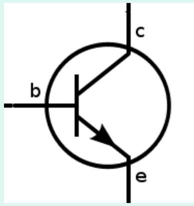
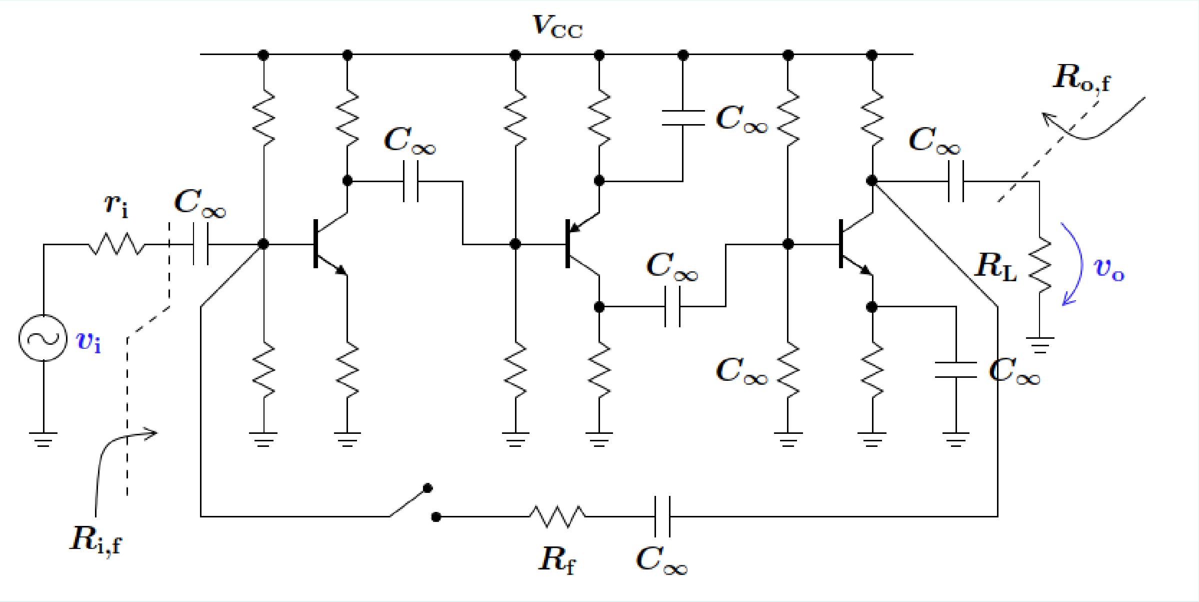
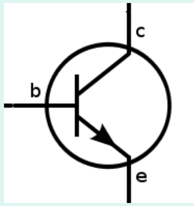


ELEC 301 - Stability and feedback

L25 - Nov 06

Instructor: Edmond Cretu

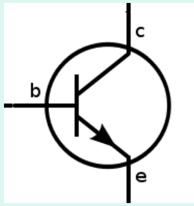




Last time

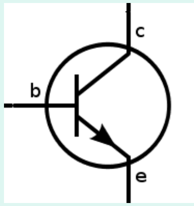
- Two methods to approach feedback circuits analysis:

1. Transform the feedback network into an ideal one, and load the open-loop amplifier (feedback diport equivalent model)
2. Loop-gain method



Stability of systems and circuits

- Feedback modifies the position of poles => potential risk to transform a stable system into an unstable one
- Stability aspects:
  - **BIBO stability - every** bounded input leads to a bounded output (<=> absolutely integrable unit impulse response h(t) for LTI)
  - In Laplace domain: no poles on the jω axis or in the rhp (strictly negative real-part)



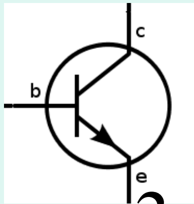
L25 Q01 BIBO stability

- If a system has a pole on the imaginary axis, is it BIBO stable?

A. Yes

B. No

C. It depends on the input excitation



Stability in state-space models

3 requirements for the design of feedback system: **stability**, steady-state errors, transient response

Alternative approaches to stability:

State-space model: $y(t) = y_{natural}(t) + y_{forced}(t)$

Stable LTI system: $y_{natural}(t) \xrightarrow{t \to \infty} 0$

Unstable LTI system: $\left| y_{natural}(t) \right| \xrightarrow{t \to \infty} \infty$

Marginally stable LTI: the natural response neither decays nor grows but remains constant or oscillates as $t \to \infty$

