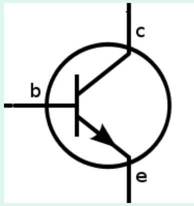
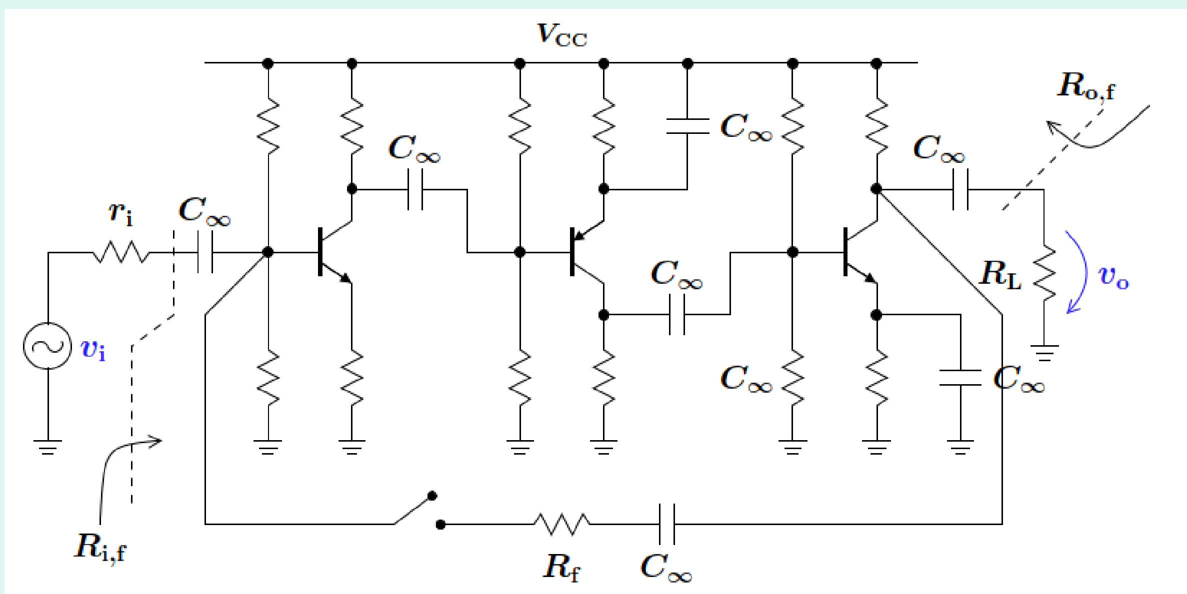


ELEC 301 - Feedback circuit analysis. Loop-gain

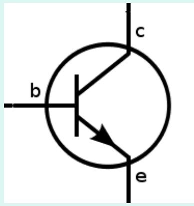
L24 - Nov 04

Instructor: Edmond Cretu



Last time

- The analysis of circuits with non-ideal feedback
- Choosing the right type of diport representation for the feedback network + loading the amplifier with the parameters of the feedback network
- Assumptions: the open-loop amplifier only transmits signal in the forward direction (only passive input port R_i), the feedback network only transmits signals in the reverse direction (we neglected h_{21} parameter)



L24 Q01 - Real feedback

- Assume that we have a non-ideal amplifier in a feedback loop with a non-ideal feedback network. How is the closed-loop gain of the system?

A. Higher than $1/\beta$

B. Smaller than $1/\beta$

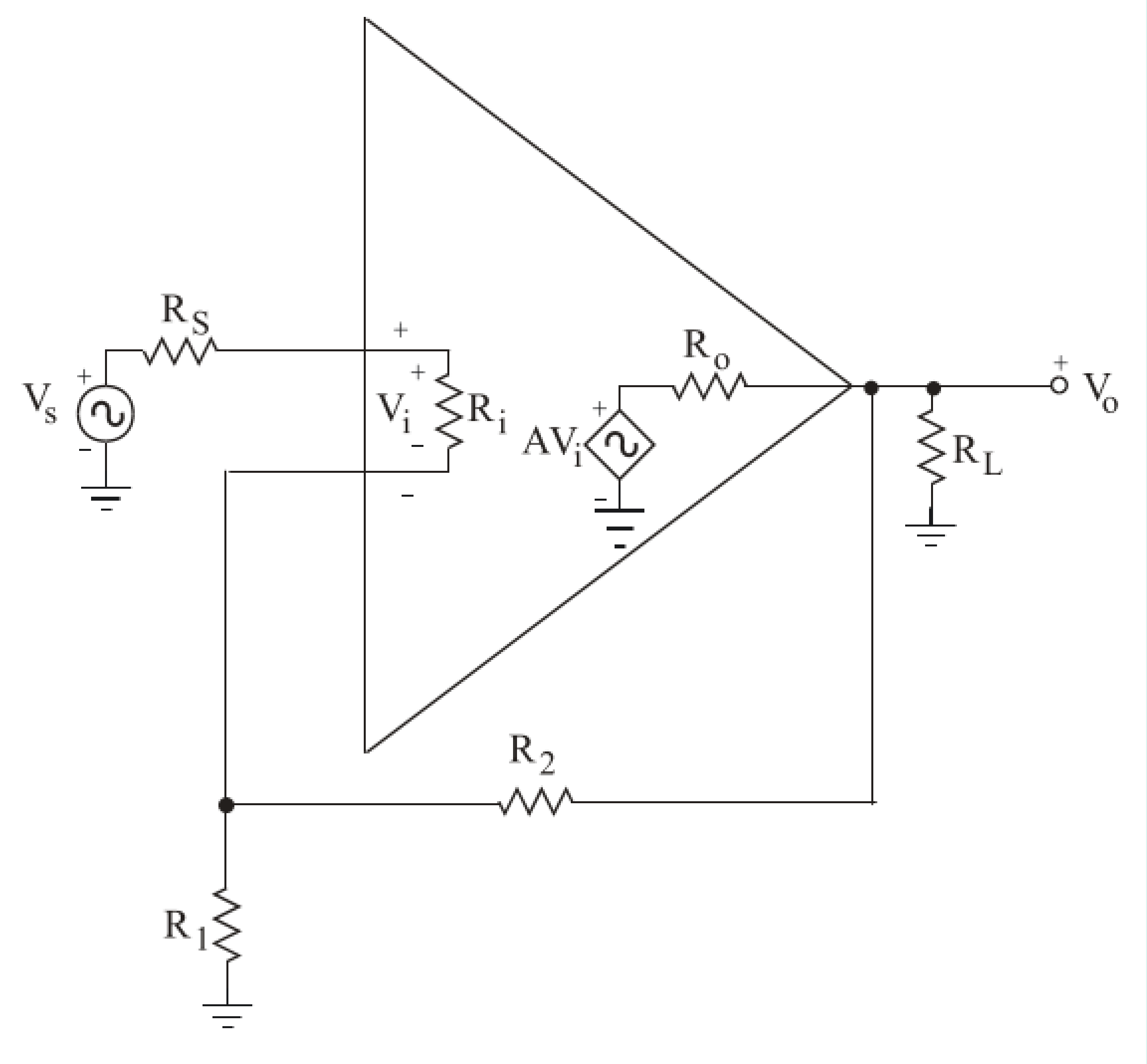
C. Equal to $1/\beta$

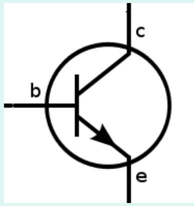
Systematic approach

- Procedure:
  1. Identify the forward path, the feedback path (A and β subsystems), and the type of feedback
  2. Build the equivalent diport model for the feedback network (usually neglect the forward path parameter)
  3. Transform the feedback network into an ideal one, by loading the forward path
  4. Compute A', R_i', R_o' (equivalent diport parameters for the loaded amplifier, neglecting the inner feedback path) and β
  5. Apply (ideal) feedback formulas to find A_f', R_{if}', R_{of}'

