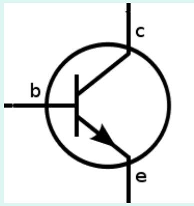
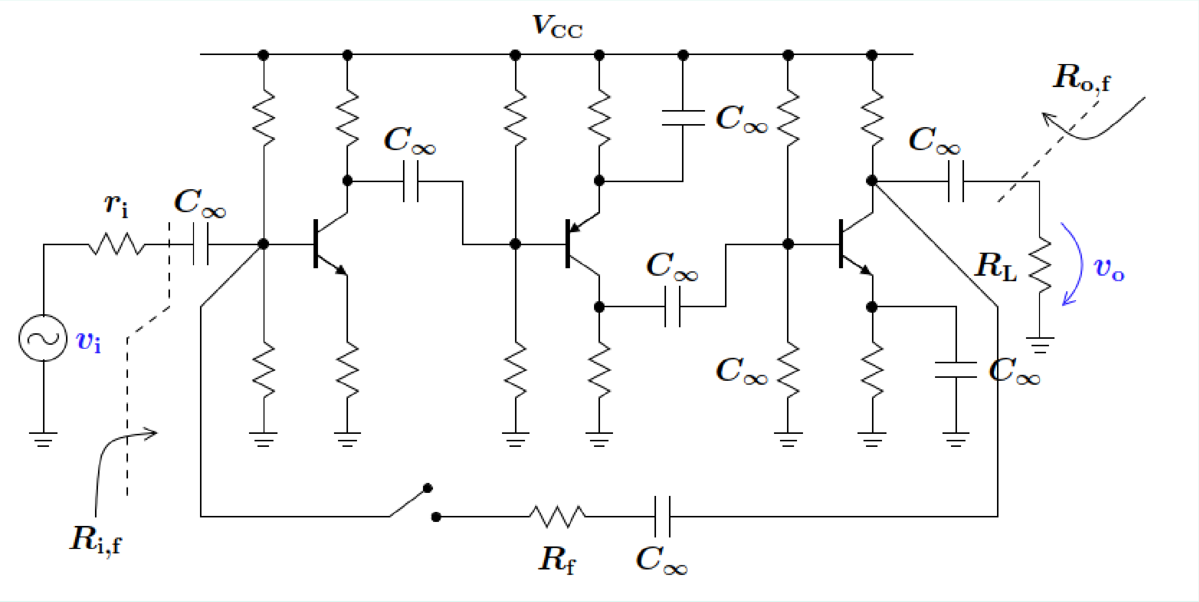


ELEC 301 - Stability and compensation techniques

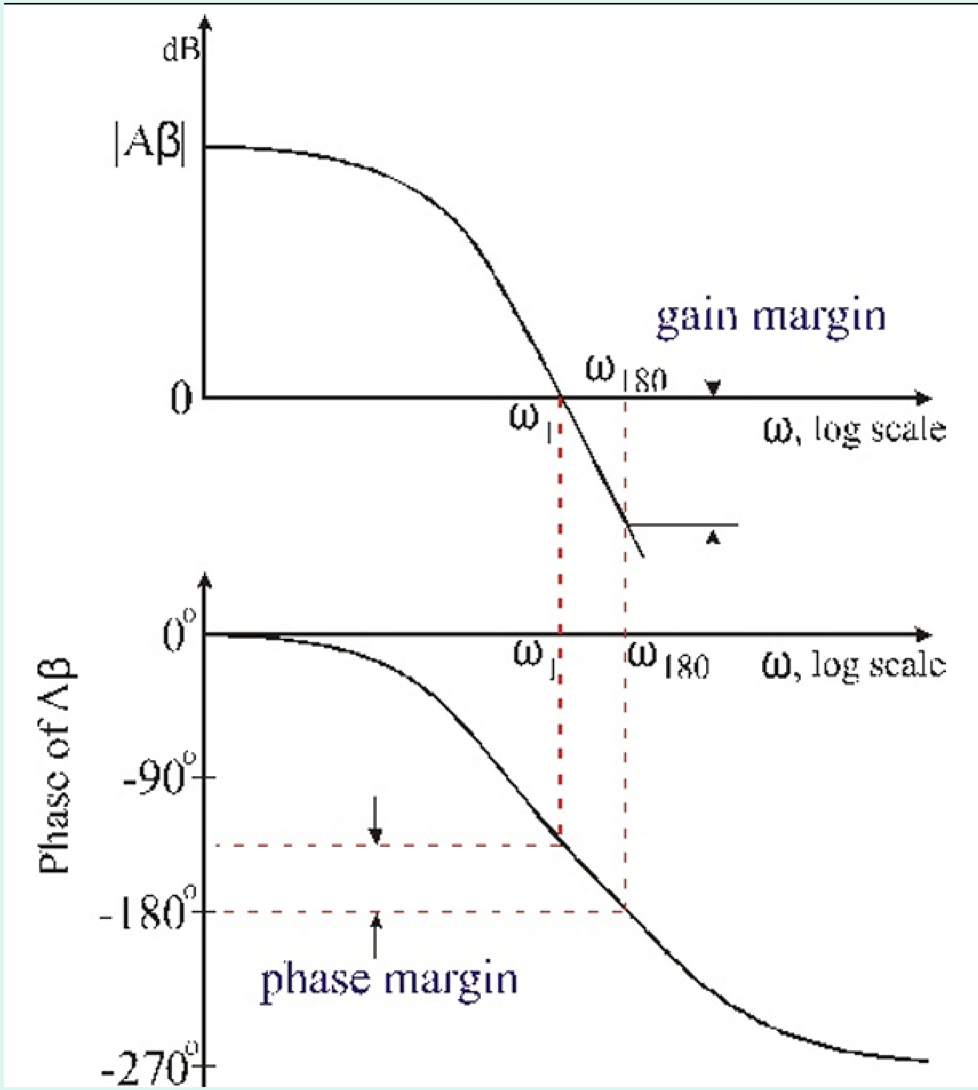
L27 - Nov 17

Instructor: Edmond Cretu



Last time

- Stability - frequency domain perspective
- Gain margin (GM)
- Phase margin (PM)
- Bode plots for $A\beta(j\omega)$
- Bode plots for $A(j\omega)$ and $1/\beta(j\omega)$
- The closure rule of thumb

