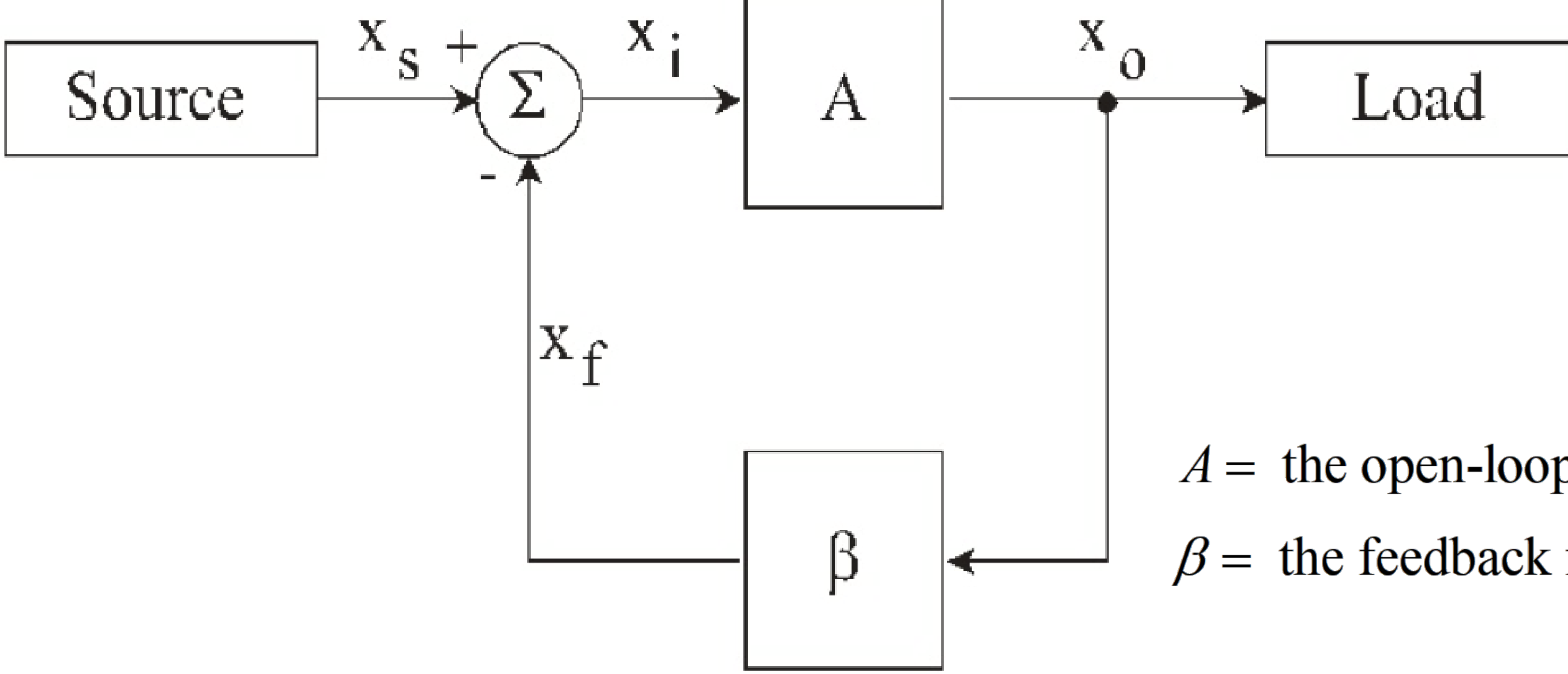
○ Harold Black (Western Electric Co.), 1928 - amplifiers with stable gain for transatlantic telephone repeaters

○ Negative (degenerative) vs. positive (regenerative) feedback

- Feedback uses:
  - de-sensitize gain (gain less sensitive to variations in the values of components)
  - reduced nonlinear distortion (constant gain, independent of signal level)
  - extend operating bandwidth
  - control input and output impedances (Zin, Zout)
  - increase signal-to-noise ratio
  - (positive feedback) - active oscillators

Information flow perspective

- The signal x_s can be a voltage or a current
- Generic signal flowchart



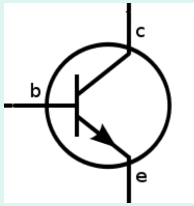
$A =$ the open-loop gain

$\beta =$ the feedback factor

$$\left.\begin{aligned} x_o &= Ax_i \\ x_f &= \beta x_o \\ x_i &= x_s - x_f \end{aligned}\right\} \Rightarrow A_f = \frac{x_o}{x_s} = \frac{Ax_i}{x_i + x_f} = \frac{A}{1 + A\beta}$$

$A\beta =$ "the loop gain"

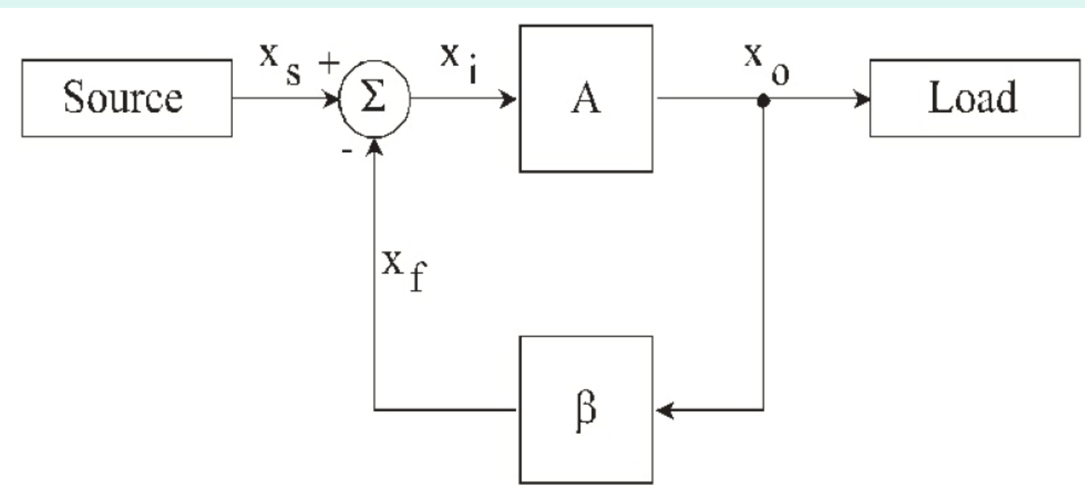
$1 + A\beta =$ "the amount of feedback"



Information flow - negative feedback amplifiers

- Usually the loop gain is very large due to A: Aβ>>1=> the gain is the feedback amplifier is entirely determined by the feedback network

$$A_f = \frac{A}{1+A\beta} = \frac{1}{\beta + \frac{1}{A}} = \frac{1}{\beta}\frac{1}{1+\frac{1}{A\beta}} \underset{A\beta \gg 1}{\approx} \frac{1}{\beta}\left(1 - \frac{1}{A\beta}\right) \approx \frac{1}{\beta}$$