Example: non-inverting amplifier

- Introduce non-idealities: R_i, R_o

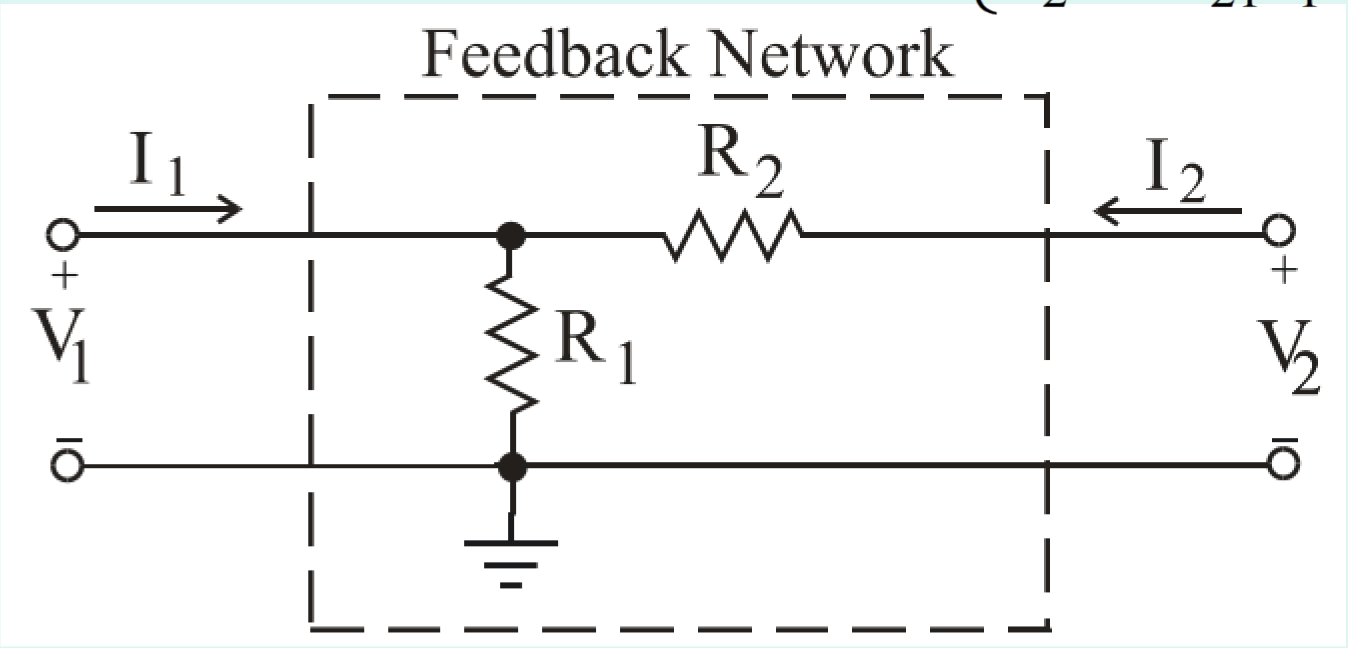


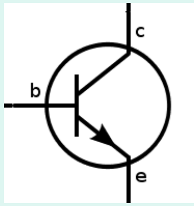


Feedback network

- Series-shunt topology => h-parameters
- Port (1) V1-dependent, I1-independent variable
- Port (2) V2- independent, I2 - dependent variable
- $\beta = V_1/V_2|_{I1=0} = h_{12}$

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$





L24 Q02 h22

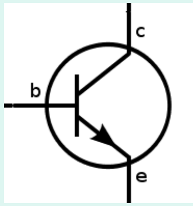
- What is the interpretation of the h_{22} parameter?

A. The input resistance at port (1) of the feedback network when (2) is SC

B. Output resistance at port (2) when (1) is OC

C. Output admittance at port (2) when (1) is OC

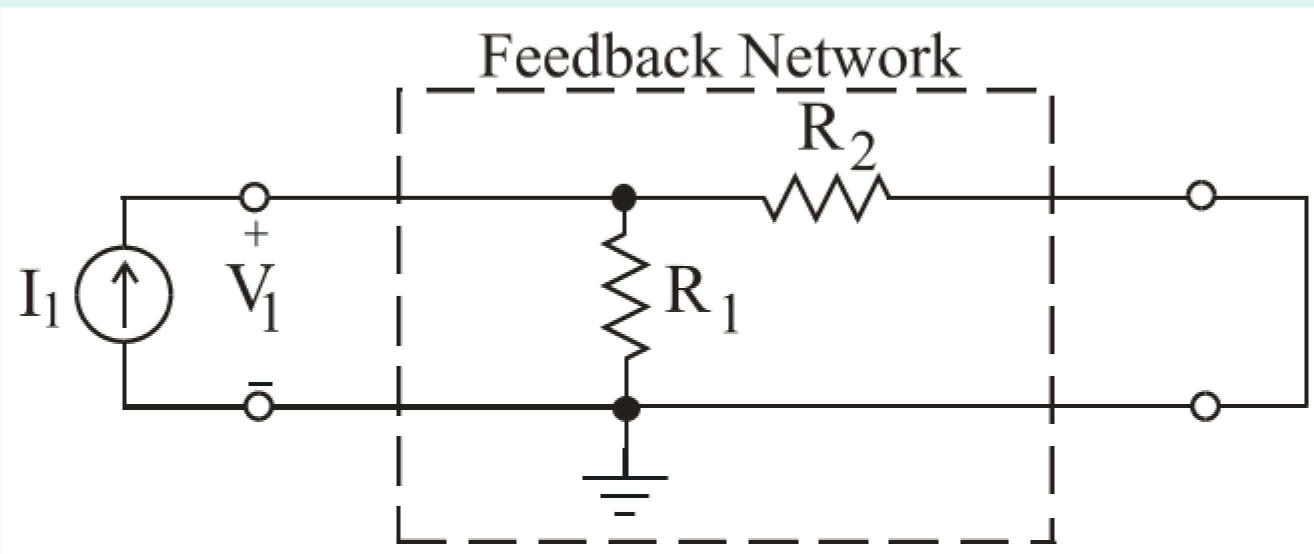
D. Output admittance at port (2) when (1) is SC

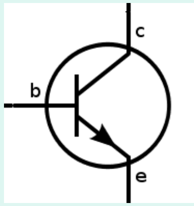


Find h_{11}

- h_{11} = input resistance of the diport

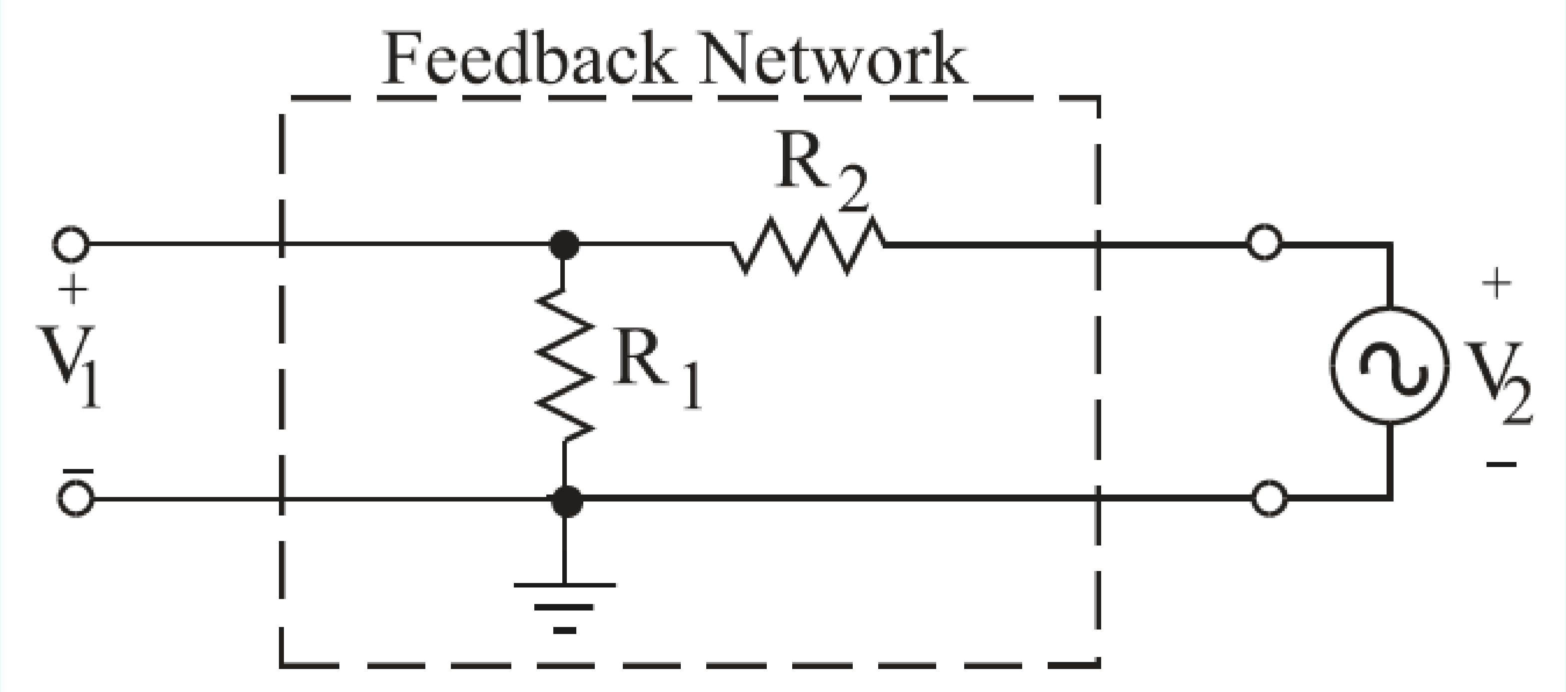
$$R_{i\beta} = h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 \parallel R_2$$