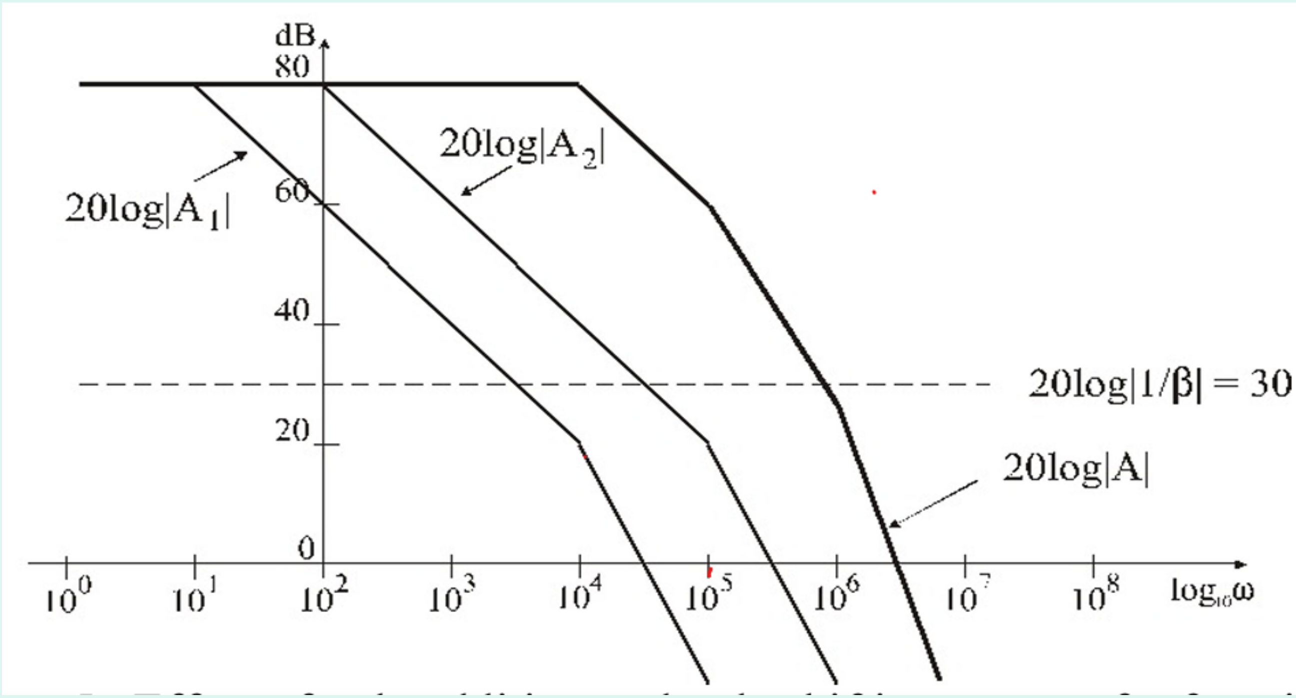


"Closure Rule of Thumb"

- The point at which ω_{180} occurs is always on the section of the Bode diagram on which the slope is -40dB/dec or greater (WHY?)
- If ω_{180} intercept is on the section with a slope of -20dB/dec the amplifier will not be unstable
- The "**Closure rule of thumb**" - if at the intersection of $20\log|1/\beta(j\omega)|$ and $20\log|A(j\omega)|$, the difference of slopes (called the **rate of closure**) should not exceed 20db/dec, then the amplifier is stable