A_f = the closed-loop gain

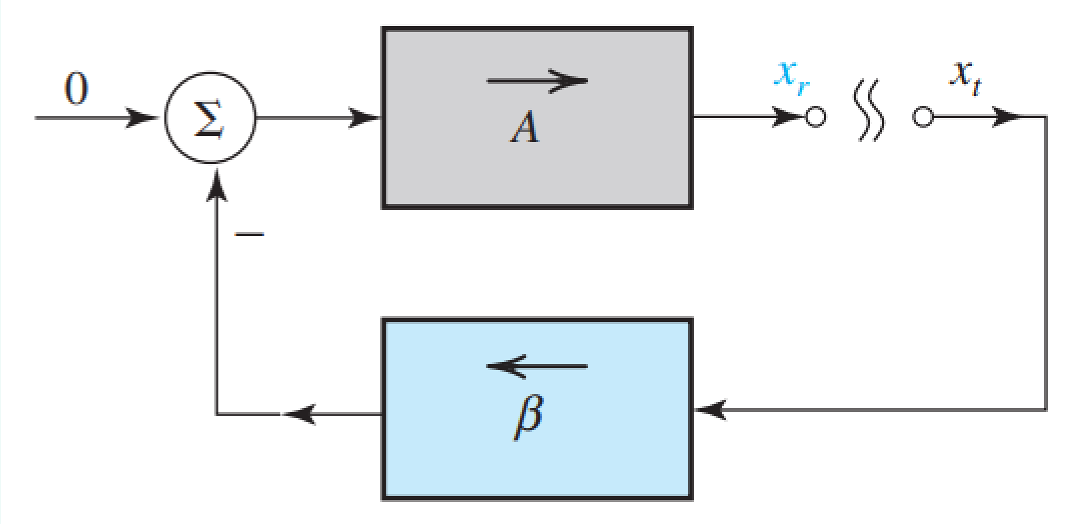
x_i = the "error signal"

$$x_i = \frac{x_o}{A} = \frac{x_s}{1+A\beta} = \frac{x_s}{A\beta}\frac{1}{1+\frac{1}{A\beta}} \underset{A\beta \gg 1}{\approx} 0$$

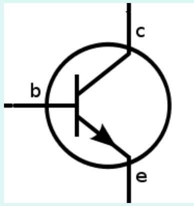
$$x_f = \beta x_o \underset{A\beta \gg 1}{\approx} \beta \frac{x_s}{\beta} = x_s$$

Loop gain

- Breaking the feedback loop to determine the loop gain
- set x_s=0



$$A\beta = -\frac{x_r}{x_t}$$



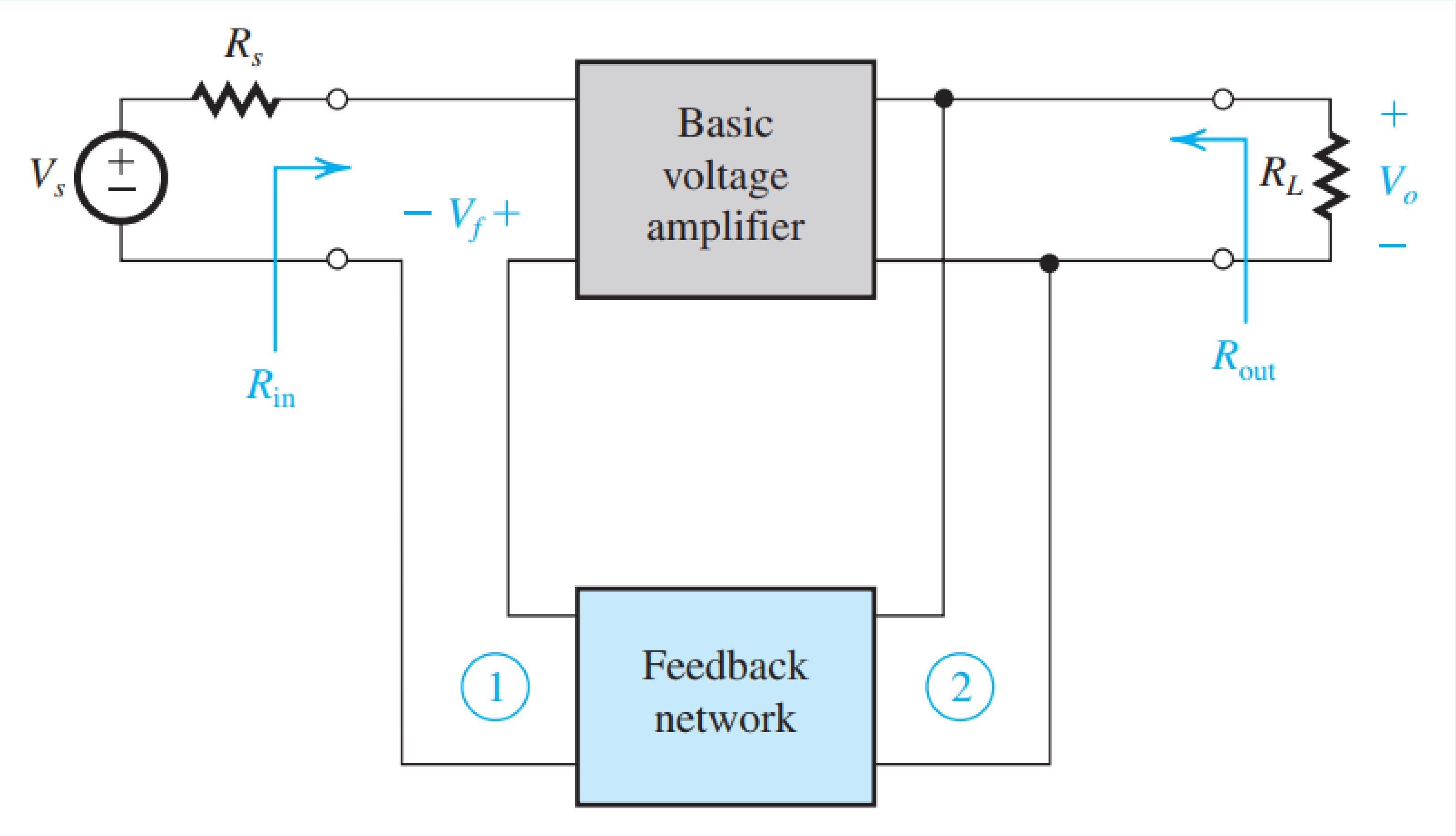
Information flow to energy flow

- signals mapped into voltages/currents => 4 categories of feedback circuit architectures

Input signal x_s	Output signal x_o	Feedback type	Remarks
voltage v_s	voltage v_o	series-shunt	Voltage amplifier A_{vf} (Z_{in} high, Z_{out} low)
current i_s	voltage v_o	shunt-shunt	Transimpedance amplifier Z_f (Z_{in} low, Z_{out} low)
voltage v_s	current i_o	series-series	Transadmittance amplifier Y_f (Z_{in} high, Z_{out} high)
current i_s	current i_o	shunt-series	Current amplifier A_{if} (Z_{in} low, Z_{out} high)

