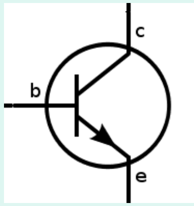
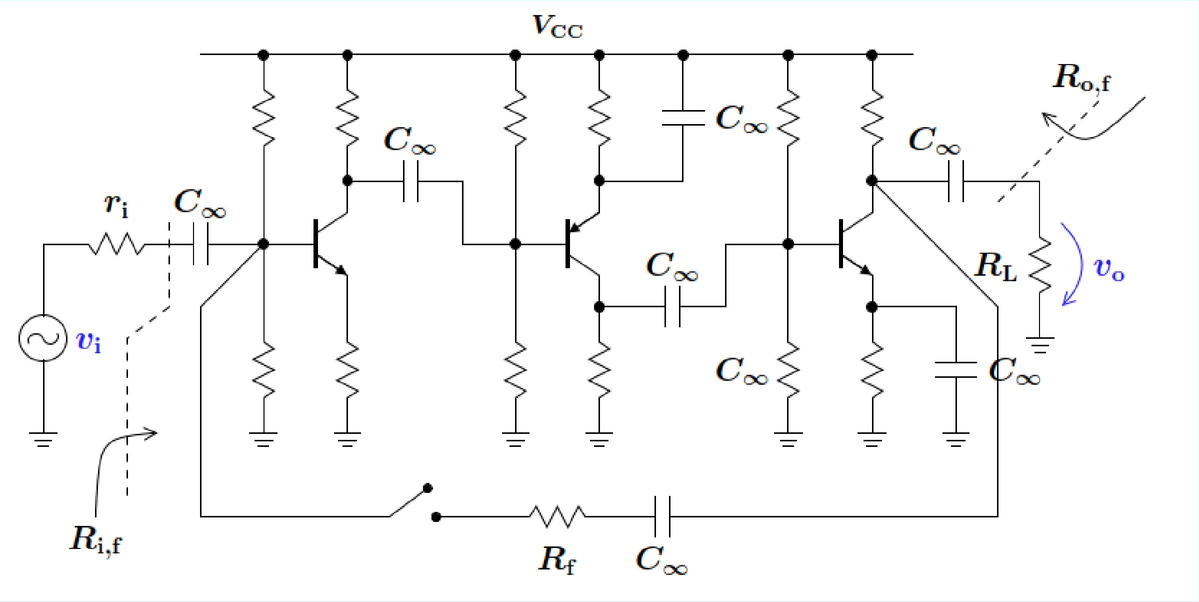
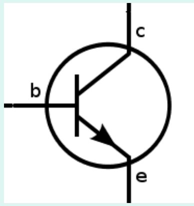


ELEC 301 - Stability - Bode plots

L26 - Nov 13

Instructor: Edmond Cretu





Last time

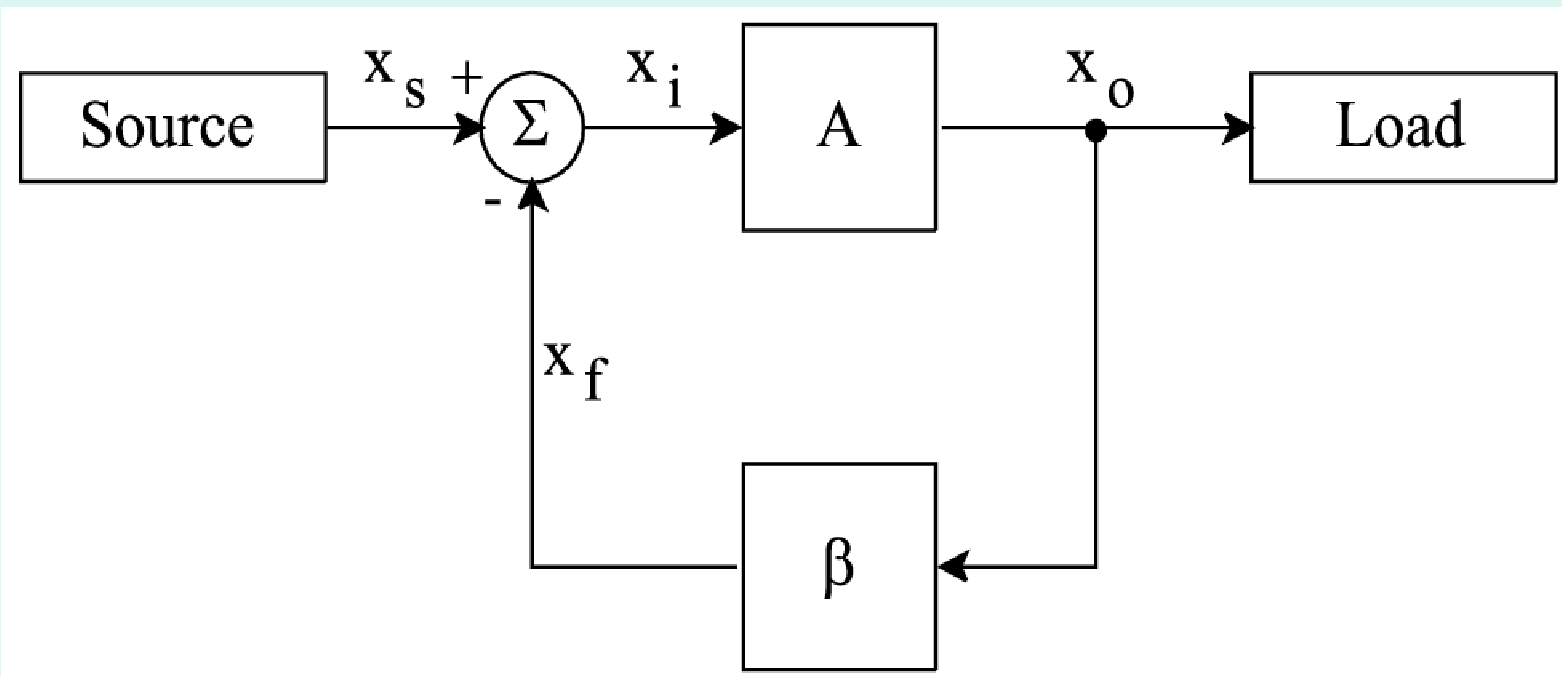
- Perspectives on stability:
  - Time-domain analysis: BIBO stability, absolutely integrable unit impulse response h(t) for LTI
  - s-domain analysis: no poles on the jω axis or in the rhp (strictly negative real-part)
  - State-space modeling perspective: the evolution of the natural response in time
  - Frequency-domain analysis (Bode plots) - exm for single-pole and two poles amplifiers with passive feedback