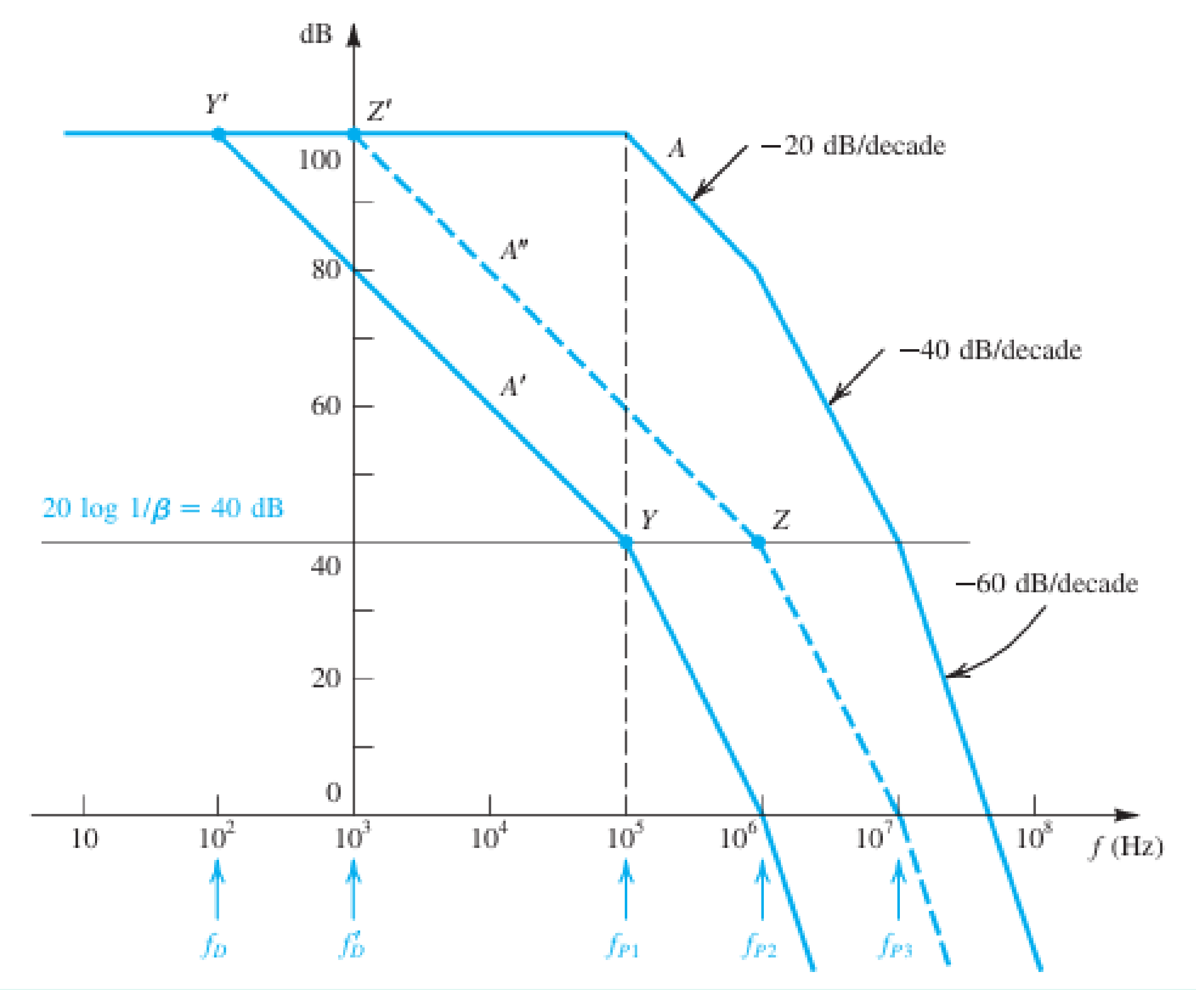
Frequency compensation

- Simplest method: introduce a new pole in A(s) at a sufficiently low frequency, so that the new open-loop gain A'(s), intersects 20log(1/|β|)with a slope difference of -20dB/dec
- Better method: shift the pole originally at 10^4rad/s to a lower frequency - move the dominant LF pole to 10^2rad/s results in $A_2(j\omega)$, improving the BW with one decade



Frequency compensation - extra pole addition

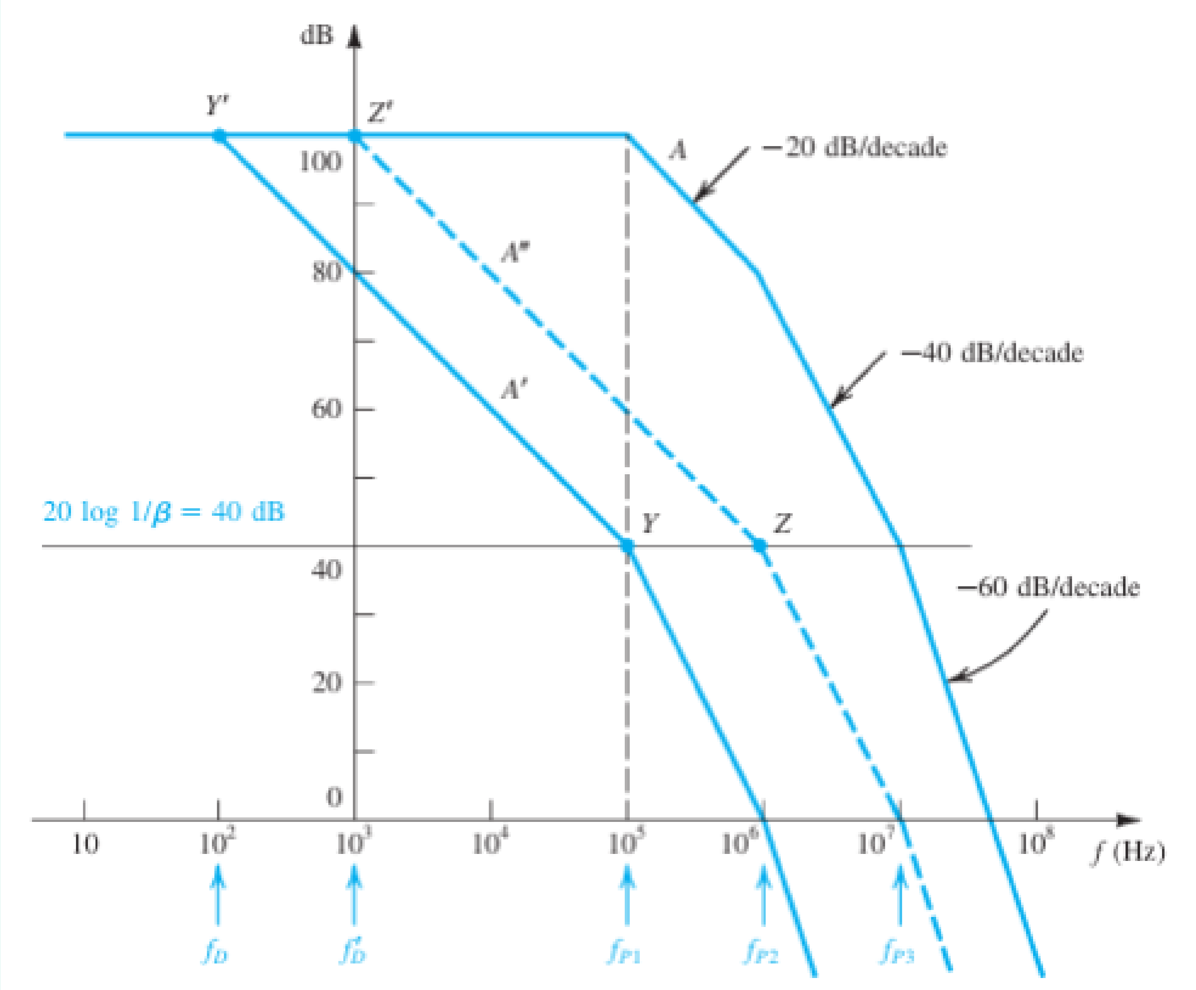
- Introduce a pole so that the A'(jω) and 1/β(jω) intersection happens before the dominant HF pole
- Significantly reduced amount of feedback at most frequencies => reduced performance