Find h12

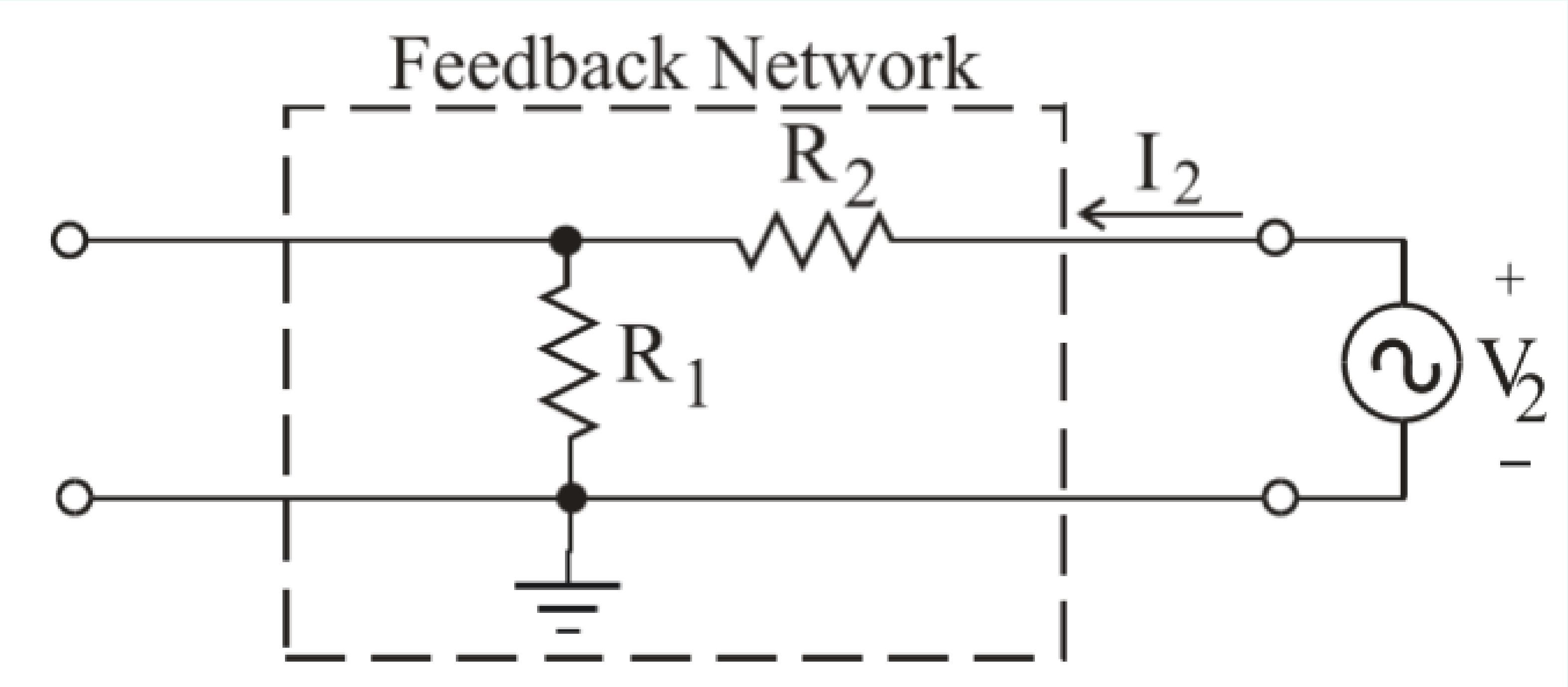
- $\beta = h_{12}$ = feedback amount



$$\beta = h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + \frac{R_2}{R_1}}$$

Find h22

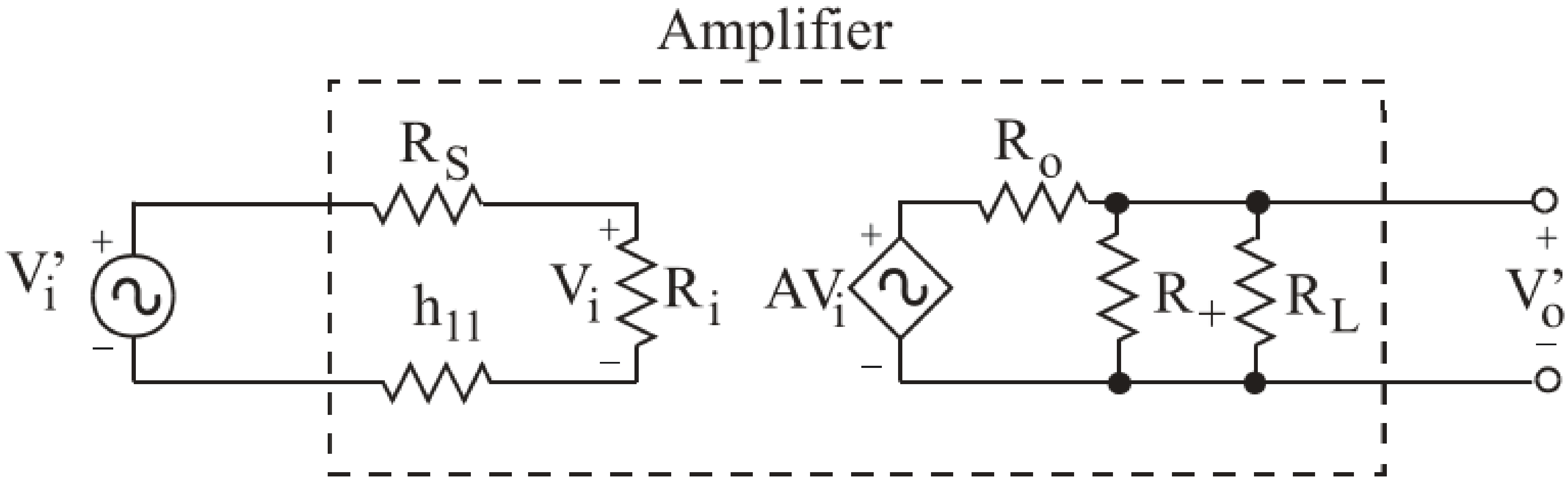
- h_{22}=output admittance



$$\frac{1}{R_{o\beta}} = h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_1 + R_2}$$

Modified amplifier + ideal feedback

- We absorb all non-ideal components of the feedback network into the amplifier

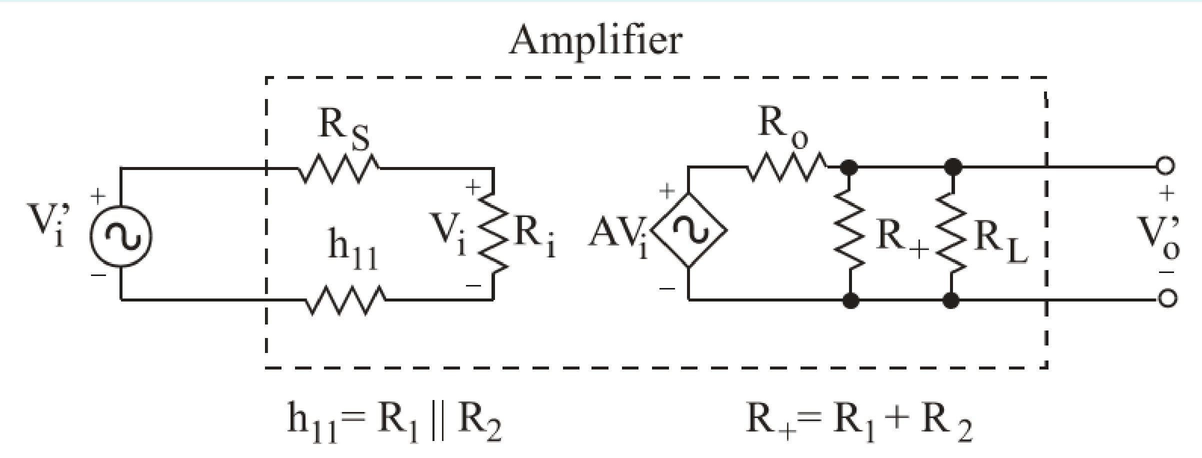


$h_{11} = R_1 \parallel R_2$ $\qquad$ $R_+ = R_1 + R_2$

- The feedback network has become an ideal one

Modified gain computation

- We compute the gain for the "loaded" forward path

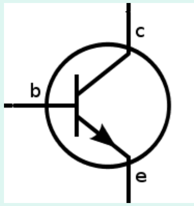


$$A' = \frac{V_o'}{V_i'} = \frac{AV_i \dfrac{R_L \| (R_1 + R_2)}{R_o + R_L \| (R_1 + R_2)}}{V_i'}$$

$$A' = A \frac{R_i}{R_s + R_i + h_{11}} \frac{R_L \| (R_1 + R_2)}{R_o + R_L \| (R_1 + R_2)}$$