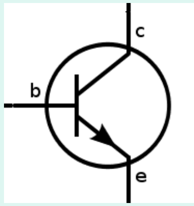
Stability - frequency response perspective

- Both the open-loop gain A(s) and the feedback factor β(s), can be functions of frequency

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \xrightarrow{s=j\omega} A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

Critical point: $1 + A(j\omega)\beta(j\omega) = 0$



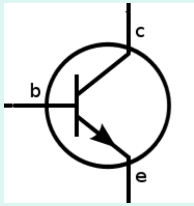


Stability - on Bode plots

Critical point: $A(j\omega)\beta(j\omega) = -1$

- The loop-gain:
- Cases when $\varphi(\omega)=180^0$:

1. $|A(j\omega)\beta(j\omega)|<1$ => increased gain $A_f(j\omega)>A(j\omega)$, but the amplifier remains stable
2. $|A(j\omega)\beta(j\omega)|=1$ => $A_f(j\omega)=\infty$ => the circuit will oscillate
3. $|A(j\omega)\beta(j\omega)|>1$ => the input signal will grow steadily until some non-linearity in the circuit will limit the loop-gain, reducing to 1 and the circuit will oscillate



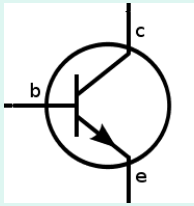
Past quiz - L25 Q03 feedback TF

- The closed-loop gain: assume $A(s)=A_0N_A(s)/D_A(s)$

$$A(s)=A_0\frac{N_A(s)}{D_A(s)},\beta(s)=\frac{N_\beta(s)}{D_\beta(s)}\Rightarrow A_f(s)=\frac{A_0\frac{N_A(s)}{D_A(s)}}{1+A_0\frac{N_A(s)}{D_A(s)}\frac{N_\beta(s)}{D_\beta(s)}}=\frac{A_0N_A(s)D_\beta(s)}{D_A(s)D_\beta(s)+A_0N_A(s)N_\beta(s)}$$

What are the poles of A_f when A_0 is (close to) zero?

A. They are identical with the poles of A(s)
B. They are identical with the zeros of A(s)
C. The are identical with the poles of β(s)
D. They are identical with the zeros of β(s)



L26 Q01 feedback TF

- The closed-loop gain: assume $A(s)=A_0N_A(s)/D_A(s)$

$$A(s)=A_0\frac{N_A(s)}{D_A(s)},\beta(s)=\frac{N_\beta(s)}{D_\beta(s)}\Rightarrow A_f(s)=\frac{A_0\frac{N_A(s)}{D_A(s)}}{1+A_0\frac{N_A(s)}{D_A(s)}\frac{N_\beta(s)}{D_\beta(s)}}=\frac{A_0N_A(s)D_\beta(s)}{D_A(s)D_\beta(s)+A_0N_A(s)N_\beta(s)}$$

What are the poles of A_f when A_0 is increasing to infinity?

A. They are identical with the poles of A(s)

B. They are identical with the zeros of A(s)

C. The are identical with the poles of β(s)

D. They are identical with the zeros of β(s)

General case - Root locus method (Evans)

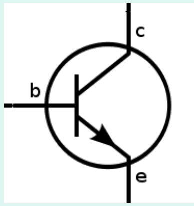
- The closed-loop gain: assume $A(s)=A_0N_A(s)/D_A(s)$

$$A(s)=A_0\frac{N_A(s)}{D_A(s)},\ \beta(s)=\frac{N_\beta(s)}{D_\beta(s)} \Rightarrow A_f(s)=\frac{A_0\dfrac{N_A(s)}{D_A(s)}}{1+A_0\dfrac{N_A(s)}{D_A(s)}\dfrac{N_\beta(s)}{D_\beta(s)}}=\frac{A_0N_A(s)D_\beta(s)}{D_A(s)D_\beta(s)+A_0N_A(s)N_\beta(s)}$$

- For $A_0=0$ => the poles of $A_f(s)$ are identical with the poles of $A(s)$
- For $A_0 \to \infty$, then the poles of $A_f(s)$ are identical with the zeros of $\beta(s)$, including zeros at infinity (similar with the **root locus method**)

$$A_f(s)\xrightarrow{A_0\to 0} A_0\frac{N_A(s)}{D_A(s)}=A(s)$$

$$A_f(s)\xrightarrow{A_0\to \infty} \frac{A_0\dfrac{N_A(s)}{D_A(s)}}{A_0\dfrac{N_A(s)}{D_A(s)}\dfrac{N_\beta(s)}{D_\beta(s)}}=\frac{D_\beta(s)}{N_\beta(s)}$$