Source: Sedra&Smith - Microelectronic circuits

Compensation alternative - eliminate or shift internal pole

- If f_{p1} pole can be eliminated, then we apply the same strategy starting from f_{p2} => increased BW for A"(jω)

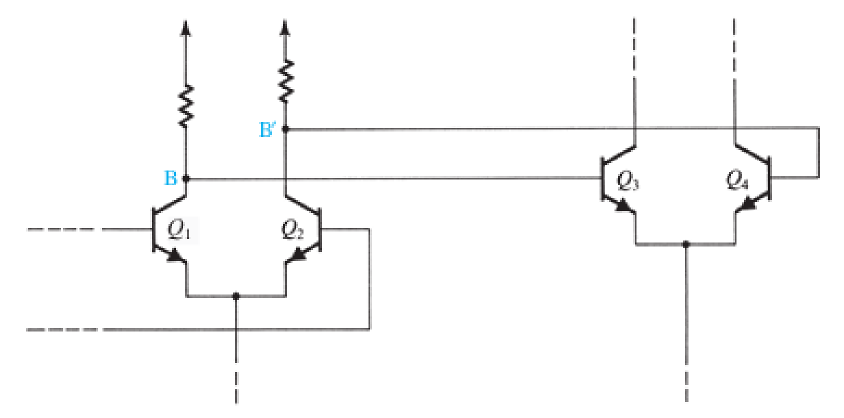


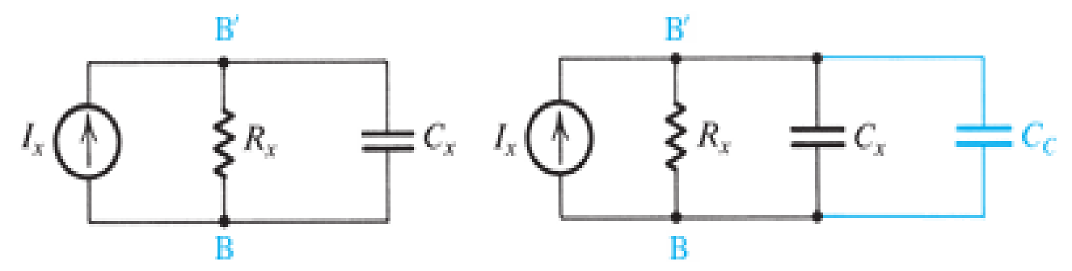
Source: Sedra&Smith - Microelectronic circuits

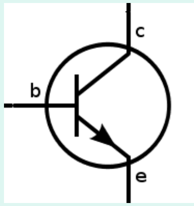
Circuit implementation

- Amplifier formed by cascaded gain blocks
- Assume f_{p1} given by the interface between the two differential stages
- Add compensating capacitor C_c
- Adding C_c might change the position of other poles => iterative approach
- C_c may be large (>100pF) => impractical in IC - Miller compensation as alternative (C_c in the feedback path)

$$\omega_{p1} = \frac{1}{R_x C_x} \xrightarrow{+C_C} \omega_{p1}^{'} = \frac{1}{R_x\left(C_x + C_c\right)}$$

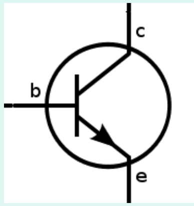






Compensators design

- Methods of improving the design of a control system
- Modify pole-zero patterns to achieve desired time or frequency responses
- **Compensation** (stabilization) = the act of modifying a system to reshape its root locus in order to improve system performance
- "Satisfactory compensation" - stable, satisfactory transient response, and large enough gain to ensure the steady-state error does not exceed the specified maximum
- Compensation devices: electric networks (filters) or mechanical equipment (levers, springs, dashpots, etc.)



Compensator design approaches

- Time-domain performance specification - PO (percentage overshoot), settling time, peak time, steady-state error
- Frequency-domain performance: peak of the closed-loop frequency response, resonant frequency, bandwidth, gain margin, phase margin - Bode plot, Nyquist plot, Nichols chart
- s-plane design - alteration of root-locus to obtain a suitable performance

PID (proportional-integral-derivative) Controller

Adv.: Good performance in a wide range of operating conditions; functional simplicity.

To implement a PID, 3 parameters must be determined.

"Academic" PID controller:

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s$$

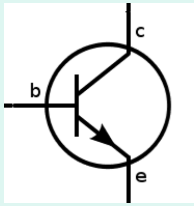
(Transfer function)

which corresponds to

$$u(t) = K_P e(t) + K_I \int e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

In practice:

$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_D s + 1}$$



PID Controller - variations

PI controller: Used extensively in process control on a broad range of applications due to simplicity and relatively good performance