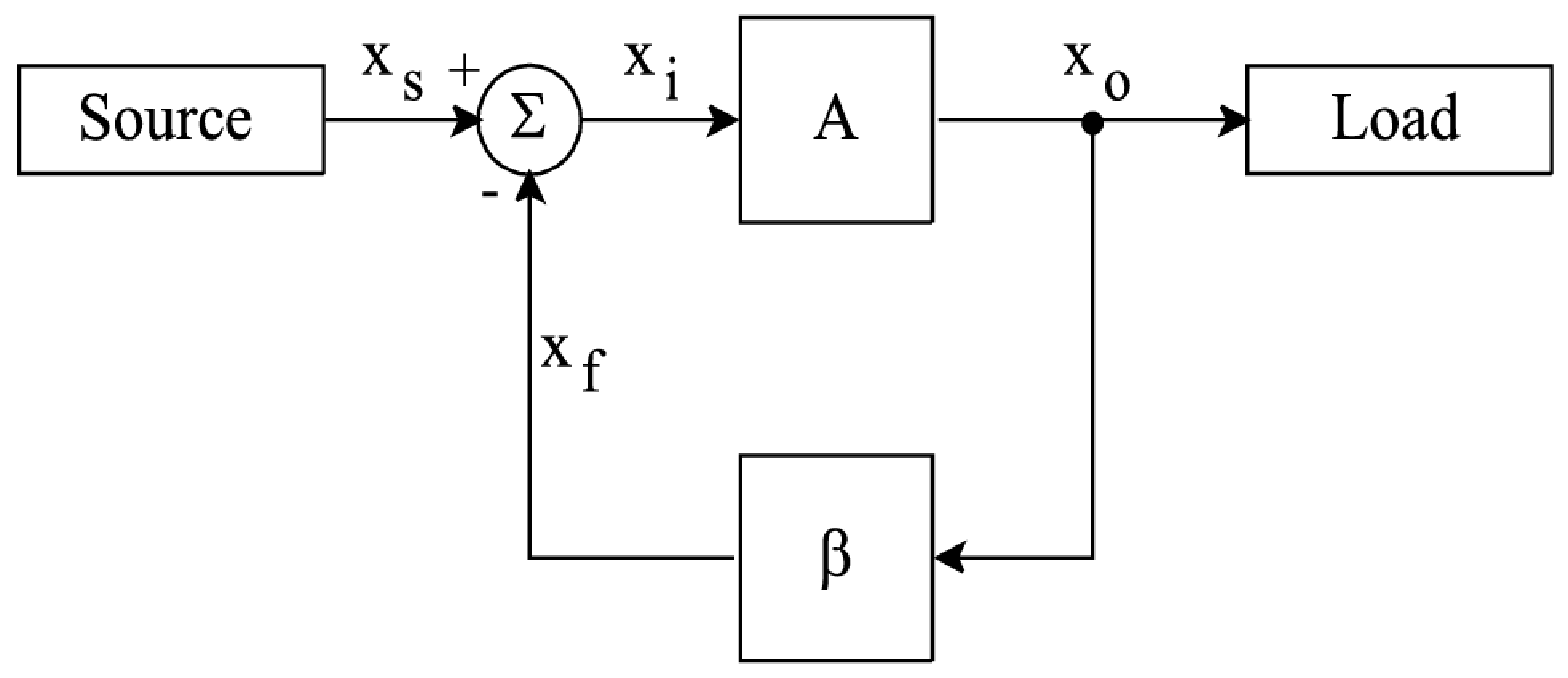
BIBO stability: every bounded input yields a bounded output (a system is unstable if there is a bounded input leading to an unbounded output)

Stability - frequency response perspective

- Both the open-loop gain A(s) and the feedback factor β(s), can be functions of frequency

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \xrightarrow{s=j\omega} A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

Critical point: $1 + A(j\omega)\beta(j\omega) = 0$

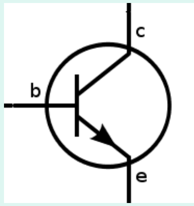


Stability - on Bode plots

Critical point: $A(j\omega)\beta(j\omega)=-1$

- The loop-gain:
- Cases when $\varphi(\omega)=180^0$:

1. $|A(j\omega)\beta(j\omega)|<1$ => increased gain $A_f(j\omega)>A(j\omega)$, but the amplifier remains stable
2. $|A(j\omega)\beta(j\omega)|=1$ => $A_f(j\omega)=\infty$ => the circuit will oscillate
3. $|A(j\omega)\beta(j\omega)|>1$ => the input signal will grow steadily until some non-linearity in the circuit will limit the loop-gain, reducing to 1 and the circuit will oscillate



Poles of the feedback amplifier

$$A_f(s)=\frac{A(s)}{1+A(s)\beta(s)} \Rightarrow Poles\left[A_f(s)\right]=Zeros\left[1+A(s)\beta(s)\right]$$