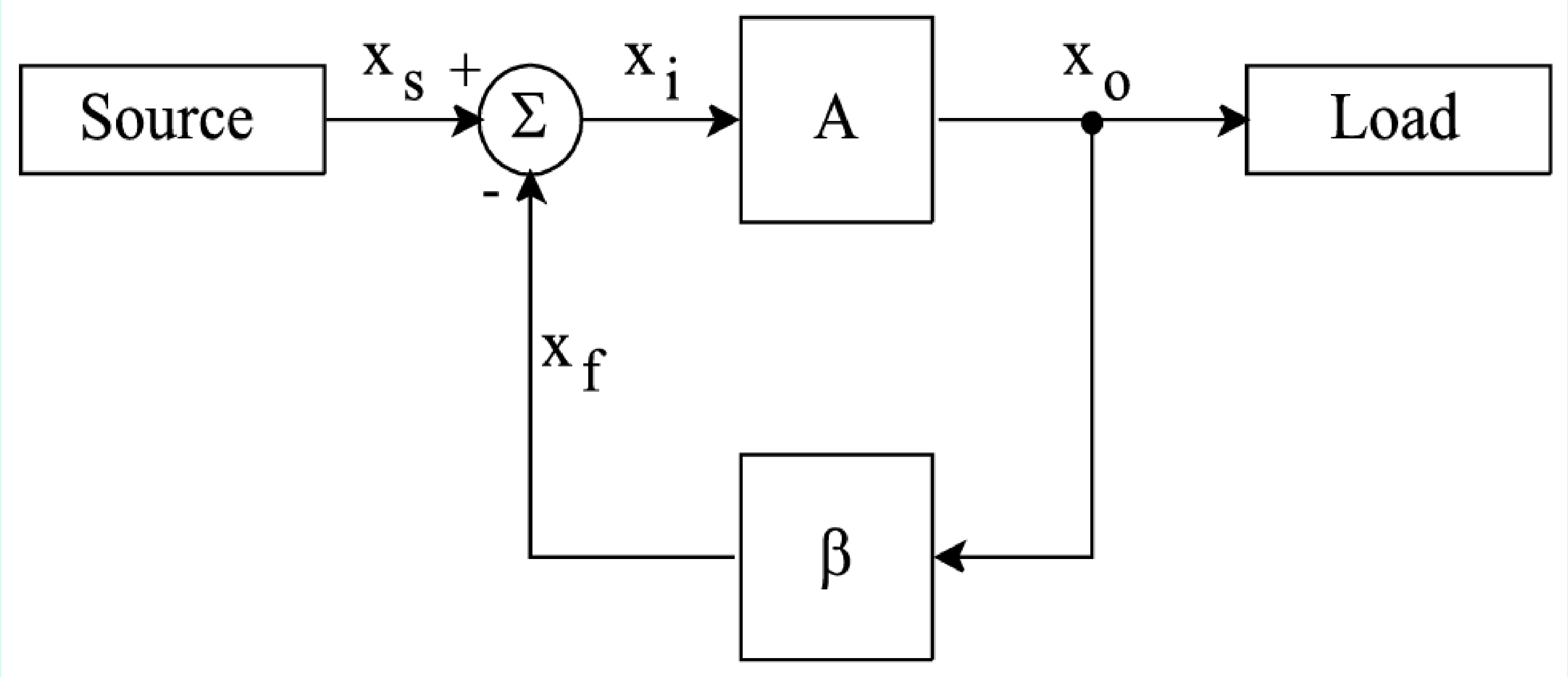


Recall - frequency response perspective

- Both the open-loop gain A(s) and the feedback factor β(s), can be functions of frequency

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \xrightarrow{s=j\omega} A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

Critical point: $1 + A(j\omega)\beta(j\omega) = 0$

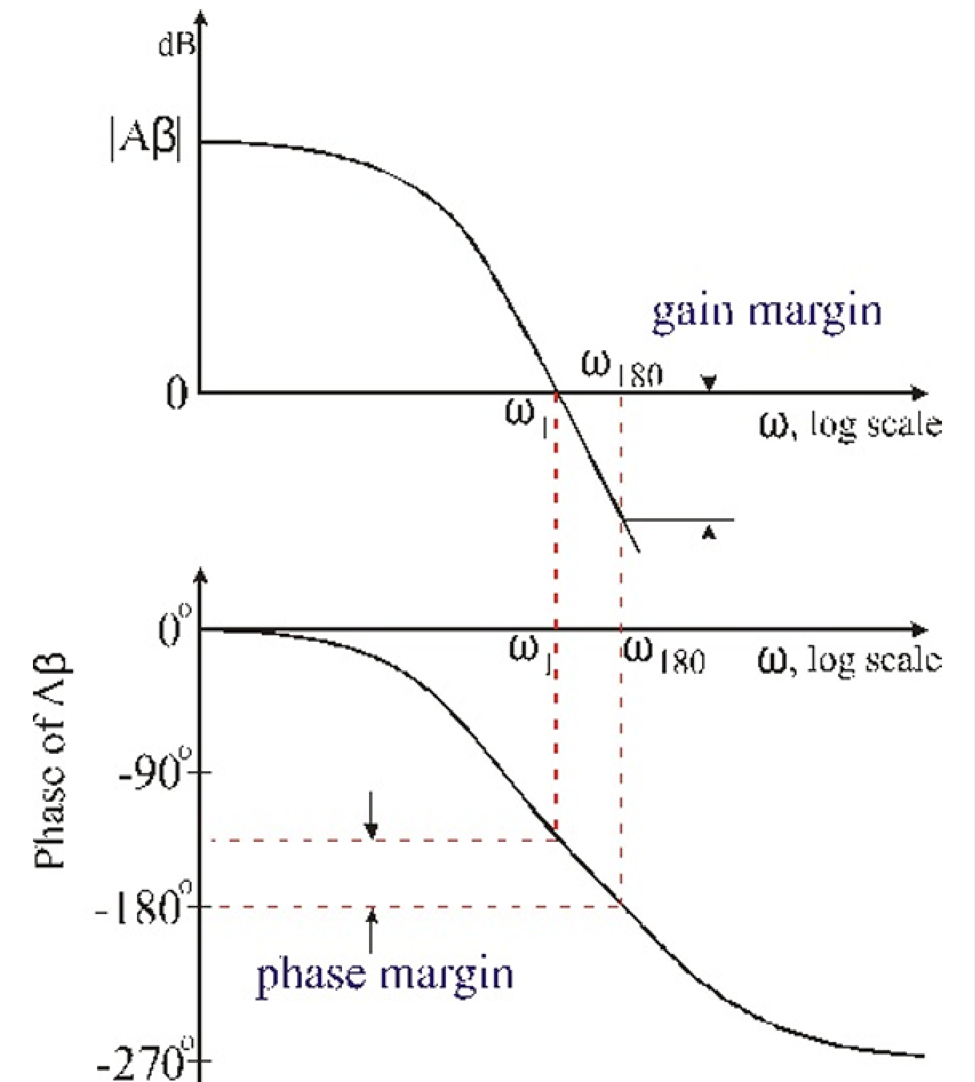


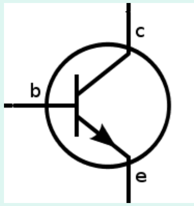
Stability analysis using Bode plots

- We use the loop-gain Aβ on a Bode plot, to determine whether an amplifier with feedback is stable
- Main criteria: we check whether the magnitude of the loop gain |Aβ| is greater than, equal to, or less than 1 at the frequency at which $\varphi=180^0$ (ω_{180})
- Alternatively: check the phase shift when the loop gain |Aβ|=1