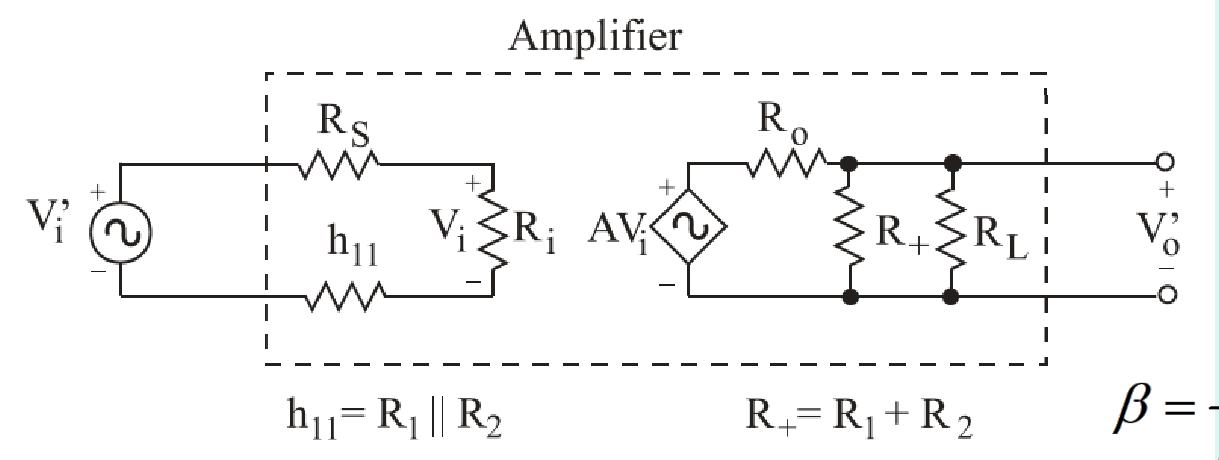
$$A' = A \frac{R_i}{R_s + R_i + R_1 \| R_2} \frac{R_L \| (R_1 + R_2)}{R_o + R_L \| (R_1 + R_2)}$$

$$\begin{cases} A = 10^4, R_i = 10k\Omega, R_o = 100\Omega \\ R_1 = 1k\Omega, R_2 = 9k\Omega, R_s = 1k\Omega \end{cases} \Rightarrow A' = 7571$$



Ideal vs. real voltage amplifier

- Compare the ideal case approximation with the real circuit:

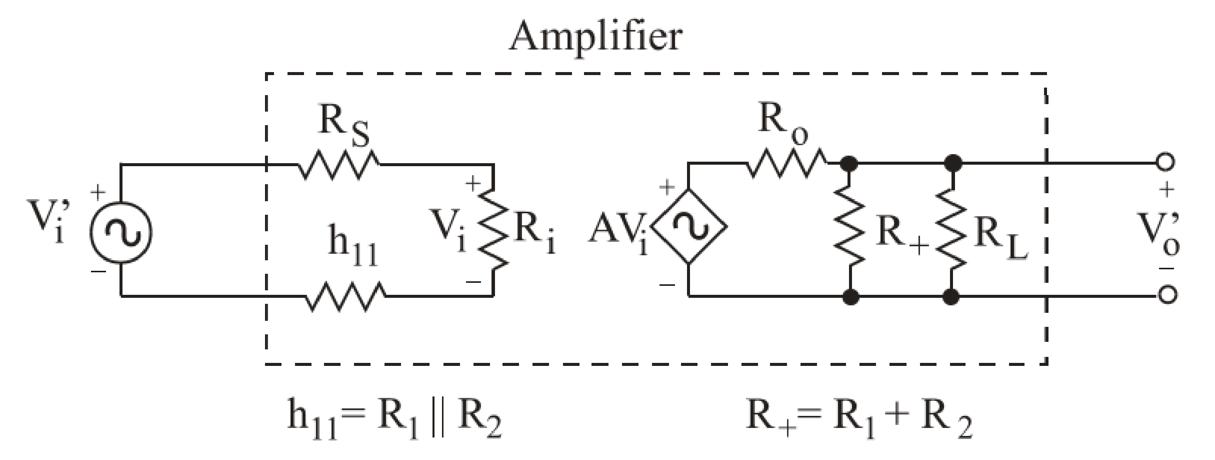


$$\beta = \frac{R_1}{R_1 + R_2} = 0.1 \Rightarrow A_{ideal} \approx \frac{1}{\beta} = 10$$

$$A_f = \frac{A'}{1 + A'\beta} = \frac{7571}{758.1} = 9.99$$

Input and output resistances

- The influence of feedback on equivalent resistances:



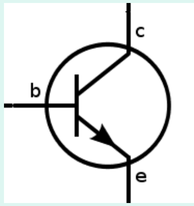
$$R_{if} = R_i'\left(1 + A'\beta\right) = \left(R_s + R_i + R_1 \,||\, R_2\right)\left(1 + A'\beta\right) = 11.9k\Omega \cdot 758.1 = 9.021M\Omega$$

$$R_{in} = R_{if} - R_s = 9.02M\Omega$$

R_{in} = Input resistance without the source resistance
R_{out} = output resistance without the load resistance

$$R_{of} = \frac{R_o'}{1 + A'\beta} = \frac{R_o \,||\, R_L \,||\, \left(R_1 + R_2\right)}{1 + A'\beta} = 0.119\Omega$$

$$R_{of} = R_{out} \,||\, R_L \Rightarrow \frac{1}{R_{out}} = \frac{1}{R_{of}} - \frac{1}{R_L} \Rightarrow R_{out} = \frac{R_{of}}{1 - \frac{R_{of}}{R_L}} = 0.119\Omega$$



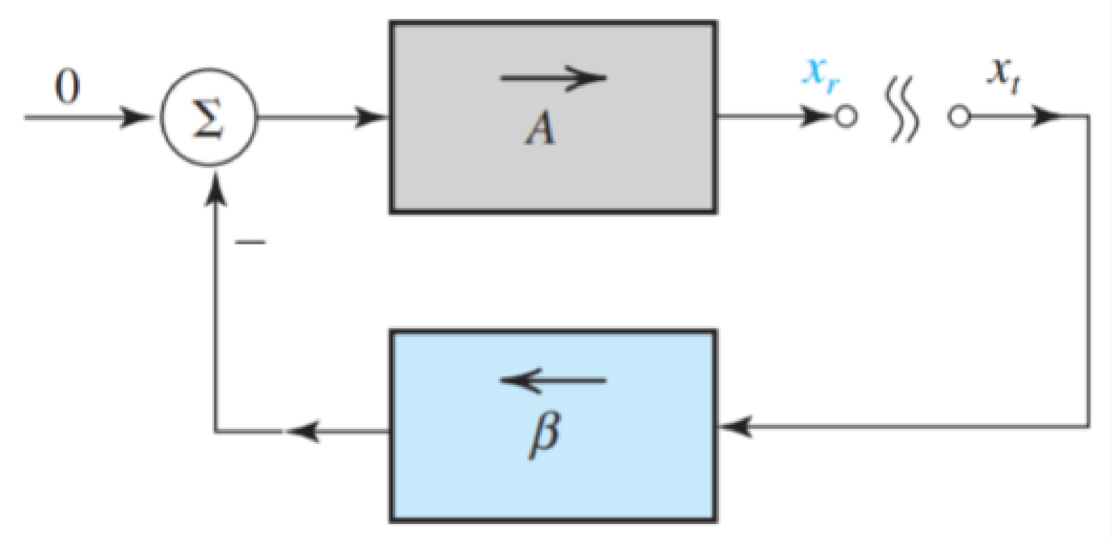
Alternative: Loop-gain method

- We have seen that a non-ideal feedback network may load a basic (non-ideal) amplifier
- Why A and β may have units, the loop gain Aβ is always a number
- Even for a non-ideal feedback network, we can isolate it and determine the feedback factor β
- For computing the total gain, it may be more practical to compute the loop gain A'β, with the condition of loading the amplifier