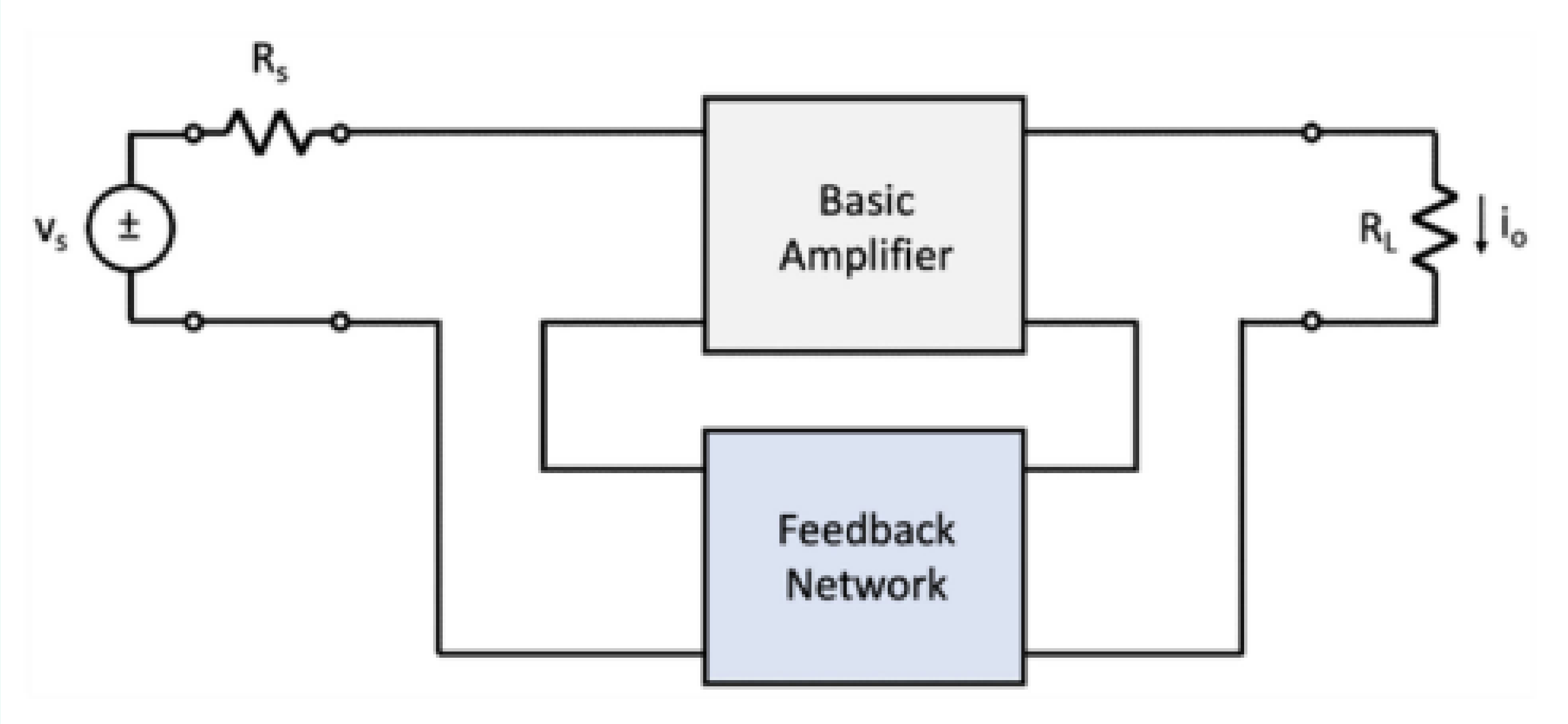
Series-shunt (series-parallel) feedback topology

- Feedback voltage amplifier (voltage-mixing, voltage sampling)



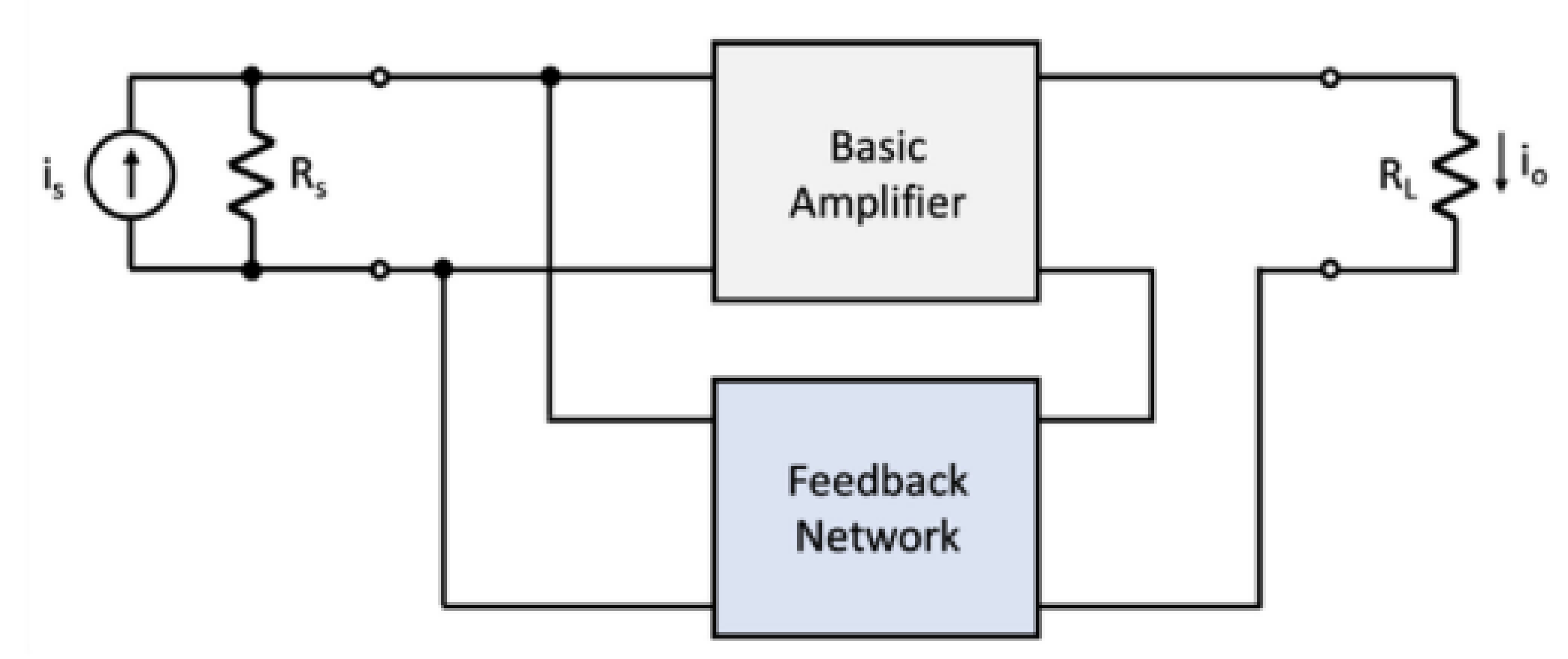
Series-series feedback

- Transadmittance amplifier (voltage-mixing/current sampling)



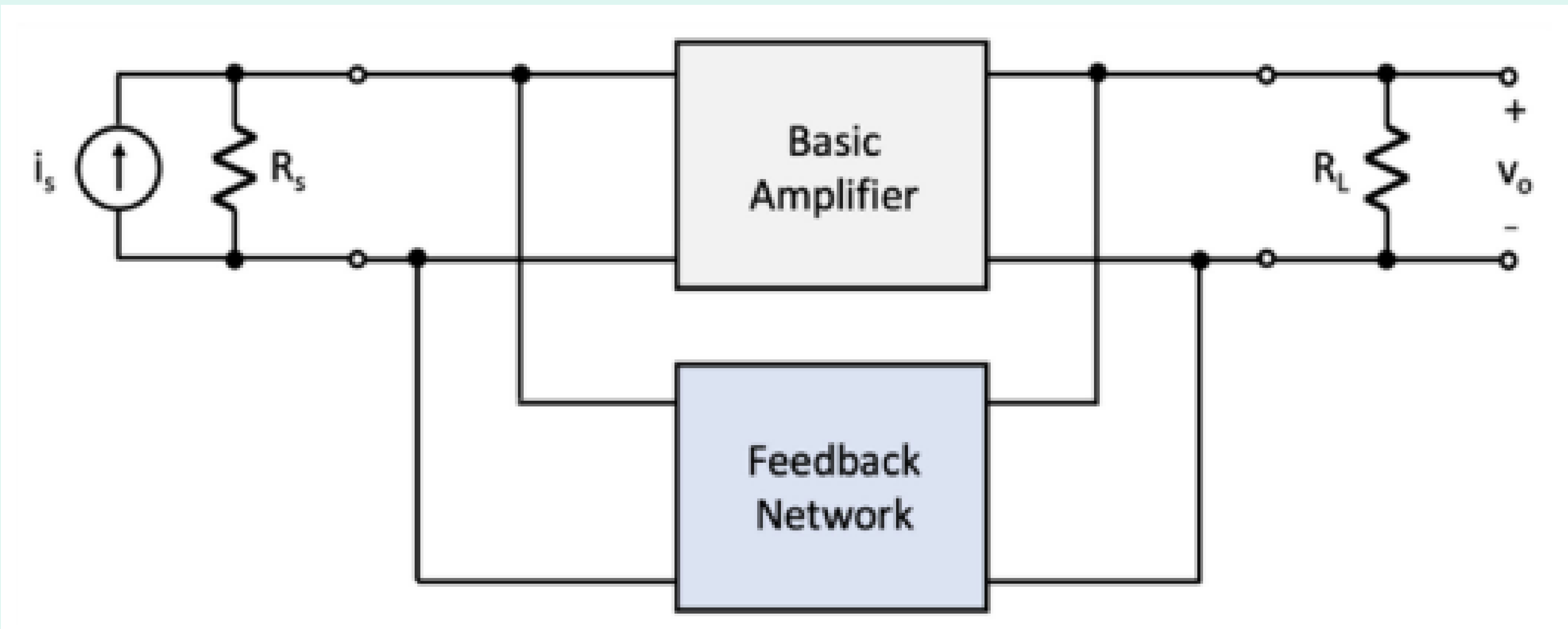
Shunt-series feedback

- Current amplifier - current-mixing/current sampling



Shunt-shunt feedback

- Transimpedance amplifier (current-mixing/voltage-sampling)