Exm: 3-poles LP filter with feedback

- We assume $\beta=\beta_0$=ct (passive feedback network





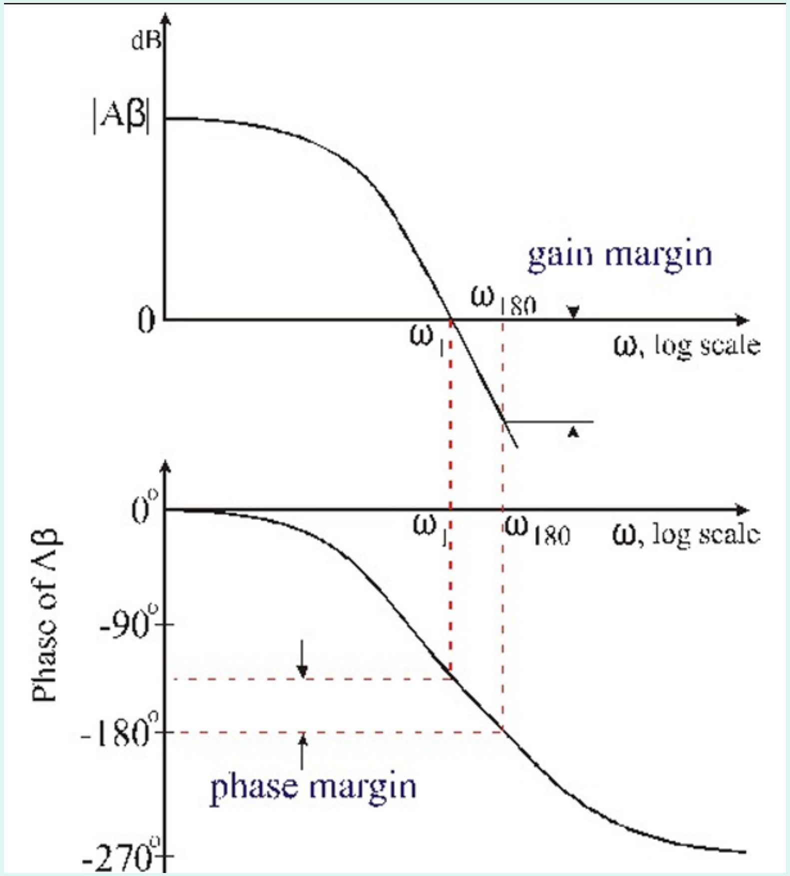
L26 Q02 LP filter

- Based on the Bode plot representation, is the system stable or not?

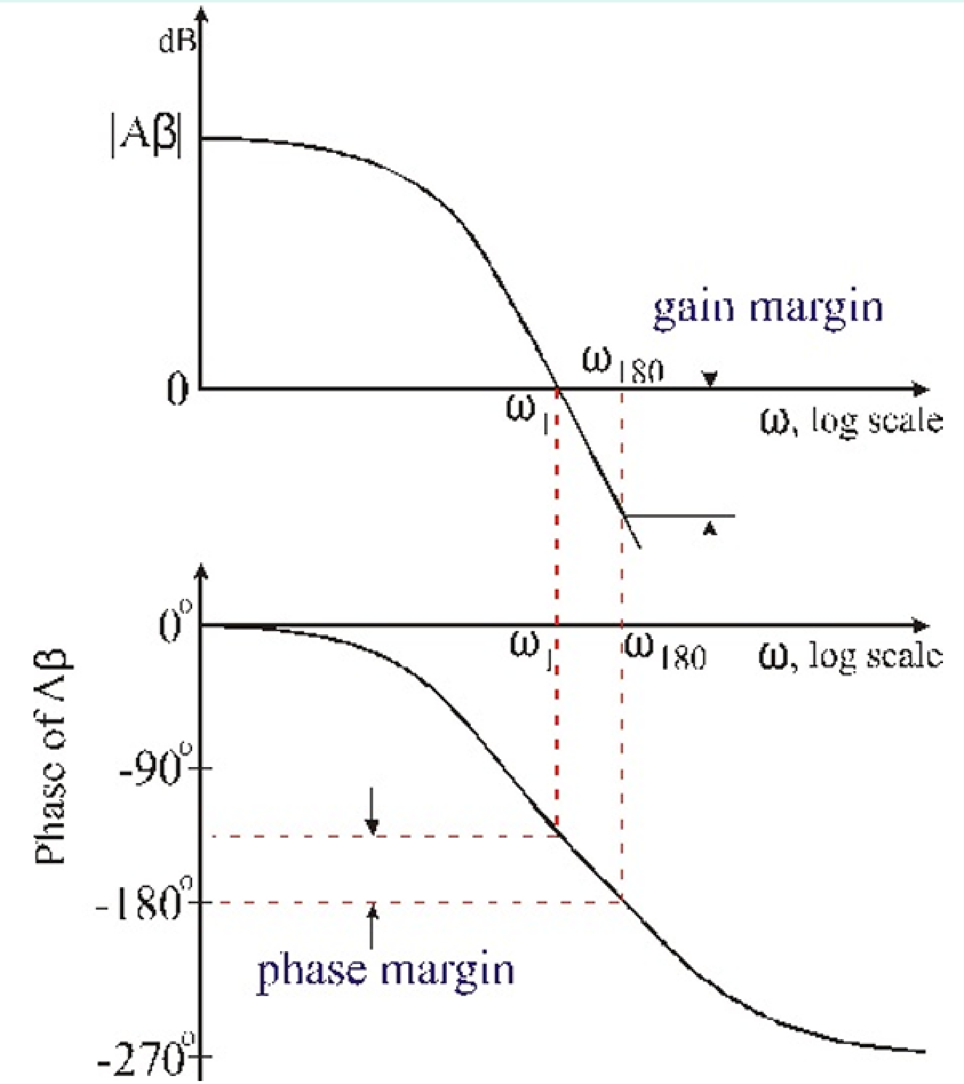
A. It is stable only for some specific input signals