Loop-gain by breaking the loop

- A practical method is to cut the loop and apply a test signal x_t:

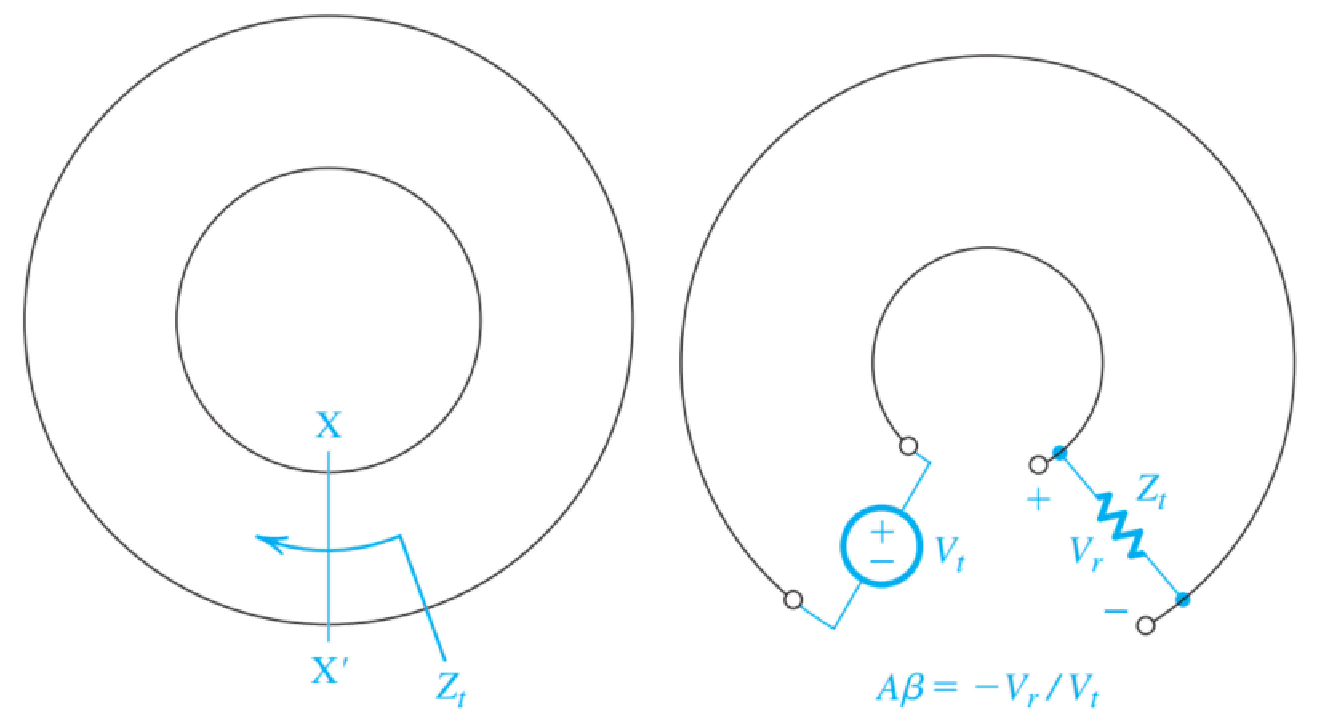
1. Set $x_s=0$
2. Break the feedback loop at a convenient location, ensuring that the values of A and β do not change
3. Apply a test signal x_t to the input of the unrolled loop (at the newly defined break port and determine the returned signal x_r at the unrolled loop output port: $x_r=-A\beta * x_t$



Source: Sedra and Smith[2014] Microelectronic circuits

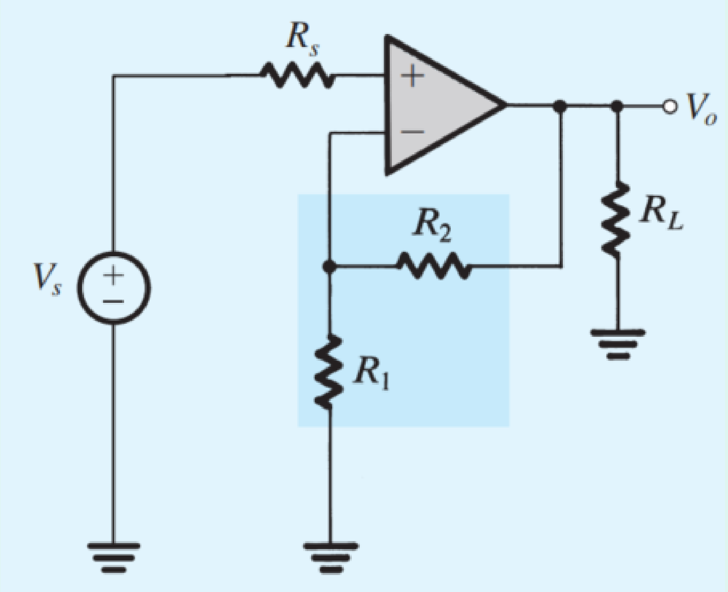
Breaking (cutting) the loop

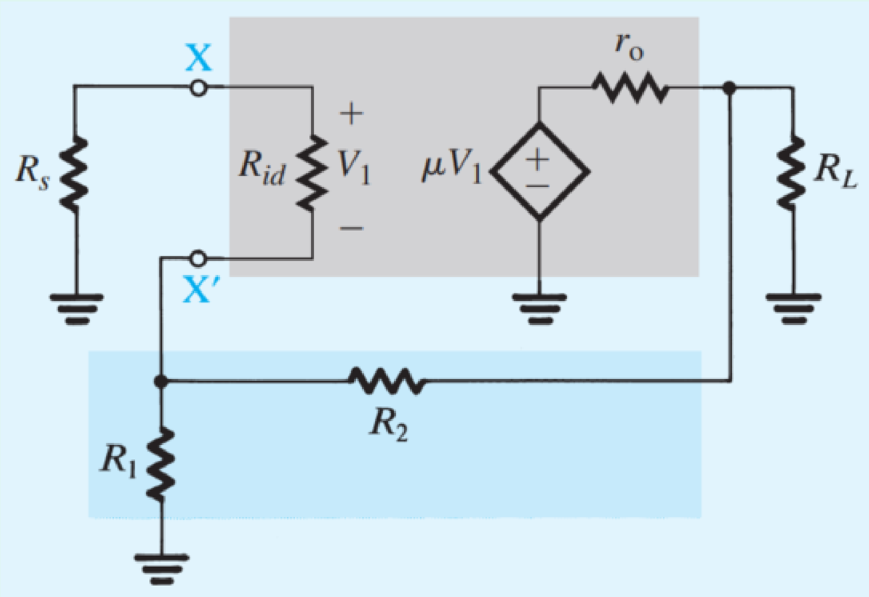
- It is important to ensure that the **loop conditions do not change when cutting it** (same loaded impedances, same biasing point)
- If cutting a loop at X-X' to apply a test voltage V_t to the newly defined left port, the right port must be connected to a loading impedance Z_t, equal to the impedance previously seen looking to the left of XX'.



Exm: non-inverting amplifier

- Loop-gain analysis method:



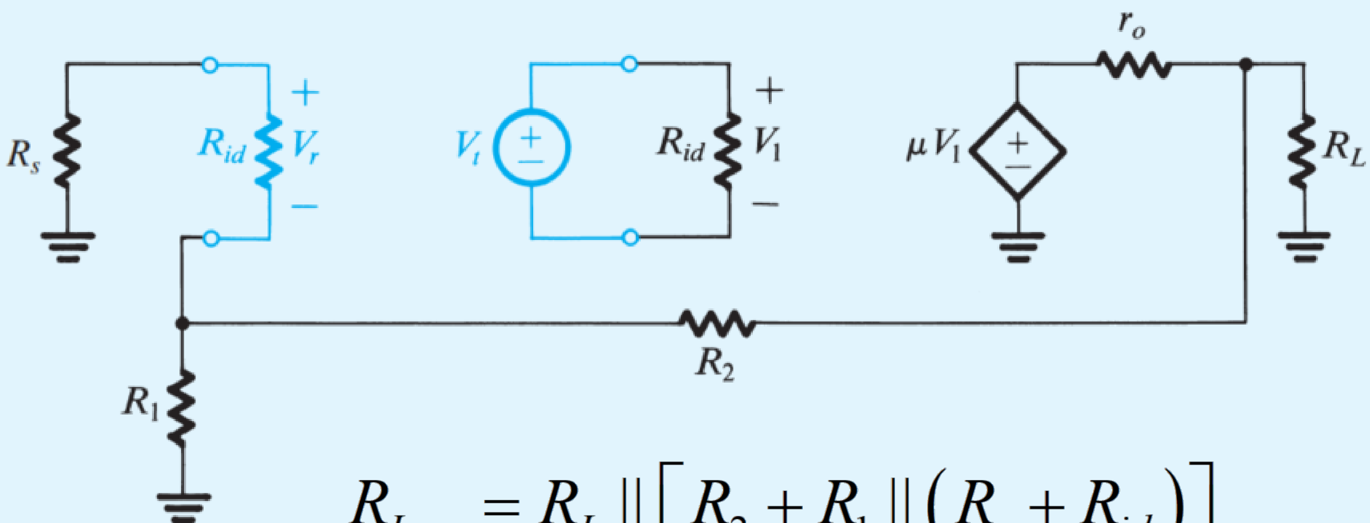


Set V_s=0; Non-ideal amplifier + feedback network

$$\beta = \frac{V_f}{V_o}\bigg|_{I_f=0} = \frac{R_1}{R_1 + R_2} \Rightarrow A_{f,ideal} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

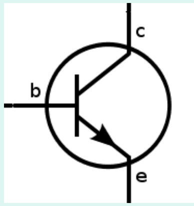
Exm: non-inverting amplifier (2)

- Cut the loop at XX' -apply V_t and preserve loading on the other side of XX' cut (same impedance as seen prior to breaking the loop)
- Homework: apply to the previous problem and compare



$$R_{L,eq} = R_L \,||\, \left[R_2 + R_1 \,||\, \left(R_s + R_{id} \right) \right]$$

$$A\beta = -\frac{V_r}{V_t} = \mu \frac{R_{L,eq}}{R_{L,eq} + r_o} \frac{R_1 \,||\, \left(R_s + R_{id} \right)}{R_1 \,||\, \left(R_s + R_{id} \right) + R_2} \frac{R_{id}}{R_s + R_{id}}$$



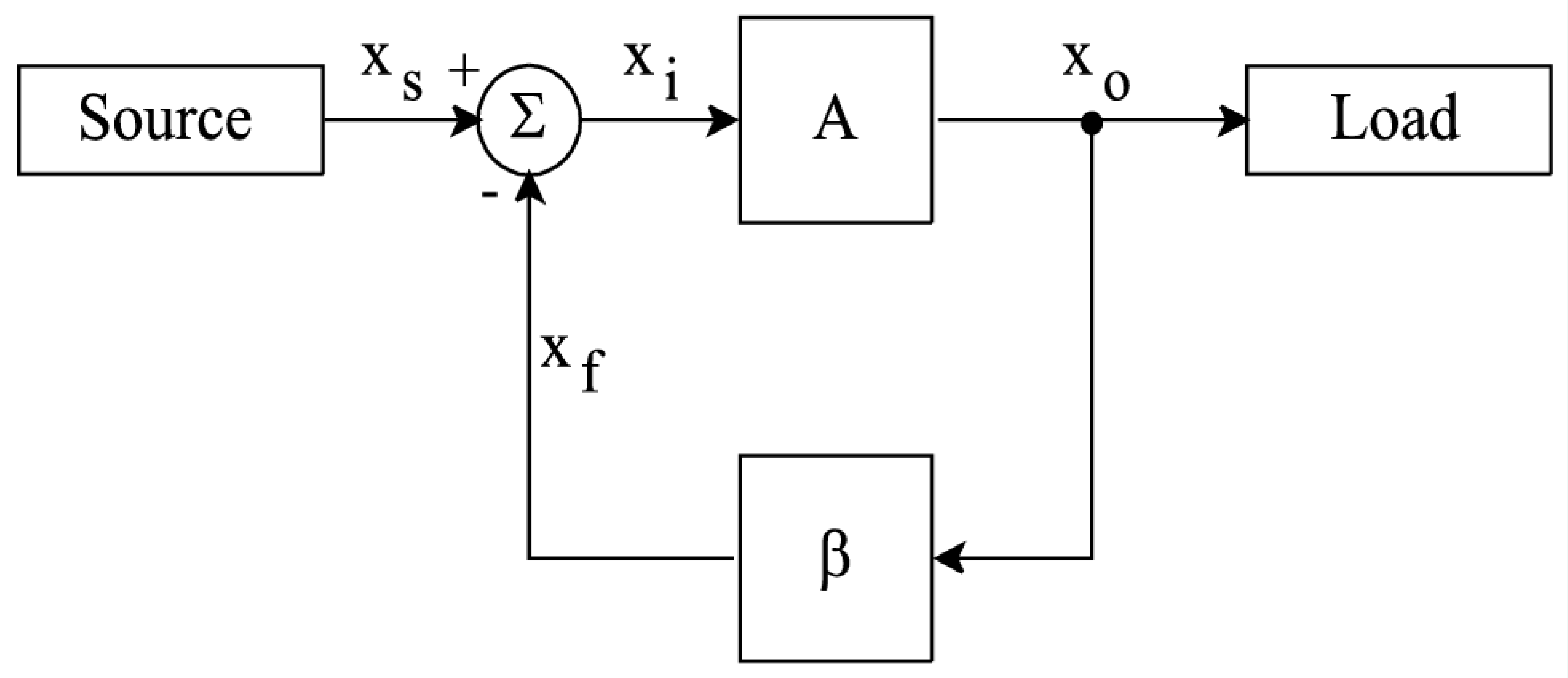
Summary: Loop-gain analysis method

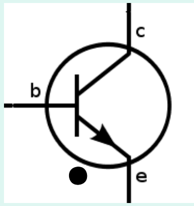
- 4 steps:

1. Identify the feedback network and use it to determine the value of β
2. Determine the ideal value of the closed-loop gain A_f as $1/\beta$ (for $A\beta >> 1$) (This can be used as an initial approximation and for checking with the actual value)
3. Break the loop at a conveninent location and determine the loop gain $A'\beta$. When breaking the loop, care should be taken to not change the conditions in the loop. The loop gain $L = A'\beta = -V_r/V_t$
4. Use the value of $A\beta$ and β to determine the open-loop gain A', and determine the closed-loop gain $A_f' = A'/(1+A'\beta)$

Stability

- Signal flow perspective - at least one pole has the real part positive: $\mathrm{Re}(p_i) \geq 0$
- Feedback modifies the position of the poles => it may cause instability





Stability aspects

Resonance concepts can be often used to describe the excitation of specific modes (poles)

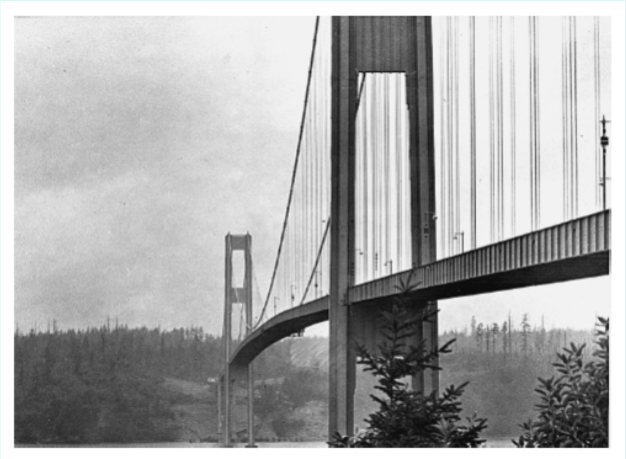
http://www.youtube.com/watch?v=eAXVa__XWZ8 (as few as 160 people could force it into "stride" vs 2000 on opening day)

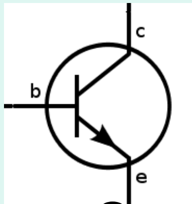
… while others are quite subtle…

Tacoma Narrows bridge: July 1, 1940 – Nov. 7, 1940 on a windy day with *steady, moderate* winds

https://www.youtube.com/watch?v=XggxeuFDaDU

Wind-induced oscillation causes increased amplitude leading to catastrophic failure. Check out "Gallopin' Gertie".





Stability is important!

Several attempts at highly maneuverable fighter aircraft that exhibit static instability ended up in crashes, delays, re-written specifications and cancelled projects.