L21 Q01 Feedback amplifier

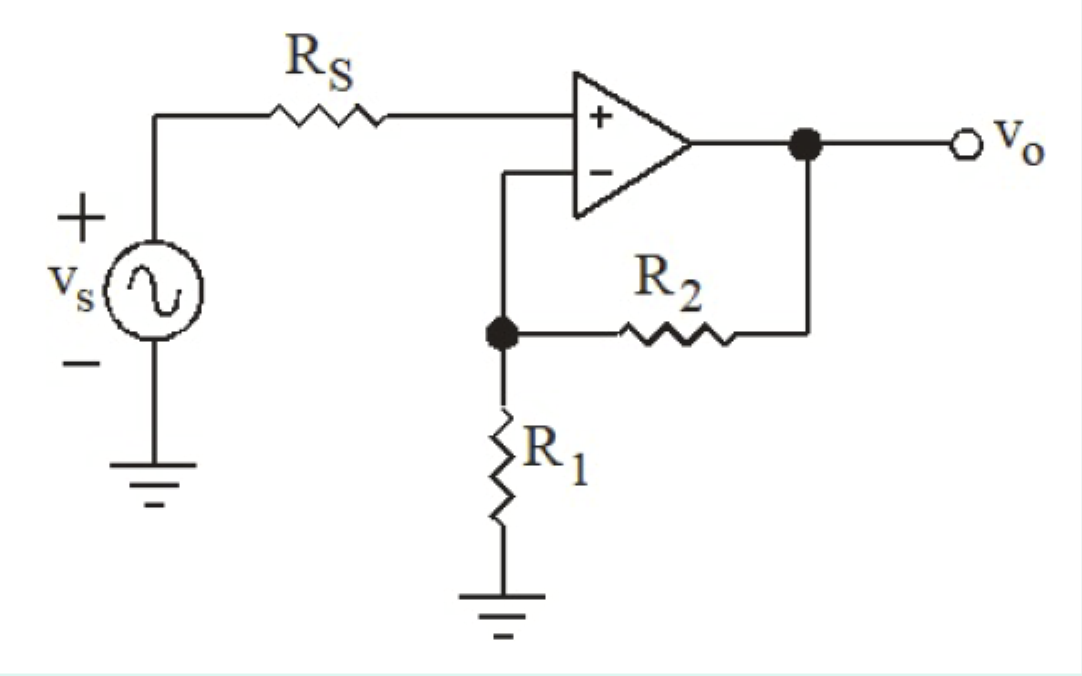
- What type of feedback the following circuit has?

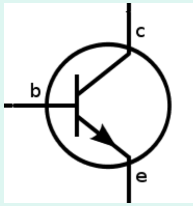
A. series-series

B. series-shunt

C. shunt-series

D. shunt-shunt





L21 Q02 inverting opamp

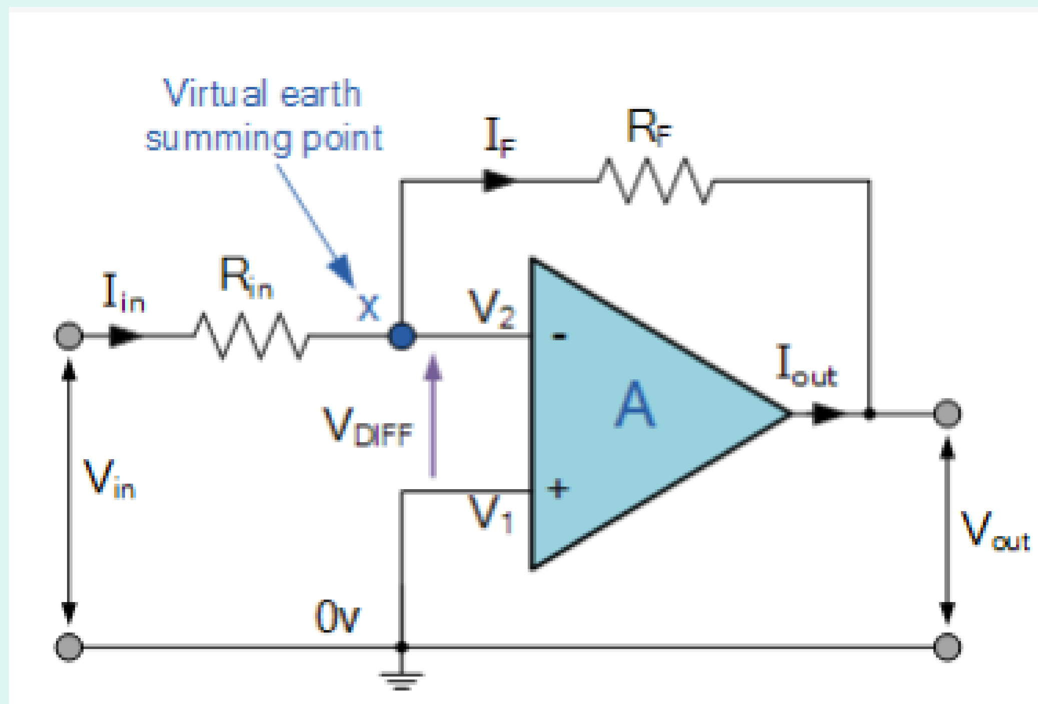
- What feedback topology is present in the following circuit?