- The "characteristic equation" of the feedback loop:

$$1+A(s)\beta(s)=0$$

$$A(s)=\frac{N_A(s)}{D_A(s)},\beta(s)=\frac{N_\beta(s)}{D_\beta(s)} \Rightarrow A_f(s)=\frac{\frac{N_A(s)}{D_A(s)}}{1+\frac{N_A(s)}{D_A(s)}\frac{N_\beta(s)}{D_\beta(s)}}=\frac{N_A(s)D_\beta(s)}{D_A(s)D_\beta(s)+N_A(s)N_\beta(s)}$$

The zeros of $A_f(s)$ are the zeros of A(s) and the poles of β(s)

- It is more complicated for the poles of $A_f(s)$

Single pole amplifier

- An usual op-amp transfer function approximation
- Assume β is independent of frequency

$$A(s) = \frac{A_0}{1 + \dfrac{s}{\omega_p}} \Rightarrow A_f(s) = \frac{\dfrac{A_0}{1 + A_0\beta}}{1 + \dfrac{s}{\omega_p(1 + A_0\beta)}}$$

- The initial pole ω_p is moved to $\omega_{pf} = \omega_p(1 + A_0\beta)$
- For frequencies $\omega >> \omega_{pf}$, the frequency response $A_f(j\omega)$ is asymptotically equal to $A(j\omega)$

$$A_f(s) \overset{\omega >> \omega_{pf}}{\approx} \frac{\dfrac{A_0}{1 + A_0\beta}}{\dfrac{s}{\omega_p(1 + A_0\beta)}} = \frac{A_0}{\left(\dfrac{s}{\omega_p}\right)} = A(s)$$

Amplifier with a two-pole response

- A more complex situation:

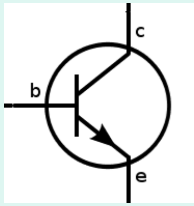
$$A(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

- The closed-loop transfer function:

$$A_f(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)+A_0\beta}$$

- The closed-loop poles: $\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)+A_0\beta = 0 \Leftrightarrow s^2 + s\left(\omega_{p1}+\omega_{p2}\right)+\left(1+A_0\beta\right)\omega_{p1}\omega_{p2} = 0$

$$s_{1,2} = -\frac{1}{2}\left(\omega_{p1}+\omega_{p2}\right) \pm \frac{1}{2}\sqrt{\left(\omega_{p1}+\omega_{p2}\right)^2 - 4\left(1+A_0\beta\right)\omega_{p1}\omega_{p2}}$$



L25 Q02 2-poles amplifier

- Can a feedback amplifier with a stable 2-pole amplifier in the open-loop become unstable when we provide a resistive feedback network?

A. Yes

B. No

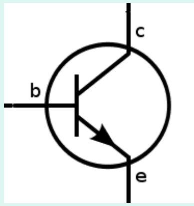
C. It depends on the resistive feedback configuration

Increasing the loop-gain $L_0=A_0\beta$

- For $L_0=0$, poles are located at open-loop poles $-\omega_{p1}$, $-\omega_{p2}$
- When $L_0=A_0\beta$ increases, the poles move together until they become co-incident at $-(\omega_{p1}+\omega_{p2})/2$

$$\frac{1}{2}\sqrt{\left(\omega_{p1}+\omega_{p2}\right)^2-4\left(1+A_0\beta\right)\omega_{p1}\omega_{p2}}=0\Rightarrow p_{pf1}=p_{pf2}=-\frac{1}{2}\left(\omega_{p1}+\omega_{p2}\right)$$

- For further increase in the loop gain $L_0=A_0\beta$ causes the poles to become complex conjugate, with the real part equal to $-(\omega_{p1}+\omega_{p2})/2$



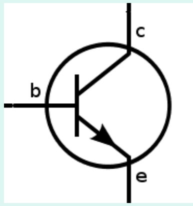
L25 Q03 feedback TF

- The closed-loop gain: assume $A(s)=A_0 N_A(s)/D_A(s)$

$$A(s)=A_0\frac{N_A(s)}{D_A(s)},\beta(s)=\frac{N_\beta(s)}{D_\beta(s)}\Rightarrow A_f(s)=\frac{A_0\frac{N_A(s)}{D_A(s)}}{1+A_0\frac{N_A(s)}{D_A(s)}\frac{N_\beta(s)}{D_\beta(s)}}=\frac{A_0 N_A(s)D_\beta(s)}{D_A(s)D_\beta(s)+A_0 N_A(s)N_\beta(s)}$$

What are the poles of A_f when A_0 is (close to) zero?

A. They are identical with the poles of A(s)
B. They are identical with the zeros of A(s)
C. The are identical with the poles of β(s)
D. They are identical with the poles of β(s)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