"Manual control of the X-29 at subsonic speeds is like balancing a one foot long broomstick! Manual over-ride is challenging…"

http://www.youtube.com/watch?v=4iToQ2FykoI

http://www.youtube.com/watch?v=vgTQ3eDkCn0

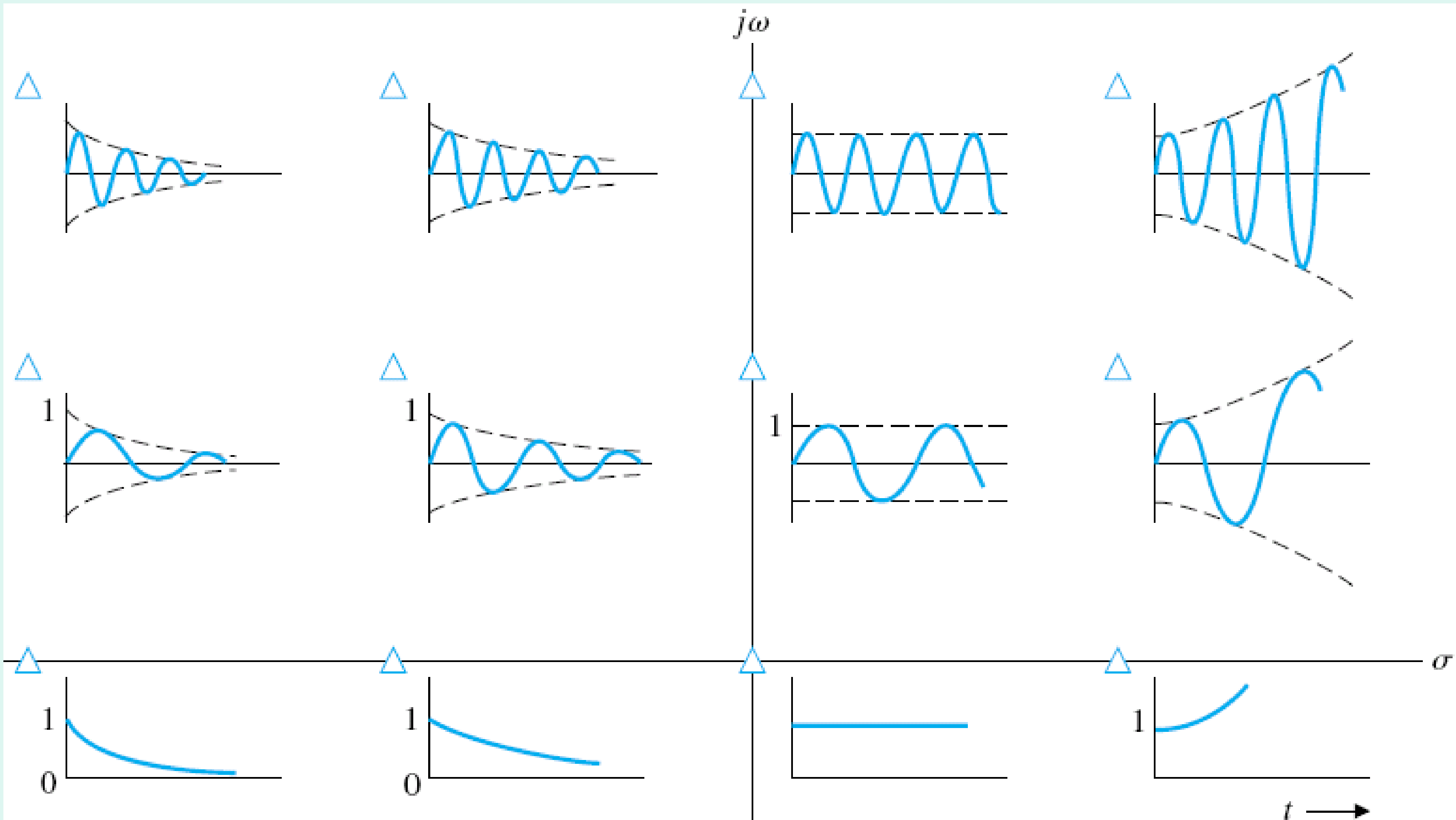
http://www.youtube.com/watch?v=r7kCSv18STI

Figure 1. *Gripen JAS39 prototype accident on 2 February 1989. The pilot received only minor injuries.*

Figure 6. *NASA X-29 forward-swept-wing aircraft (photo courtesy of NASA).*

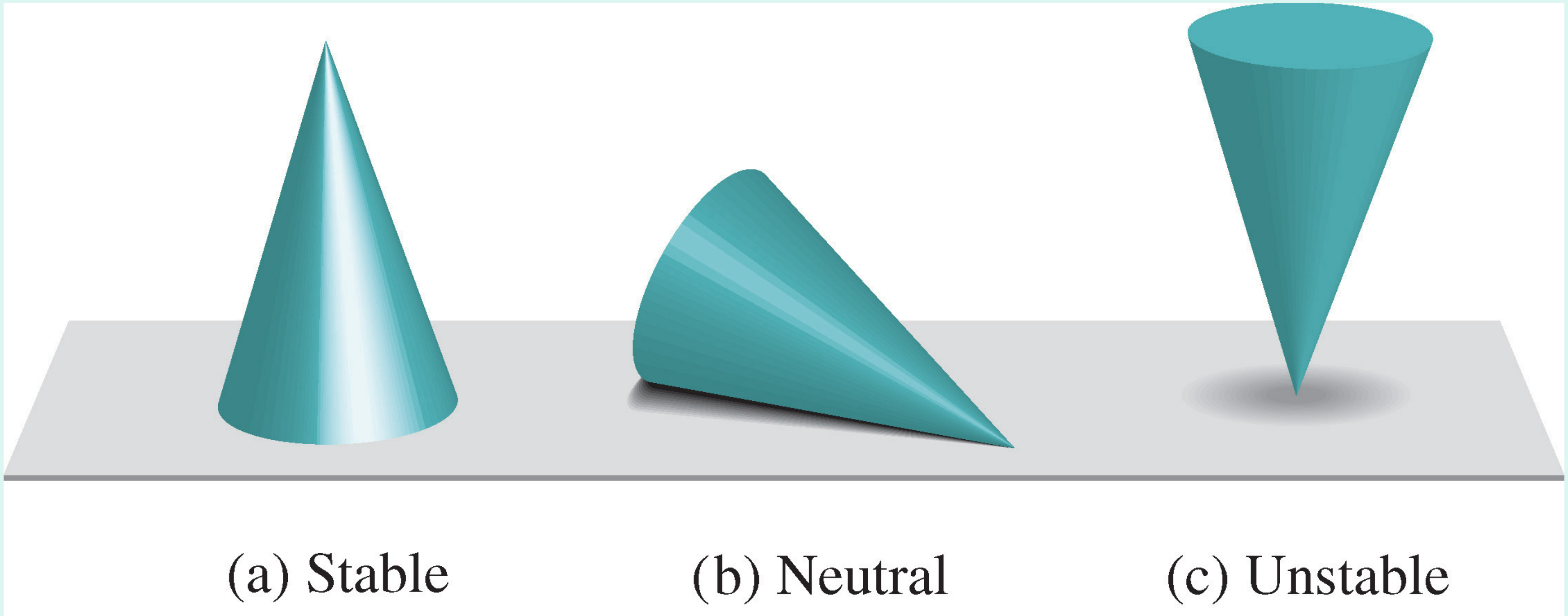
S-Plane and Transient Response

$$\text{Recall}: \frac{1}{s-a} \Leftrightarrow e^{at}$$

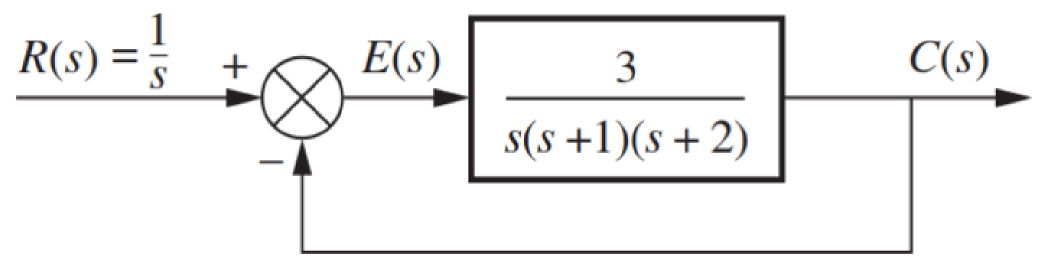


stable... unstable!