$$\text{PI controller: } G_c(s) = K_P + \frac{K_I}{s}$$

PD controller: Used extensively in controlling electromechanical systems

$$\text{PD controller: } G_c(s) = K_P + K_D s$$

PID Controller

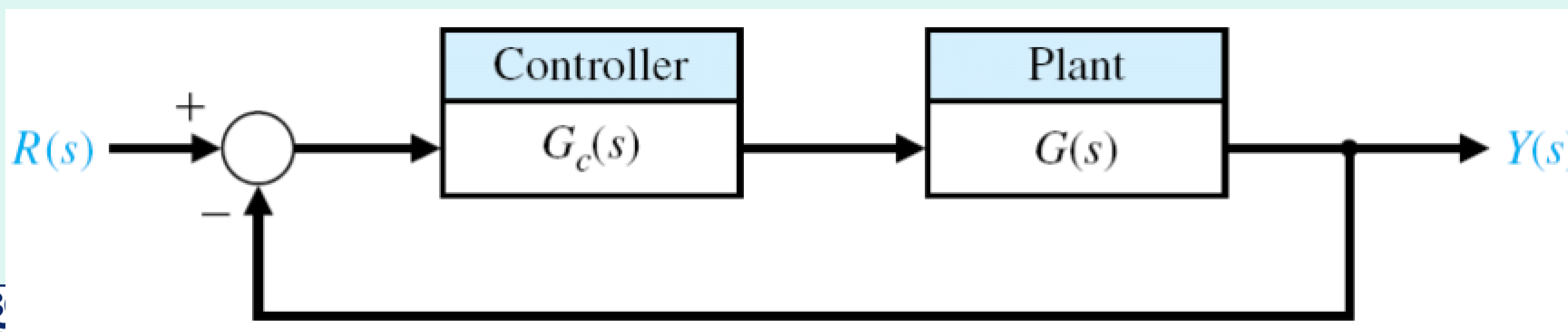
Consider the PID controller

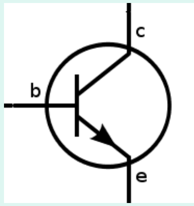
$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3 s^2 + K_1 s + K_2}{s}$$

$$= \frac{K_3(s^2 + as + b)}{s} = \frac{K_3(s + z_1)(s + z_2)}{s}$$

The PID controller introduces a pole at the origin and two zeros

A closed-loop system with a PID controller:





Compensators - roles

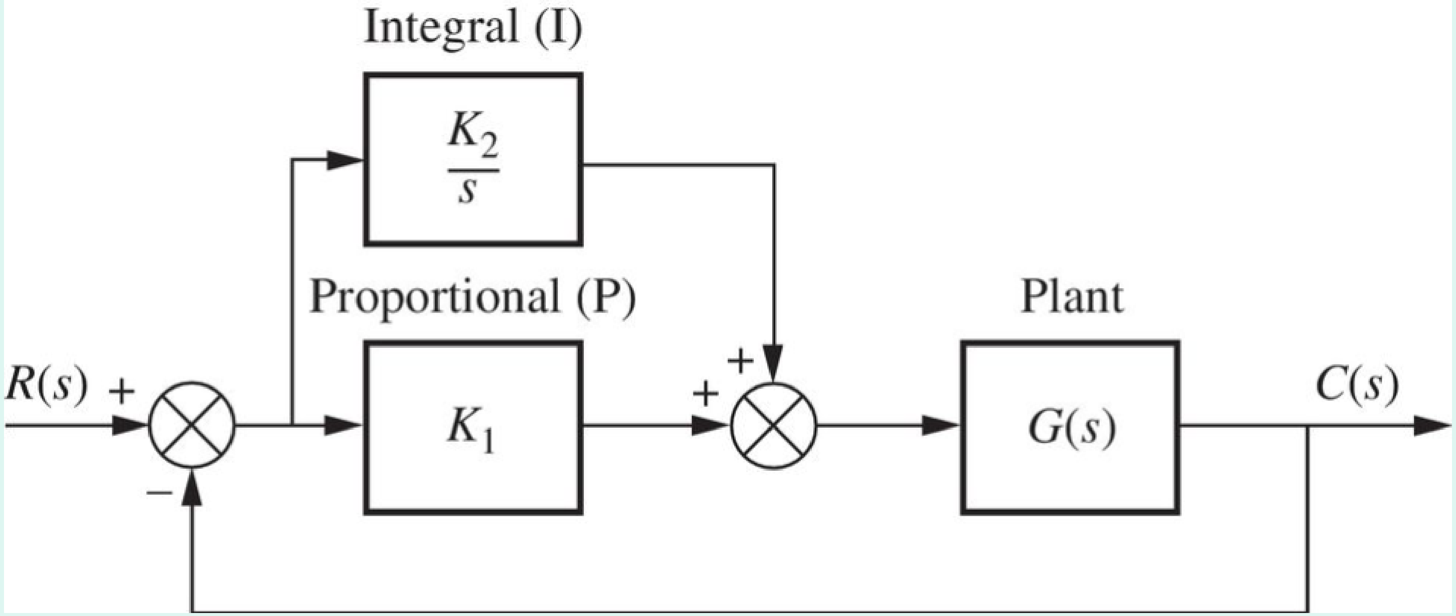
- Compensating for transient response – insert a differentiator in the forward path (derivative control)=> faster response
- Compensators – used to decouple as well between transient and steady-state error performance
- Proportional control: the higher the gain, the smaller the ε_{ss} but the larger PO => compromise
- Improving steady-state error: adding an open-loop pole at the origin in the forward path (integral control)

Ideal (active) vs. passive compensation

- **Ideal compensators** (e.g. 1/s) – pure integration can only be achieved with **active** components (amplifiers) – higher costs + power consumption
- **Passive compensators** – made with passive components => no pole at zero (like an ideal integral compensator), but close enough to meet requirements

PI controller

Generic case for ideal proportional-integral (PI) compensator - eliminates the steady-state error