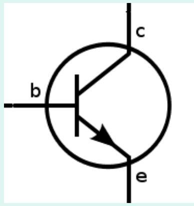
A. series-series

B. series-shunt

C. shunt-series

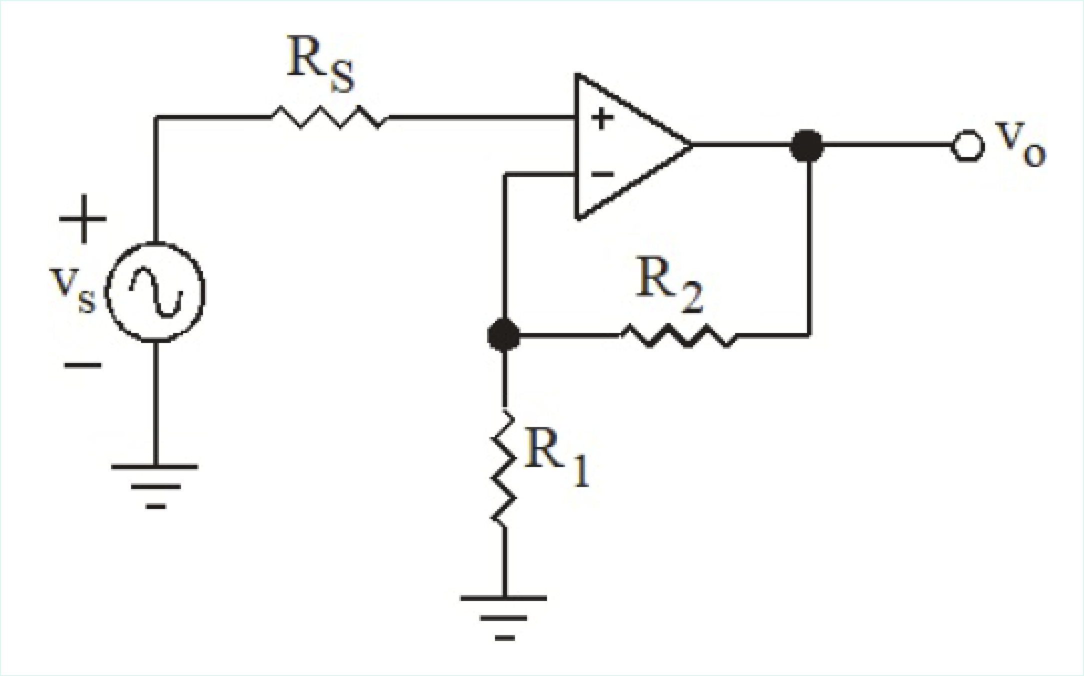
D. shunt-shunt





Circuit example

- (a) For $A=\infty$, find an expression for A_f



$$v_+ = v_s, v_- = \frac{R_1}{R_1+R_2} v_o$$

$$A_f = \frac{v_o}{v_s} = \frac{v_- \dfrac{R_1+R_2}{R_1}}{v_+} = \frac{R_1+R_2}{R_1} = \frac{1}{\beta}$$

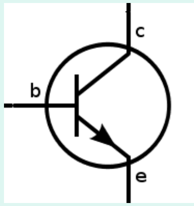
$$\beta = \frac{R_1}{R_1+R_2}$$

(b) For $A=10^4$, find the value of R_2/R_1 for which A_f equals 100

$$100 = \frac{A}{1+A\beta} \Rightarrow \beta = \frac{\dfrac{A}{100}-1}{A} = \frac{1}{100} - \frac{1}{A}$$

$$\frac{R_1}{R_1+R_2} = \frac{1}{100} - \frac{1}{10000} = \frac{99}{10000}$$

$$\frac{1}{1+\dfrac{R_2}{R_1}} = \frac{99}{10000} \Rightarrow \frac{R_2}{R_1} = \frac{10000}{99} - 1 = 100.01$$



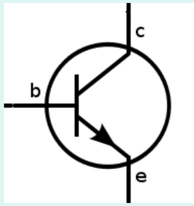
Circuit example (2)

- (c) Parameter variation: What happens to A_f if A decreases by 25%?

$$A_f = \frac{A}{1+A\beta} = \frac{7500}{1+\dfrac{7500}{1+100/01}} = 99.67$$

(d) The amount of feedback for (b)

$$1+A\beta = 1+\frac{10000}{101.01} = 100 \rightarrow 20\log\left(1+A\beta\right) = 40dB$$



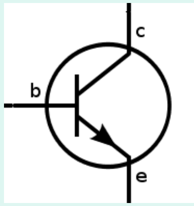
Gain de-sensitivity

- In the previous exm: for A=10000 and β=1/101.01, a 25% change in A gave a 0.33% change in A_f
- In general:

$$A_f = \frac{A}{1+A\beta} \Rightarrow \frac{dA_f}{dA} = \frac{1}{1+A\beta} - \frac{A\beta}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2}$$

$$\frac{dA_f}{A_f} = \frac{1+A\beta}{A}\frac{dA}{(1+A\beta)^2} = \frac{1}{1+A\beta}\frac{dA}{A}$$

The amount of feedback (1+Aβ) is also called the **de-sensitivity factor**



Bandwidth extension

- We consider a single-pole LP amplifier (open-loop gain):

$$A(s) = A_M \frac{1}{1+\frac{s}{\omega_H}}$$

The closed-loop gain: extended BW

$$A_f(s) = \frac{A(s)}{1+A(s)\beta} = \frac{\frac{A_M}{1+\frac{s}{\omega_H}}}{1+\frac{A_M}{1+\frac{s}{\omega_H}}\beta} = \frac{A_M}{1+\frac{s}{\omega_H}+A_M\beta}$$

$$A_f(s) = \underbrace{\frac{A_M}{1+A_M\beta}}_{A_{Mf}} \frac{1}{1+\frac{s}{\omega_H(1+A_M\beta)}}$$

$$A_{Mf} = \frac{A_M}{1+A_M\beta}, \omega_{Hf} = \omega_H(1+A_M\beta)$$

Bandwidth extension (2)

- The case of a high-pass single-pole amplifier

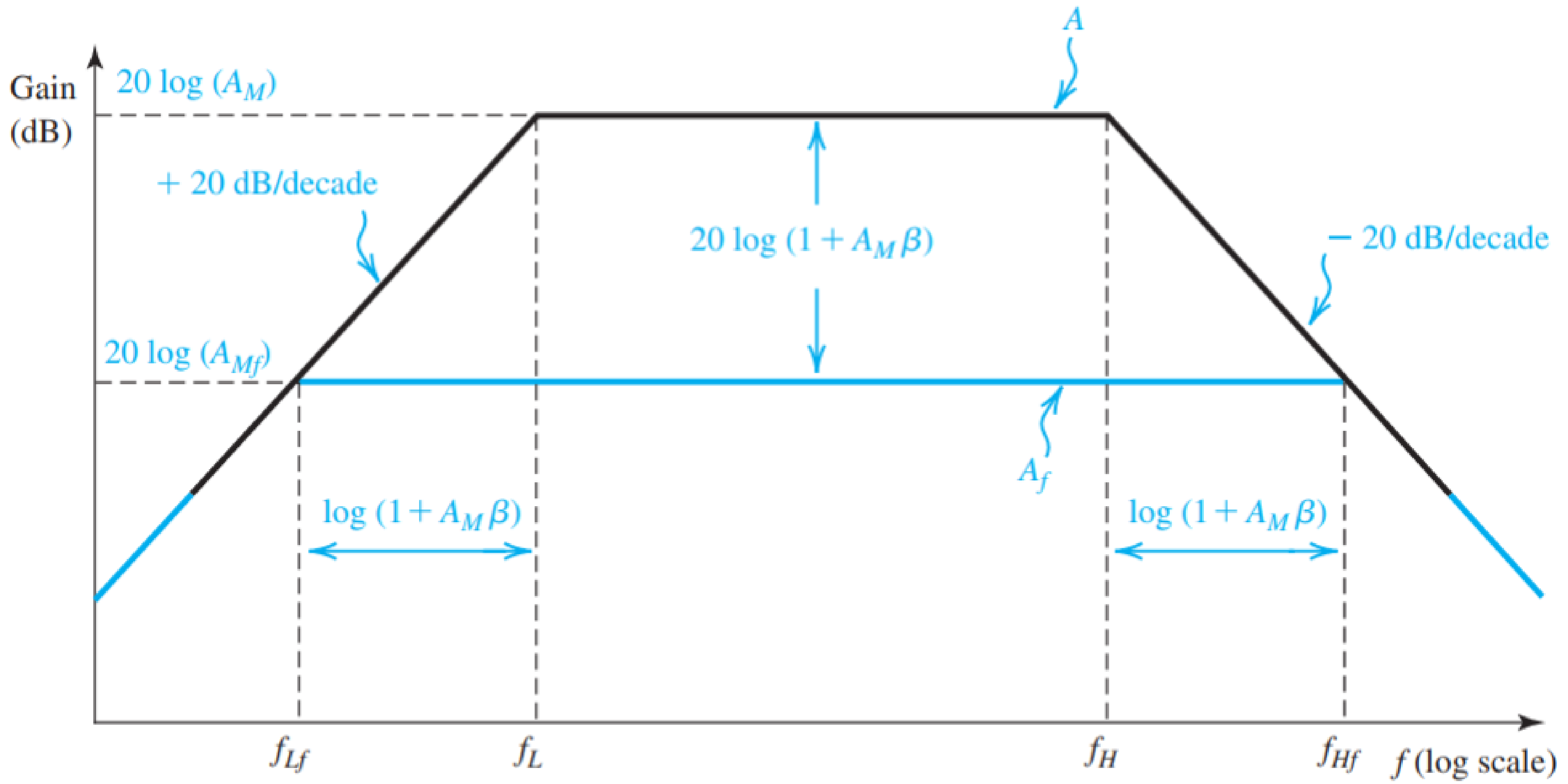
$$A(s) = A_M \frac{s}{s+\omega_L}$$

The closed-loop gain: again, the BW is extended

$$A_f(s) = \frac{A(s)}{1+A(s)\beta} = \frac{A_M \dfrac{s}{s+\omega_L}}{1+A_M\beta \dfrac{s}{s+\omega_L}} = \underbrace{\frac{A_M}{1+A_M\beta}}_{A_{Mf}} \frac{s}{s+\underbrace{\dfrac{\omega_L}{1+A_M\beta}}_{\omega_{Lf}}}$$