B. Yes, it is stable

C. No, it is not stable

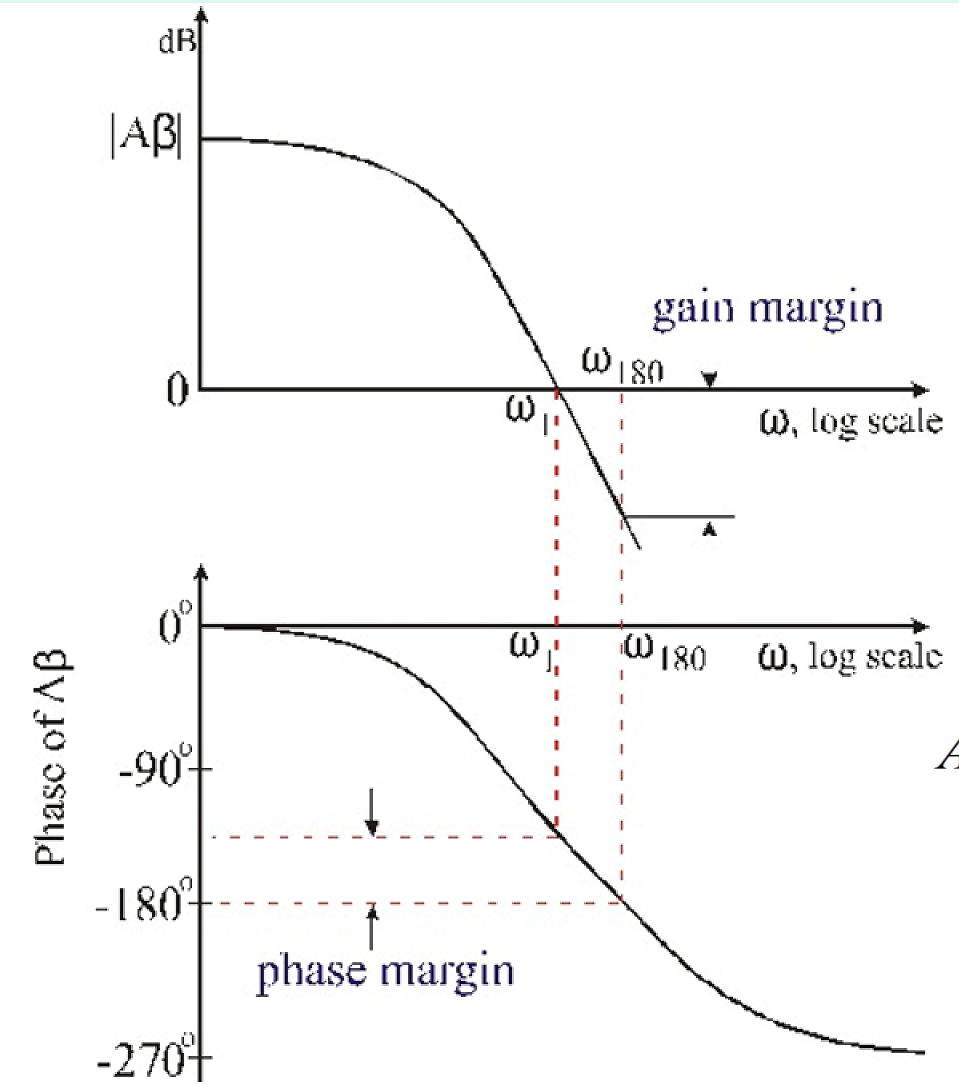


Exm: LP filter (2) - gain margin



- $\varphi=145^0$ when $20\log|A\beta|=0$ ($|A\beta|=1$) => the amplifier is stable
- **Gain margin** = the difference between the value of $|A\beta|$ at ω_{180} and 1 (0dB) - how much can we still increase the gain without becoming unstable
- 3-poles => 270^0 total phase change
- If $\omega_1=\omega_{180}/2$ (one octave difference) => gain margin = -18dB (-6dB/octave*3)

Exm: LP filter - phase margin



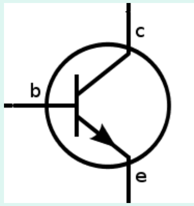
- Equivalently, we can examine the phase plot
- **Phase margin** = the difference between the phase value at ω_1 (where $|A\beta|=1$) and 180^0
- Typical designs: ensure phase margins $>45^0$

$$If\ A_0\beta >> 1 \Rightarrow A_{fLF} = \frac{A_0}{1+A_0\beta} \approx \frac{1}{\beta}$$

$$A(j\omega_1)\beta = 1e^{-j\theta}, \theta = 180^0 - \text{phase\_margin}$$

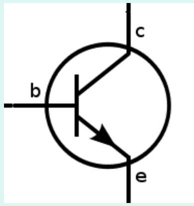
$$\mathrm{A}_f(j\omega_1) = \frac{A(j\omega_1)}{1+A(j\omega_1)\beta} = \frac{\frac{1}{\beta}e^{-j\theta}}{1+e^{-j\theta}}$$

$$\left|A_f(j\omega_1)\right| = \frac{1}{\beta}\frac{1}{\left|1+e^{-j\theta}\right|}$$



Phase margin aspects

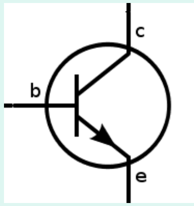
- As phase margin approaches 0^0, the closed-loop gain approaches infinity => the amplifier oscillates
- Typical amplifiers have margins of 45^0 or higher



Alternative Bode plot approach

- Draw the Bode plots (magnitude and phase) for the open-loop gain A(jω)
- Impose upon it the Bode plot of 1/β(jω)
- The log() difference (in the magnitude plot):

$$20\log\left|A(j\omega)\right| - 20\log\left|\frac{1}{\beta(j\omega)}\right| = 20\log\left|A(j\omega)\beta(j\omega)\right|$$



Exm: 3-poles amplifier + feedback

- Assume an amplifier with A_0=80dB, and poles at 10^4, 10^5, and 10^6rad/s. Let β be a real constant, $20\log|1/\beta|=70$dB
- Remark: if β is a real constant (horizontal line in the magnitude plot and zero phase shift in the phase plot), then the phase plot of the loop gain $A(j\omega)\beta$ is the same as that for $A(j\omega)$