$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{s}$$

Adjust the zero by varying K_2/K_1

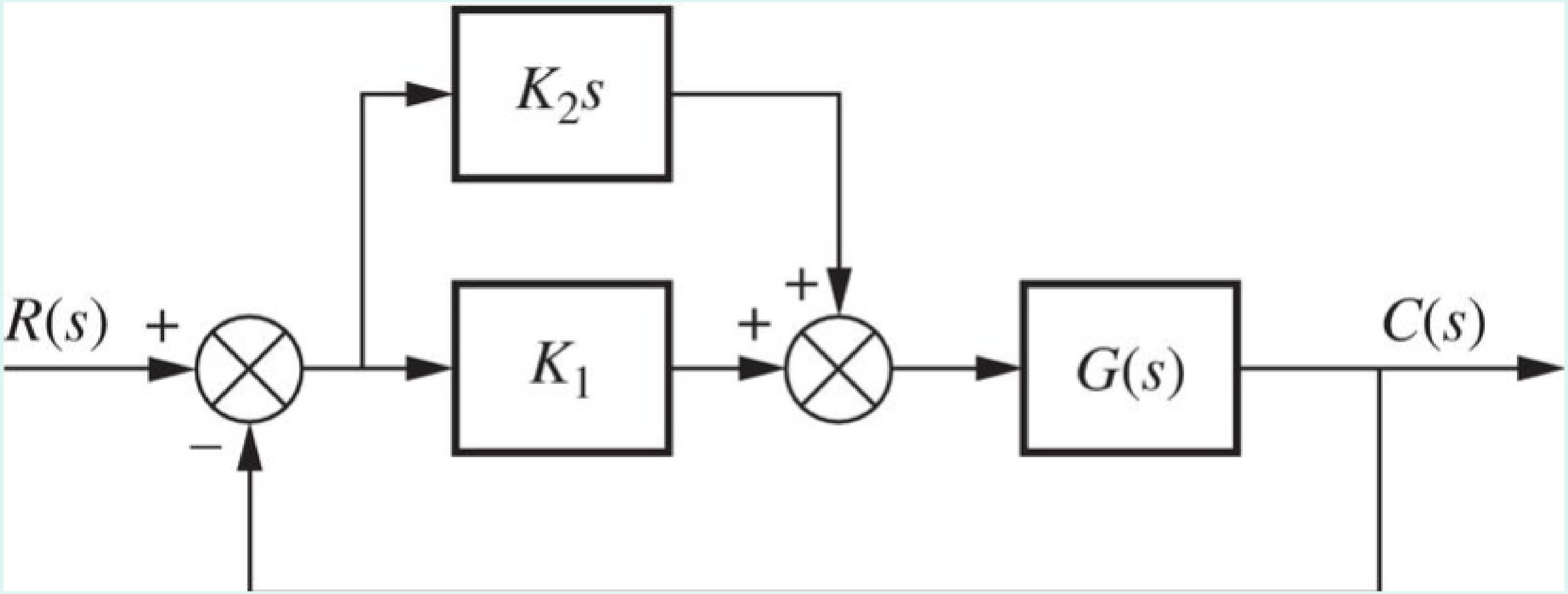
PD controller – improve transient response

- Typical goals: improve PO and settling time
- Technique: reshape RL so that it goes through the desired closed-loop location
- Simplest way: add a zero to the forward path
- Ideal derivative or PD controller:

$$G_c(s) = s + z_c$$

- Concl: transient responses unattainable by a simple gain tuning can be obtained by augmenting the system with an ideal derivative compensator