Bandwidth extension (3)

- Negative feedback => A_M reduces, f_H increases, f_L reduces by the same $(1+A_M\beta)$ factor

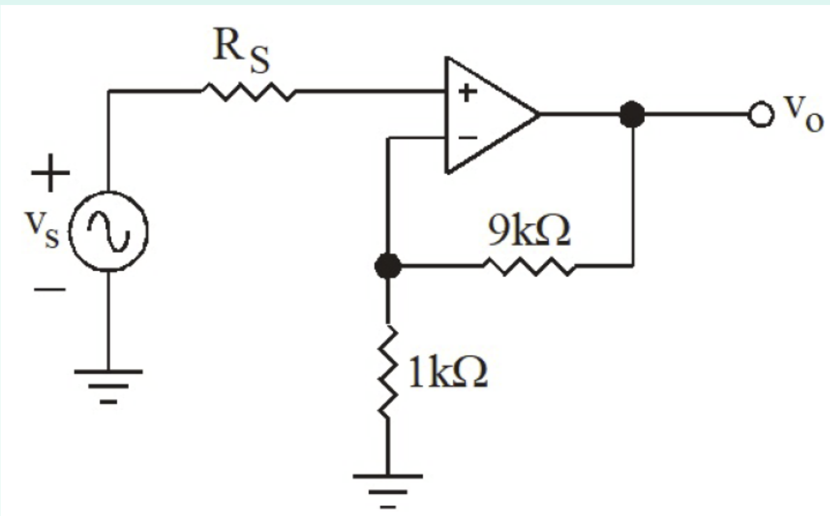


$$f_{Lf} = \frac{f_L}{1 + A_M\beta} \qquad A_{Mf} = \frac{A_M}{1 + A_M\beta} \qquad f_{Hf} = f_H(1 + A_M\beta)$$

(Source: Sedra and Smith[2014]Microelectronic circuits)

Example

- Assume A=10^6, 10Hz bandwidth and a roll-off of -20dB/decade (single-pole LP)



$$A(s)=\frac{A_M}{1+\frac{s}{\omega_H}}=\frac{10^6}{1+\frac{s}{2\pi 10}}$$

$$A_{fM}=\frac{v_o}{v_s}=1+\frac{9k\Omega}{1k\Omega}=10\approx\frac{1}{\beta}$$

$$A_f(s)=\frac{A_M}{1+A_M\beta}\frac{1}{1+\frac{s}{\omega_H\left(1+A_M\beta\right)}}=\frac{10^6}{1+0.1\cdot 10^6}\frac{1}{1+\frac{s}{2\pi 10\left(1+0.1\cdot 10^6\right)}}$$

$$A_{Mf}=\frac{10^6}{1+0.1\cdot 10^6}\approx 9.9999,\ f_{Hf}=10\left(1+0.1\cdot 10^6\right)=1.00001MHz$$

Nonlinear distortion reduction

- An open-loop amplifier with a piecewise linear transfer function, and feedback gain β=0.05

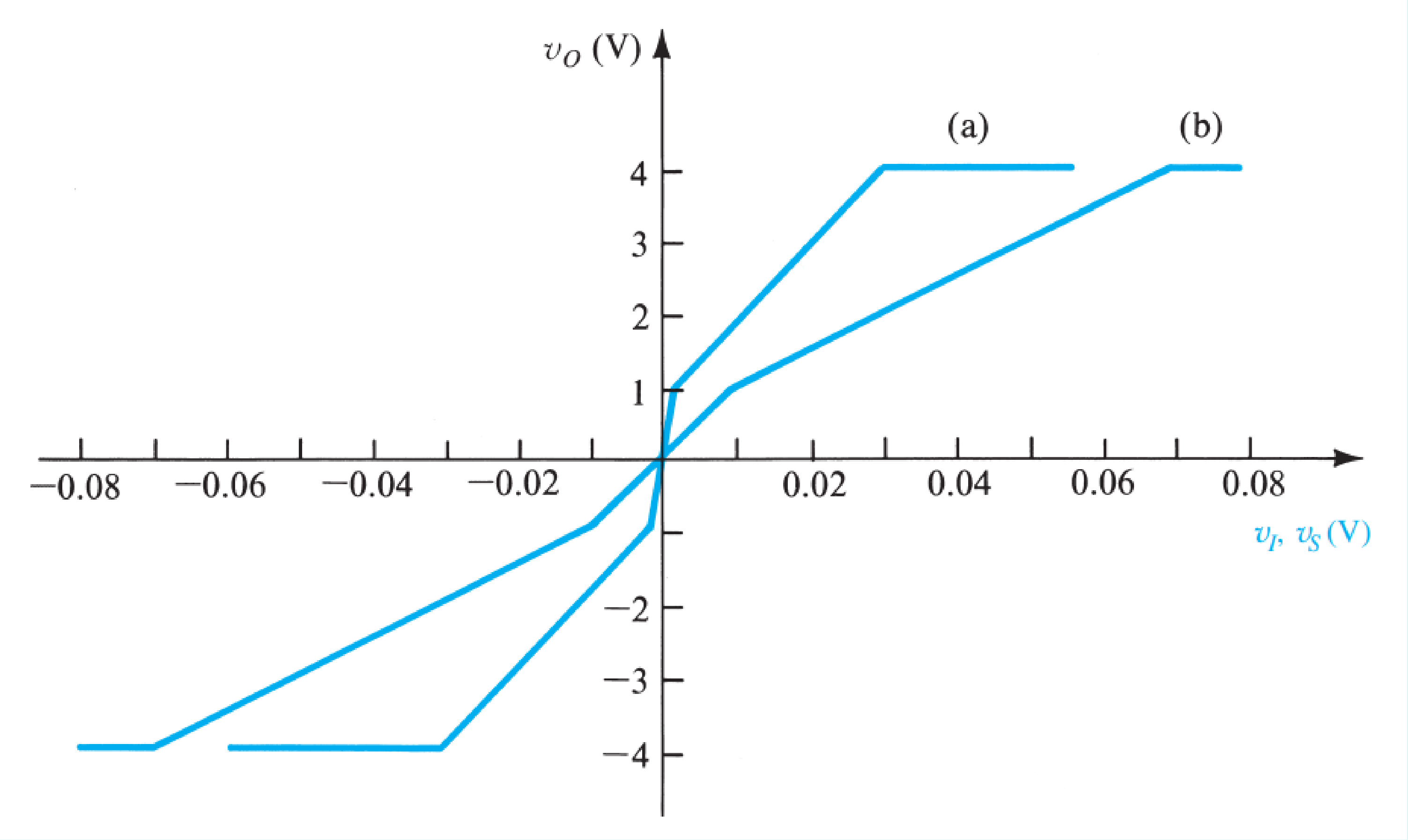
$$A=\begin{cases}10^4 & if \quad 0\le |v_o|<8V\\ 10^3 & if \quad 8V\le |v_o|<11V\\ 10^2 & if \quad 11V\le |v_o|<14V\\ 0 & if \quad 14V\le |v_o|\end{cases} \Rightarrow A_f=\begin{cases}\dfrac{10^4}{1+0.05\cdot 10^4}=19.96 & if \quad 0\le |v_o|<8V\\ \dfrac{10^3}{1+0.05\cdot 10^3}=19.61 & if \quad 8V\le |v_o|<11V\\ \dfrac{10^2}{1+0.05\cdot 10^2}=16.67 & if \quad 11V\le |v_o|<14V\\ 0 & if \quad 14V\le |v_o|\end{cases}$$

$$\mathrm{A}_f=\frac{A}{1+A\beta}=\frac{1}{\beta+\dfrac{1}{A}}=\frac{1}{\beta}\frac{1}{1+\dfrac{1}{A\beta}}$$