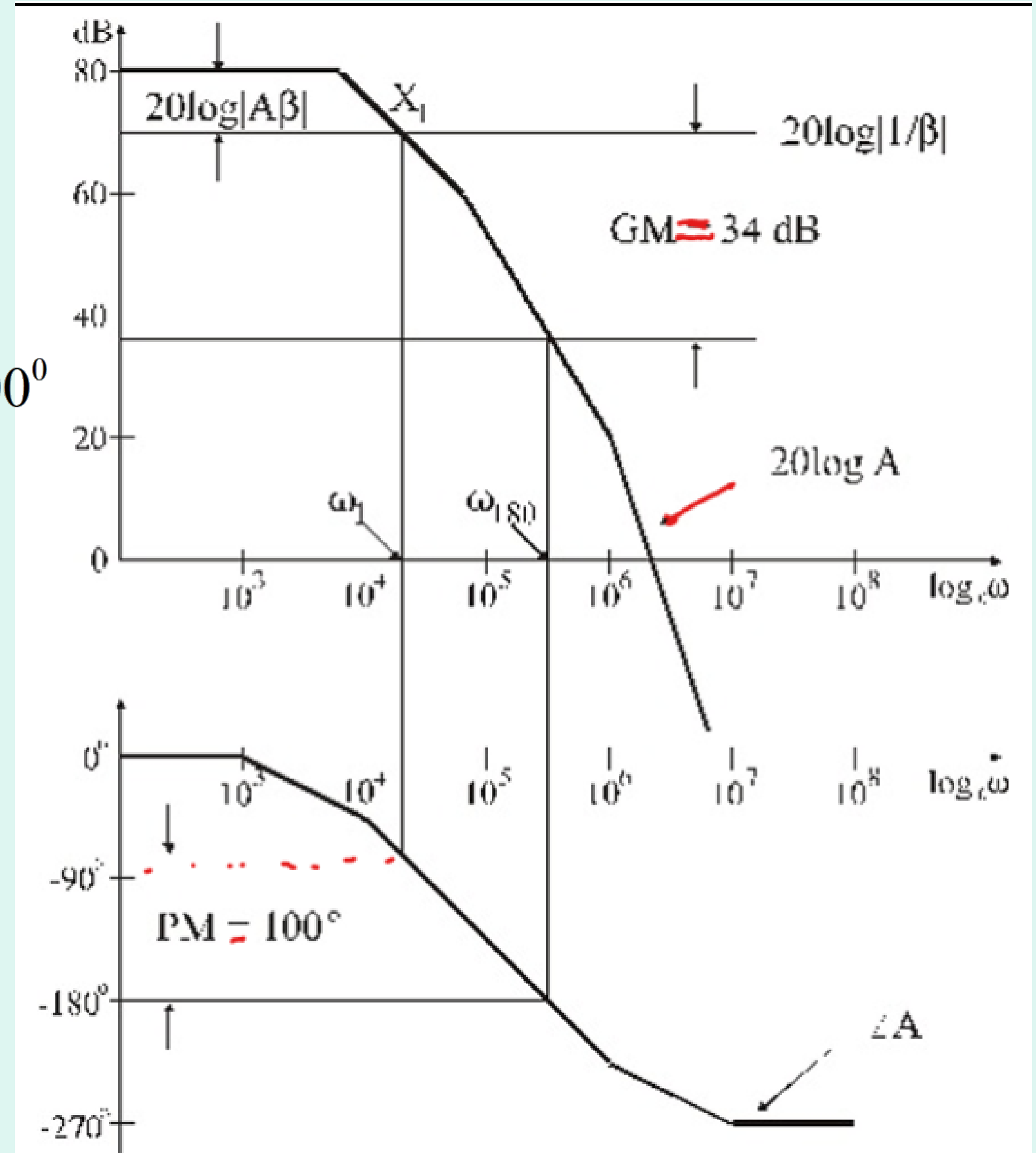
Exm - Bode plots

- Graphical solution:

$$\text{Point } x_1 : 20\log|A\beta| = 0 \Leftrightarrow |A\beta| = 1$$

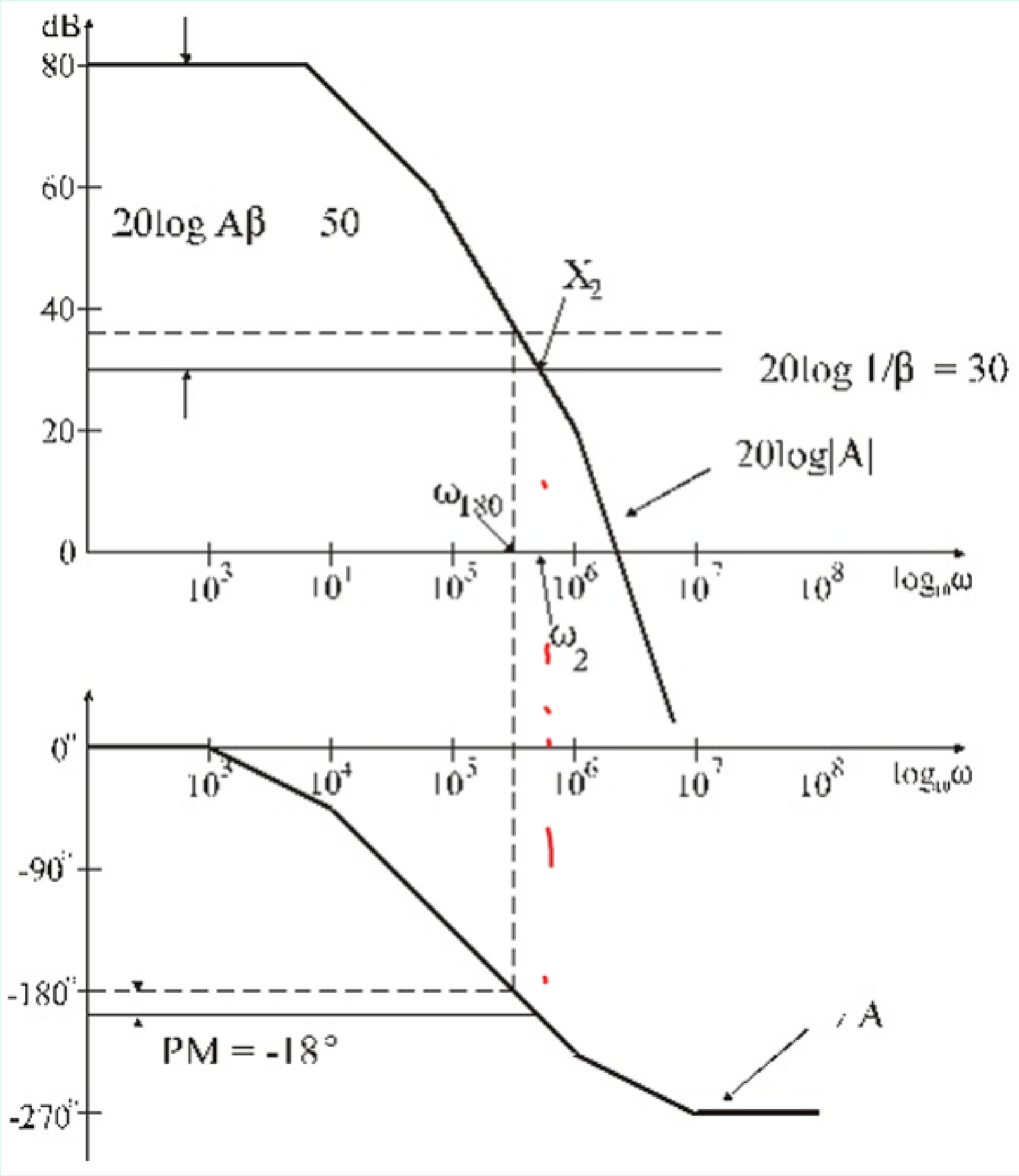
$$\theta(\omega_1) \approx -80^0 \Rightarrow \text{phase margin} \approx 100^0$$