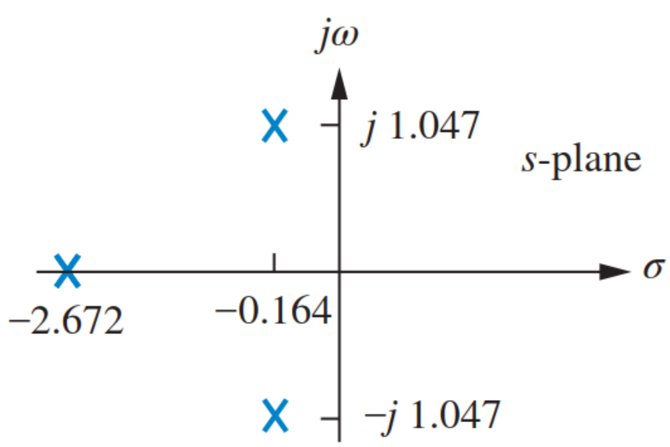
Mechanical stability



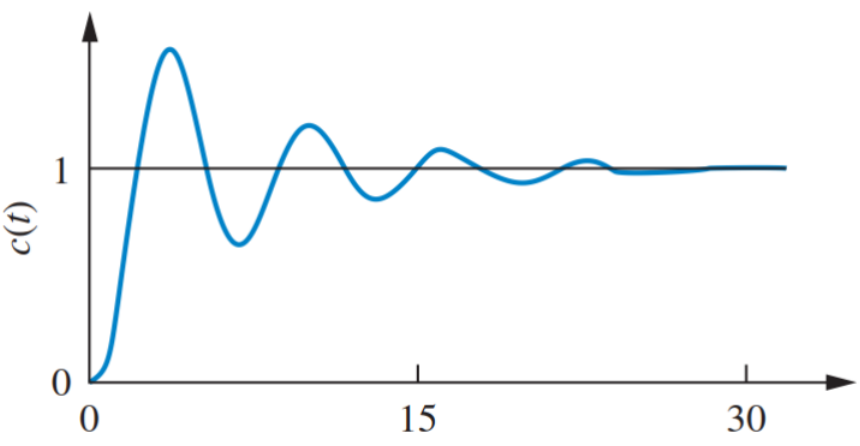
Stable system



Stable system

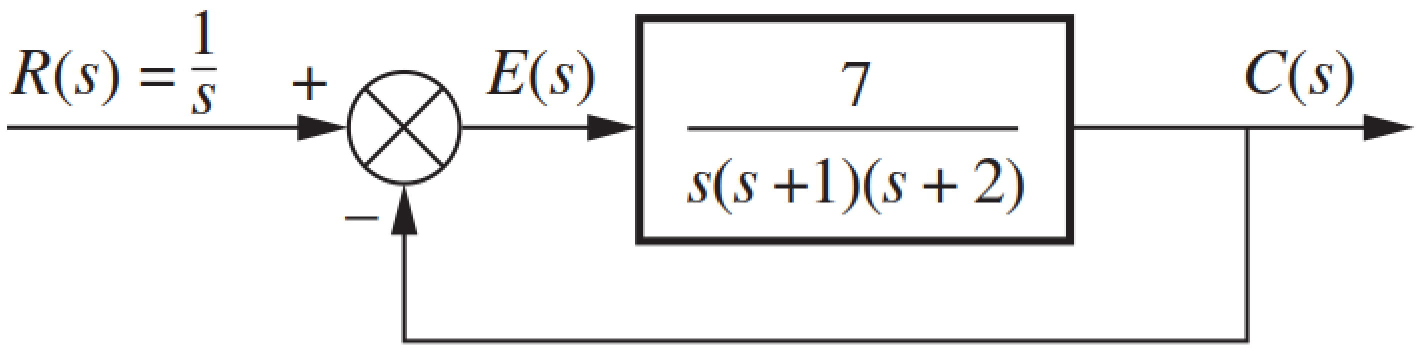


Stable system's
closed-loop poles
(not to scale)

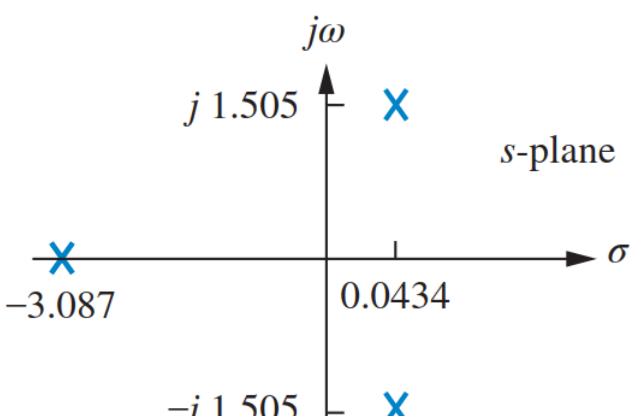


Time (seconds)

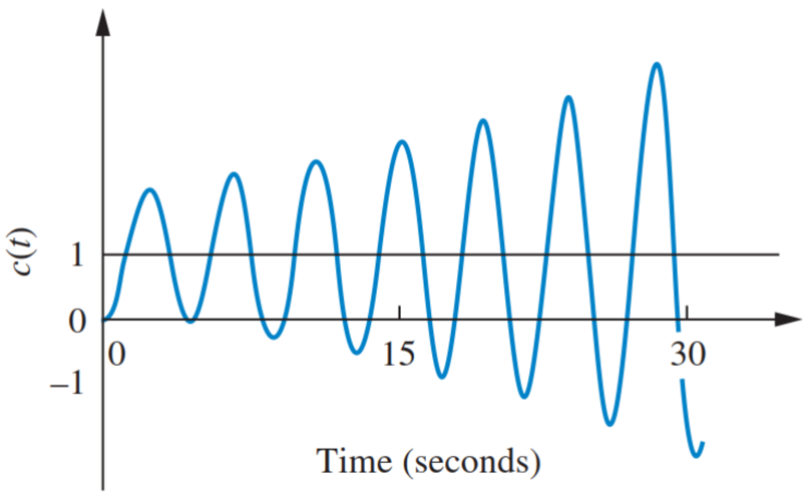
Unstable system when gain is increased

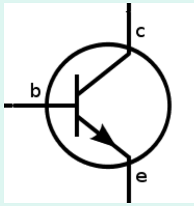


Unstable system

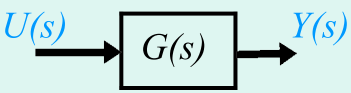


Unstable system's closed-loop poles (not to scale)





Stability – input-output perspective in time domain



We assume piecewise continuous input signals.
We consider continuous-time time-invariant causal systems described by:

$$y(t) = \int_{-\infty}^{\infty} g(t-\tau)u(\tau)d\tau = \int_{0}^{t} g(t-\tau)u(\tau)d\tau = \int_{0}^{t} u(t-\tau)g(\tau)d\tau$$

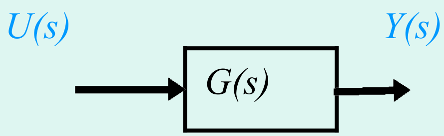
Definition: For any signal u let $\| u \|_{\infty} = \| u \| = \sup_{t \geq 0} | u(t) |$.

The signal *u(t)* is said to be **bounded** if $\| u \| < \infty$.

Definition: A system is *bounded-input-bounded-output* (BIBO) stable or has "finite gain" if there exists a K such that $\| y \| \leq K \| u \|$.

BIBO stable system <=> Every bounded input yields a bounded output
Unstable system - if there is a bounded input that yields an unbounded output

Stability - BIBO , time domain perspective (LTI systems)



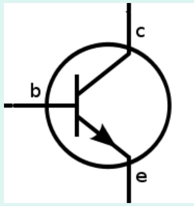
Theorem : The system described by the convolution integral

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau$$

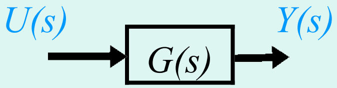
is bounded-input bounded-output stable if and only if

g (*unit impulse response*) is absolutely integrable, i.e.

$$\int_0^\infty |\, g(\tau) \,|\, d\tau = K < \infty$$



Stability – Laplace domain perspective



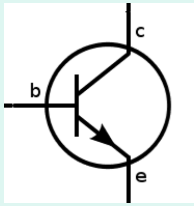
Theorem : The system described by the convolution integral

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau \qquad Y(s) = G(s)U(s) = \frac{N(s)}{D(s)}U(s)$$

with $G(s)$ a rational function with deg $D(s) \geq$ deg $N(s)$ is BIBO stable if and only if all the roots of $D(s)$ (poles of $G(s)$) have strictly negative real parts.

$$\frac{N(s)}{D(s)} = \sum_i \sum_k \frac{K_k}{(s-p_i)^k}$$

$$g(t) = \sum_i \sum_k t^k e^{p_i t}$$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