The differences in the slopes between regions (1)-(3) have been much reduced through the feedback action

Distortion reduction (2)

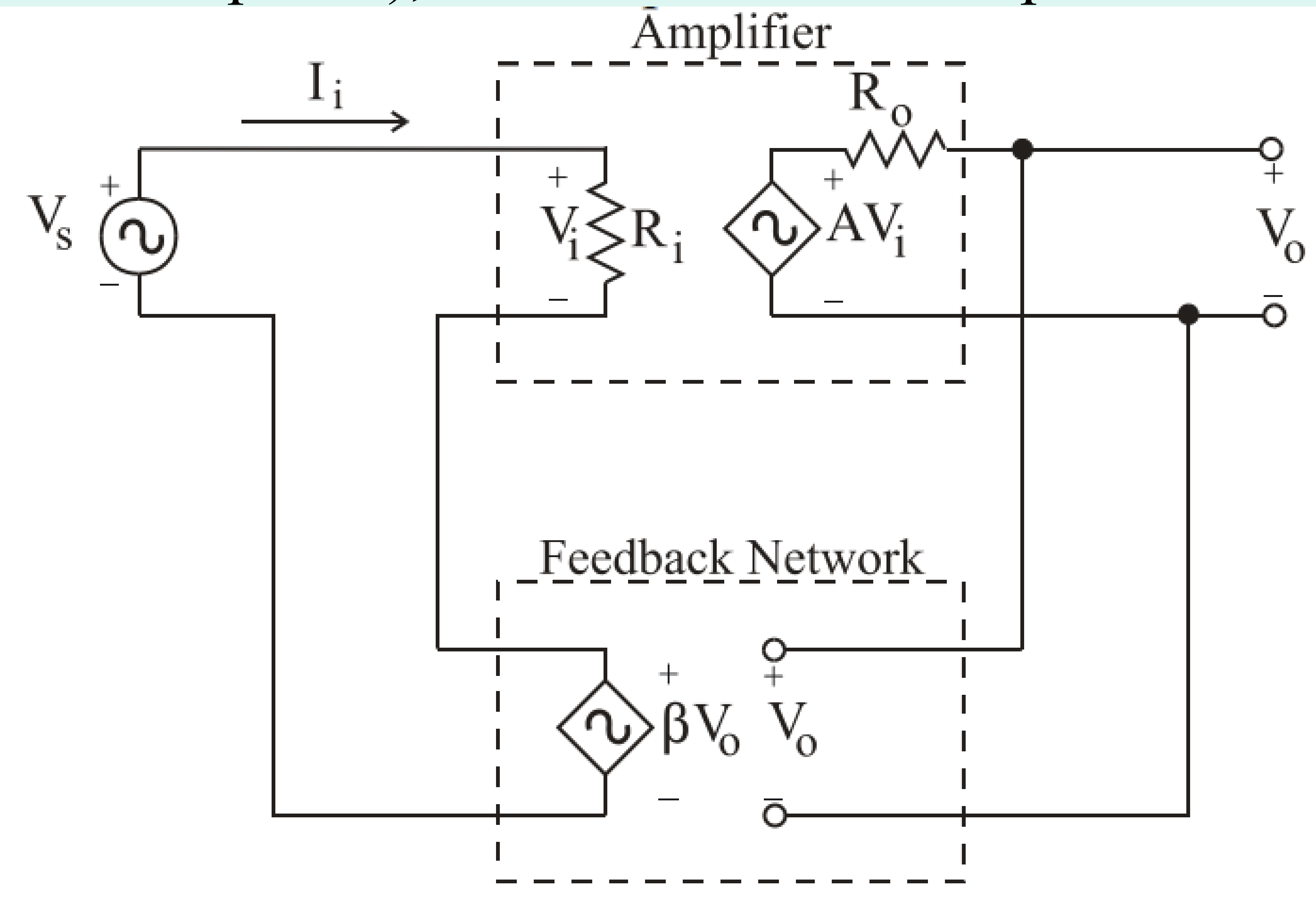
- The amplifier transfer characteristics can be linearized (made less nonlinear) through negative feedback



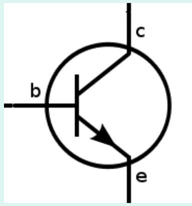
(a) without feedback; (b) with negative feedback (β=0.01)

Feedback effect on impedances

- Consider the series-shunt feedback amplifier (voltage amplifier), with ideal feedback diport

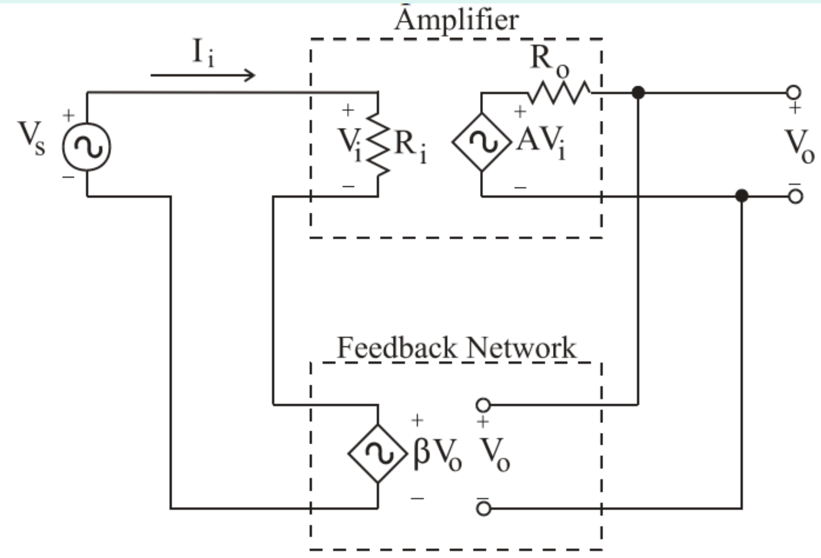


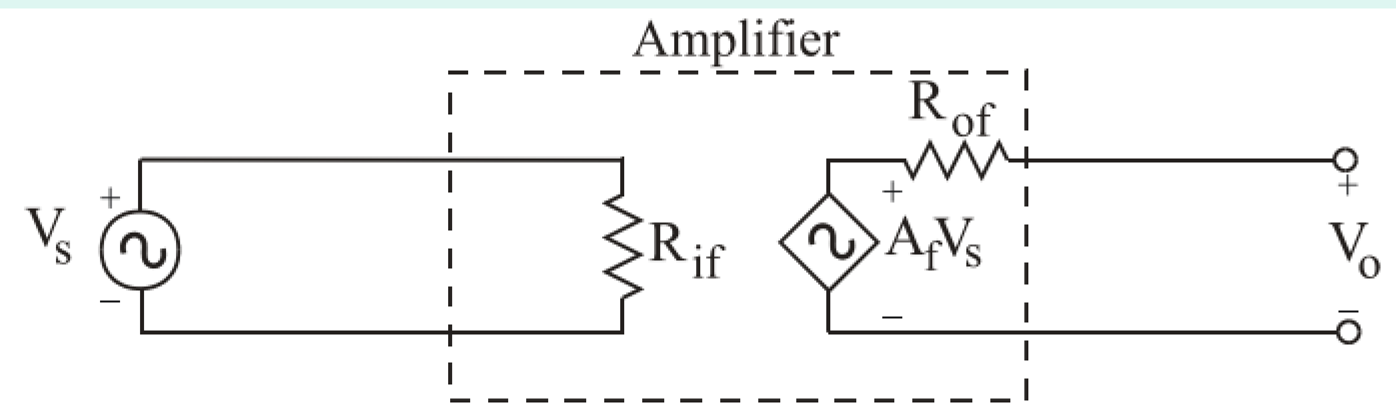
$$A_{vf} = \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$