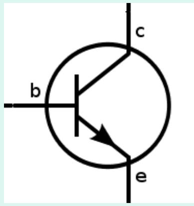
$$\text{Gain margin} \approx 34\text{dB}$$



Trade-offs gain vs bandwidth

- LF closed-loop gain $A_f \approx 1/\beta = 70dB$
- Suppose we wanted to trade-off gain for BW - reduce the closed-loop gain to 30dB
- When $|A\beta|=1$ (point X_2), the phase is less than 180^0 => the amplifier is unstable
- By inspection: the minimum closed-loop gain (marginal stability) is -36dB



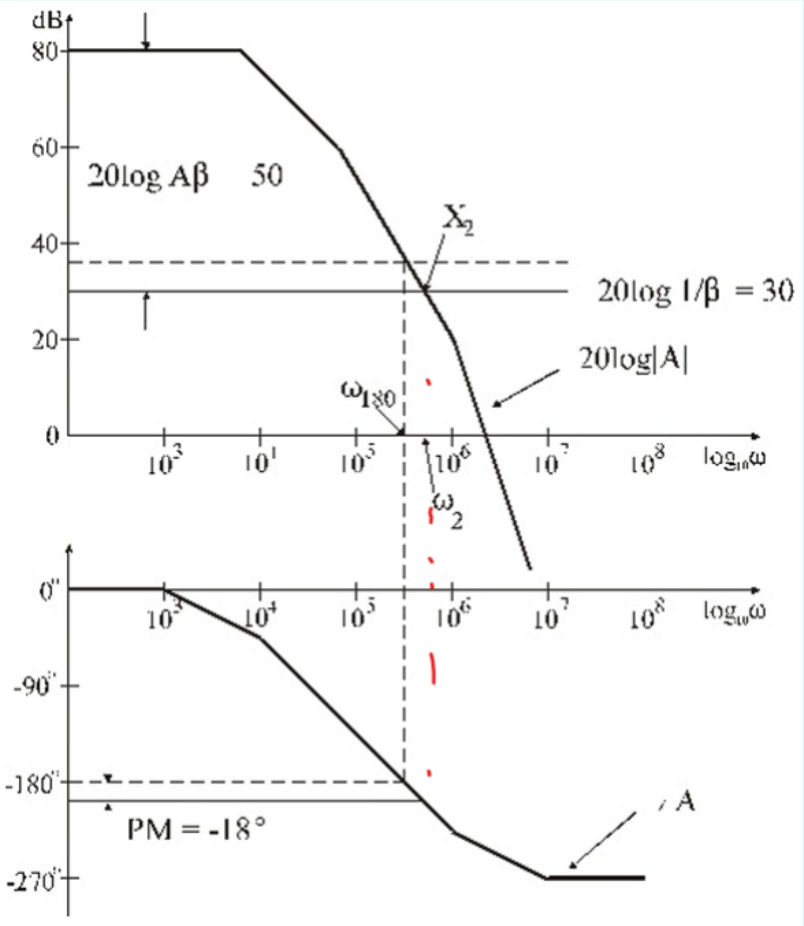


"Closure Rule of Thumb"

- The point at which ω_{180} occurs is always on the section of the Bode diagram on which the slope is -40dB/dec or greater (WHY?)
- If ω_{180} intercept is on the section with a slope of -20dB/dec the amplifier will not be unstable
- The "**Closure rule of thumb**" - if $20\log|1/\beta(j\omega)|$ closes on $20\log|A(j\omega)|$ with a difference of less than 20db/dec, the amplifier will not be unstable