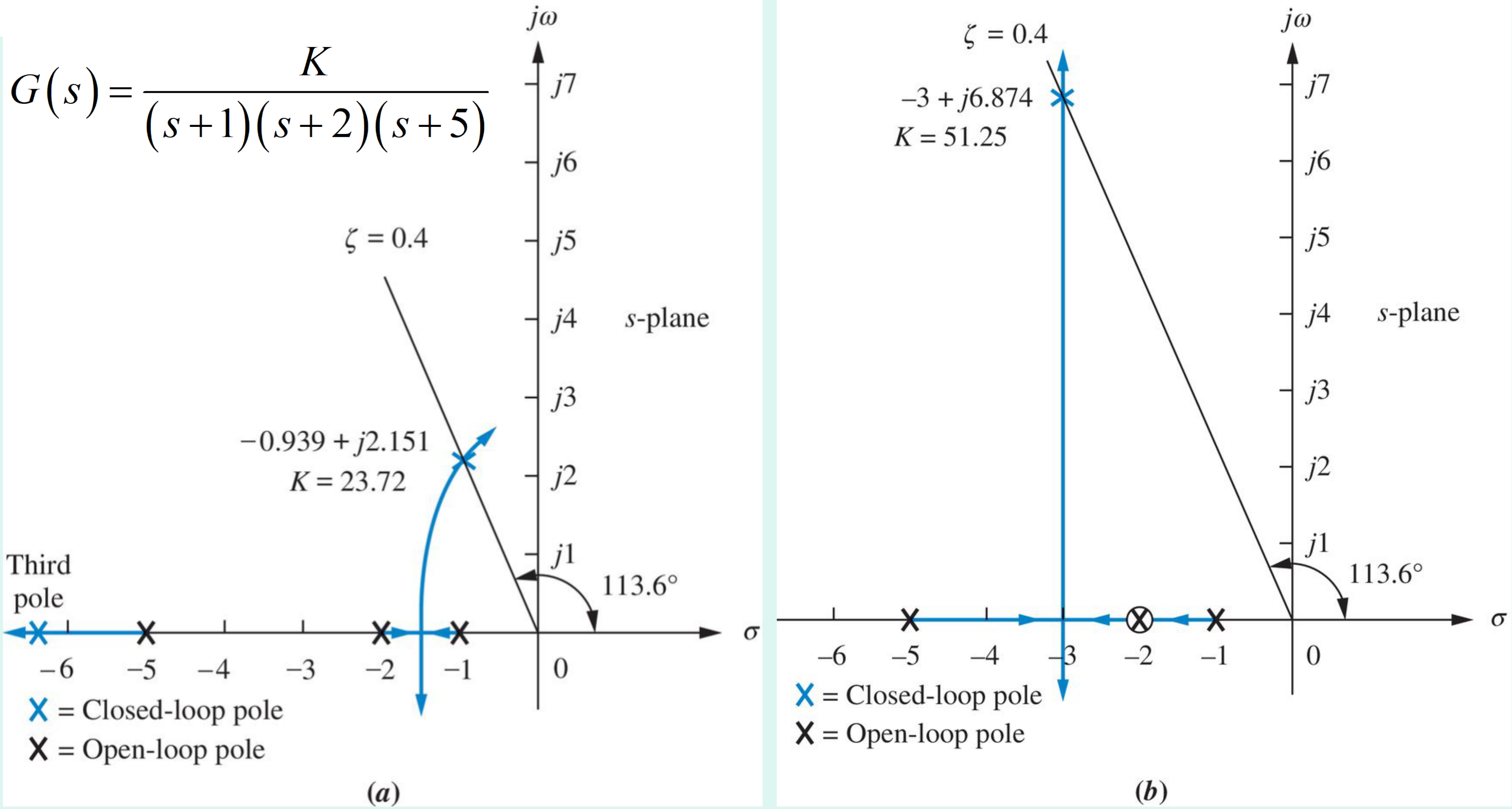
Generic PD controller



$$G_c(s) = K_2 s + K_1 = K_2\left(s + \frac{K_1}{K_2}\right)$$

Examples – effects on RL

$$G(s)=\frac{K}{(s+1)(s+2)(s+5)}$$

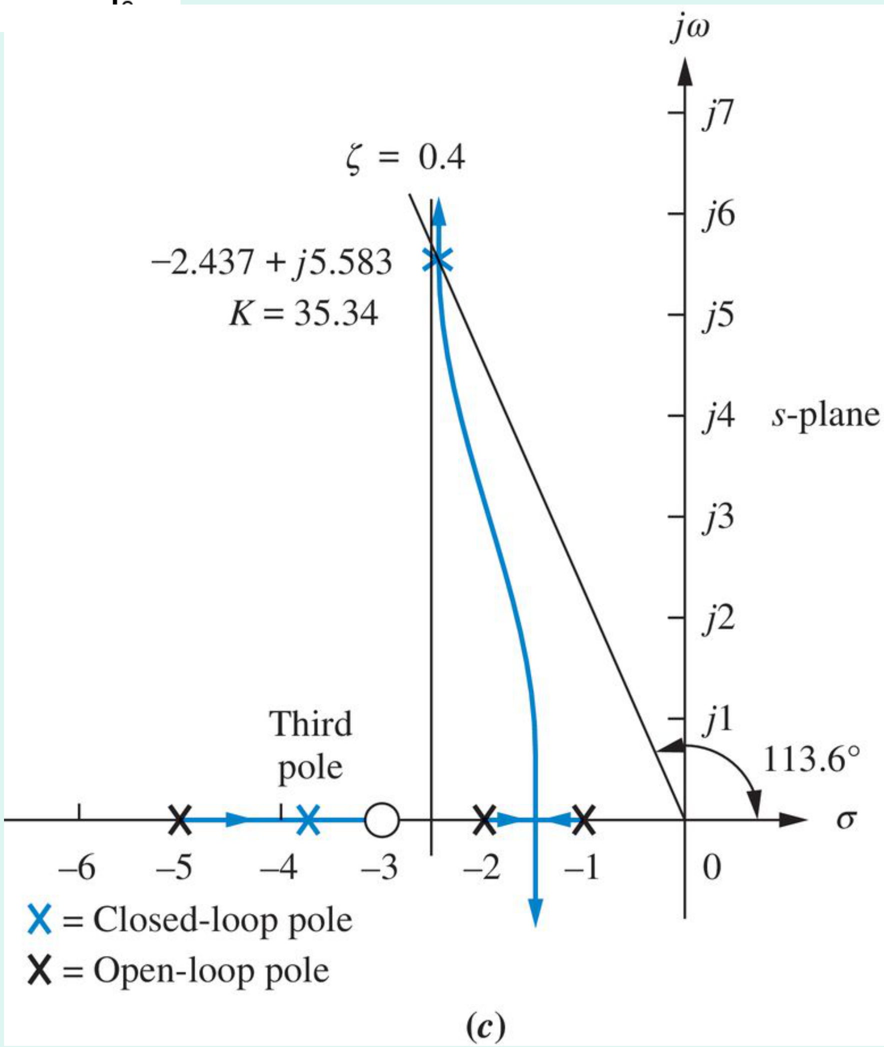


Uncompensated system

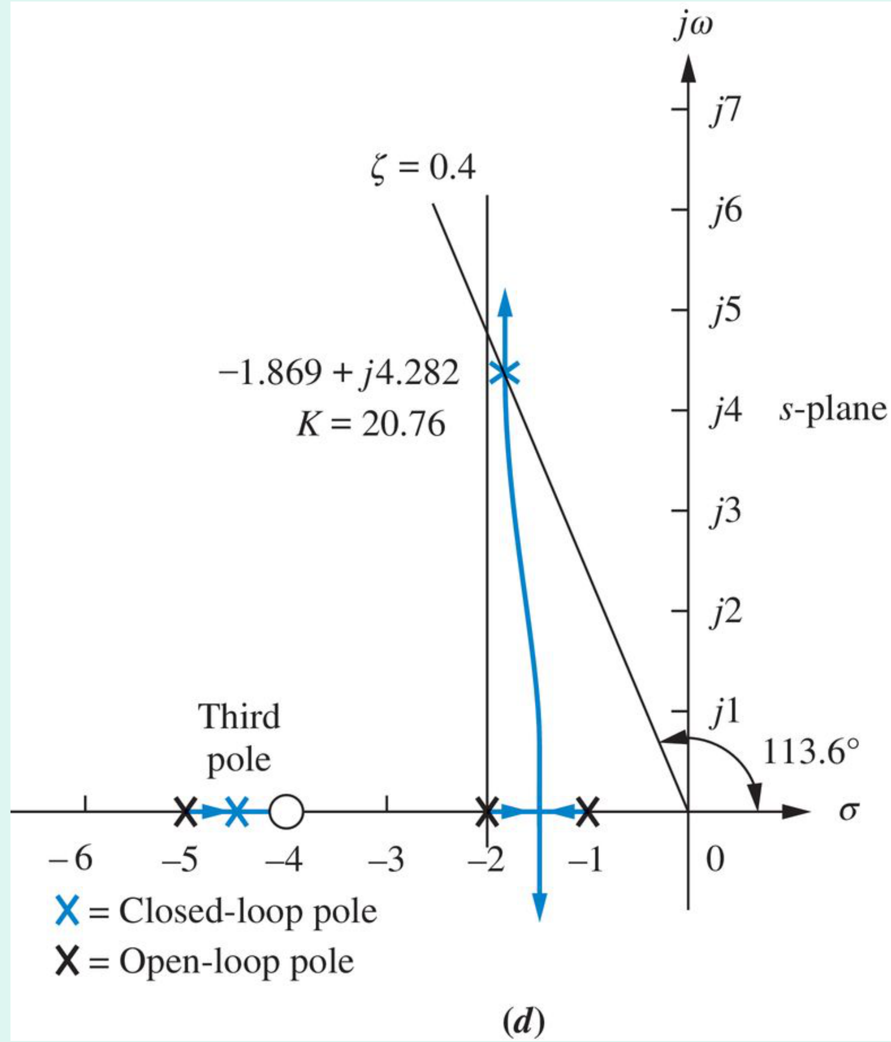
+ Zero z=-2

$$G_b(s)=\frac{K(s+2)}{(s+1)(s+2)(s+5)}$$

Example (cont.)

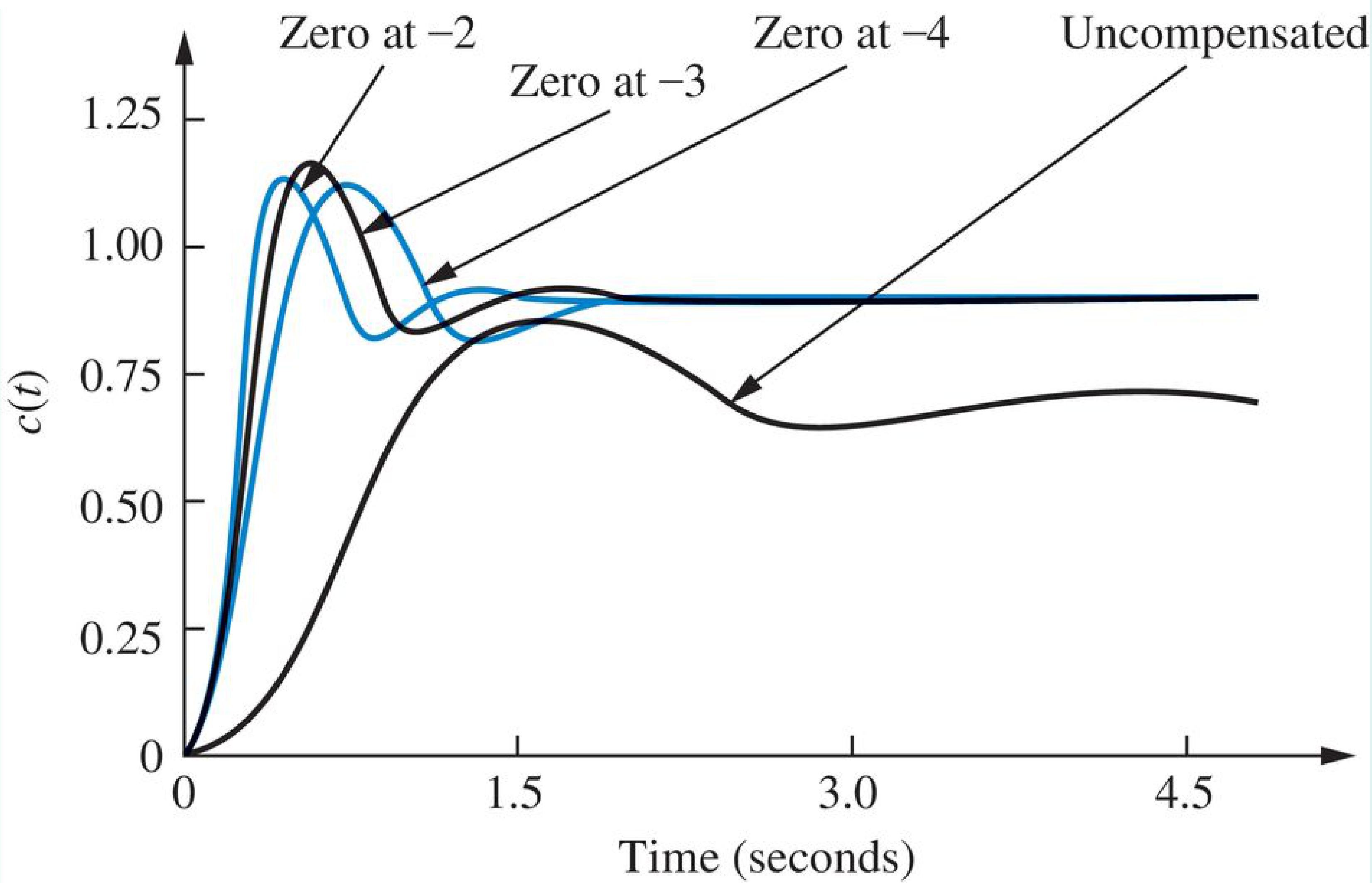


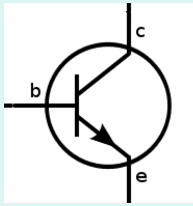
Compensator z=-3



Compensator z=-4

Transient simulations





Conclusions for PID compensators/controllers

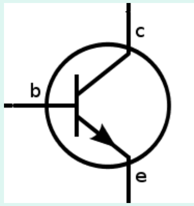
- Classic compensators - PI, PD, PID - structural modifications in the feedback structure (addition of zeros and poles)
- Design of PI, PD compensators by root locus approach

strategy: maintain one gain parameter and add zero+pole so that you optimize the feedback system performance

- PI compensators - address the improvements in the steady-state response
- PD compensators - address improvements in the transient response

Frequency compensation techniques

- Some common methods:
  - Phase-lag and phase-lead compensation - introduce additional phase lag or lead at low frequencies to stabilize the circuit
  - Miller effect compensation - add a feedback capacitor to reduce the closed-loop gain
  - Isolation resistor placement - resistors to dampen the output before reaching a capacitive load or set a zero in TF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