Voltage amplifier

- equivalent diport model

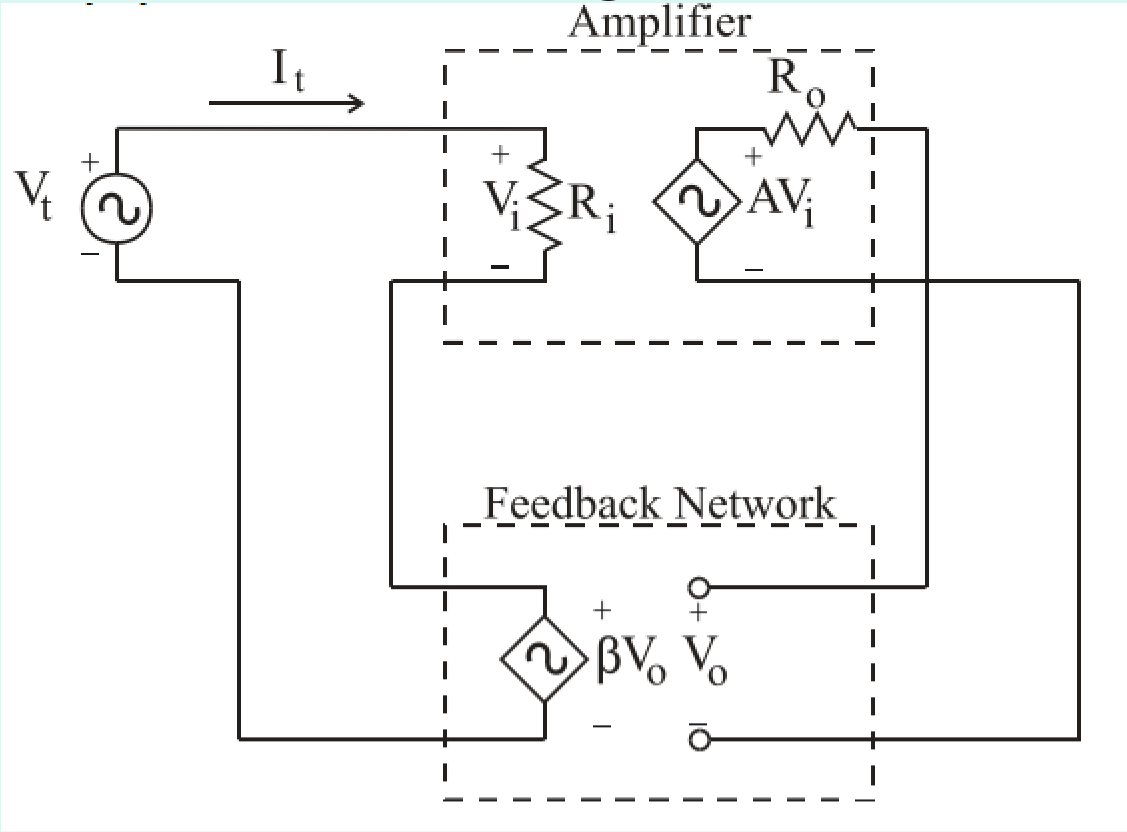




$R_{if}, R_{of} = ?$

Finding R_{if}

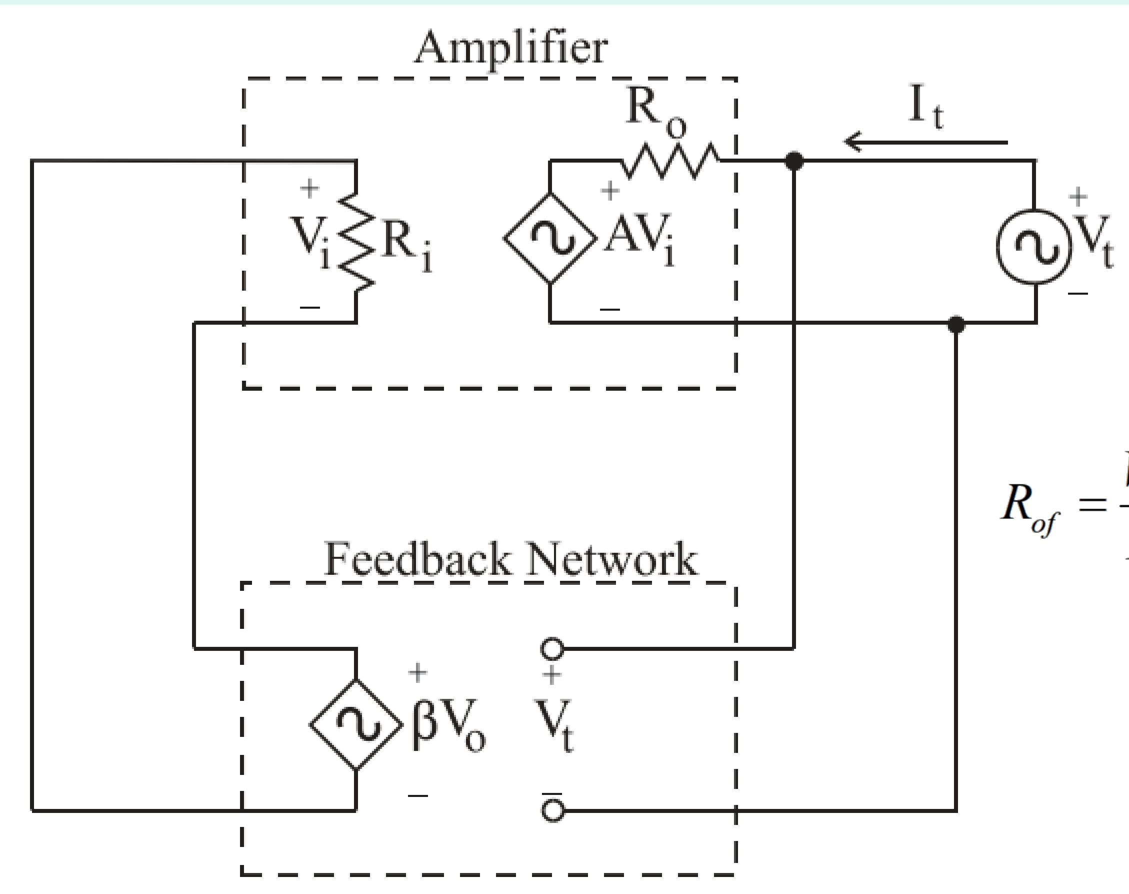
- Replace source with a test voltage source



$$R_{if} = \frac{V_t}{I_t} = \frac{V_t}{\frac{V_i}{R_i}} = R_i \frac{V_i + \beta V_o}{V_i} = R_i \left(1 + A\beta\right)$$

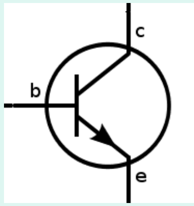
Finding R_{of}

- Apply a test voltage source at the output



$$R_{of} = \frac{V_t}{I_t} = \frac{V_t}{\dfrac{V_t - AV_i}{R_o}} = R_o \frac{V_t}{V_t + A\beta V_t} = \frac{R_o}{1 + A\beta}$$

Concl: negative feedback brings the diport closer to an ideal diport (voltage amplifier)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