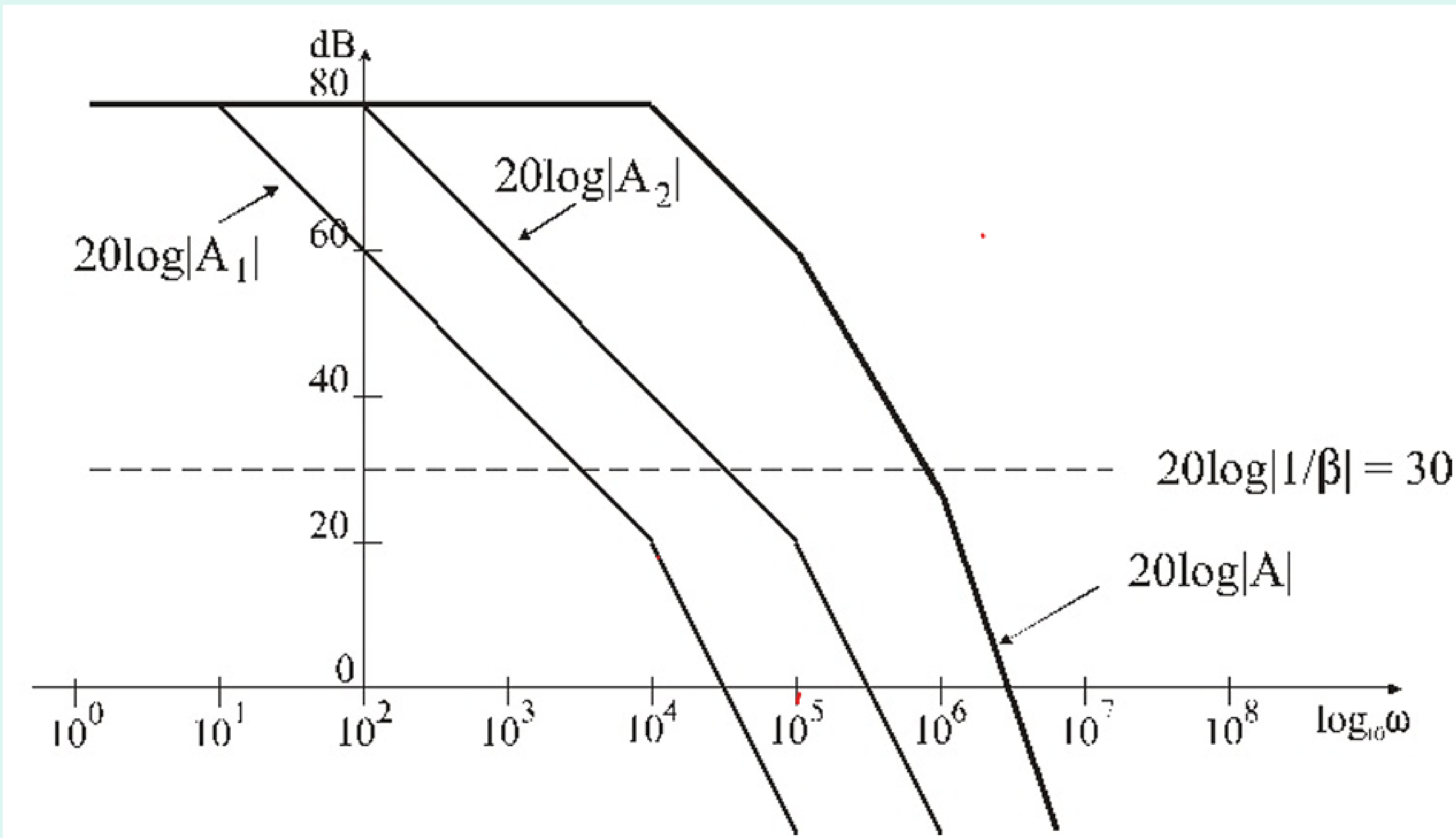
Frequency compensation

- It means adjusting the poles position to ensure stability
- Assume we want a circuit which will be stable for any amount of feedback => we must design our circuit so that the line 20log|1/β| intersects the plot 20log|A(jω)| with a closure of 20db/dec
- For 20log|1/β|=30dB, the amplifier will be unstable



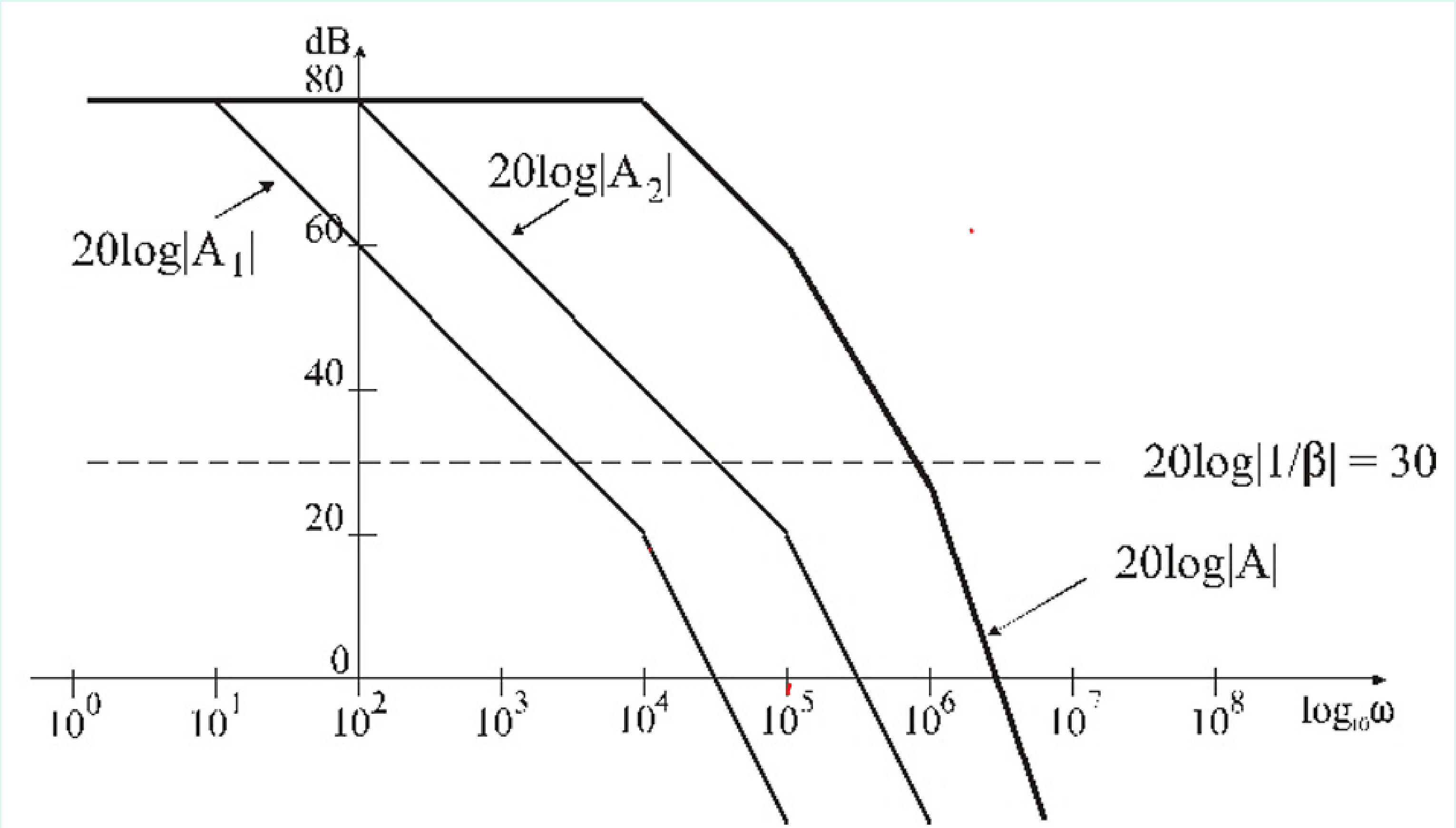
Frequency compensation - pole addition and shifting

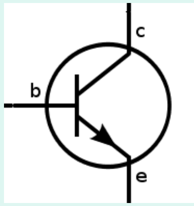
- Bring the intersection earlier - compensate by introducing a pole at 10rad/s => new gain $A_1(j\omega)$, stable, but with a significantly reduced BW



Frequency compensation (3)

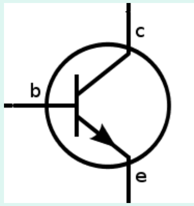
- Better method: shift the pole originally at 10^4rad/s to a lower frequency - move the dominant LF pole to 10^2rad/s results in $A_2(j\omega)$, improving the BW with one decade





Frequency compensation techniques

- Some common methods:
  – Phase-lag and phase-lead compensation - introduce additional phase lag or lead at low frequencies to stabilize the circuit
  – Miller effect compensation - add a feedback capacitor to reduce the closed-loop gain
  – Isolation resistor placement - resistors to dampen the output before reaching a capacitive load or set a zero in TF
  – Root-locus techniques - add poles and zeros so that the root locus is shaped to remain in the LHP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