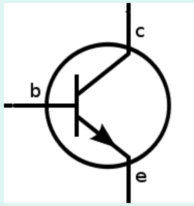
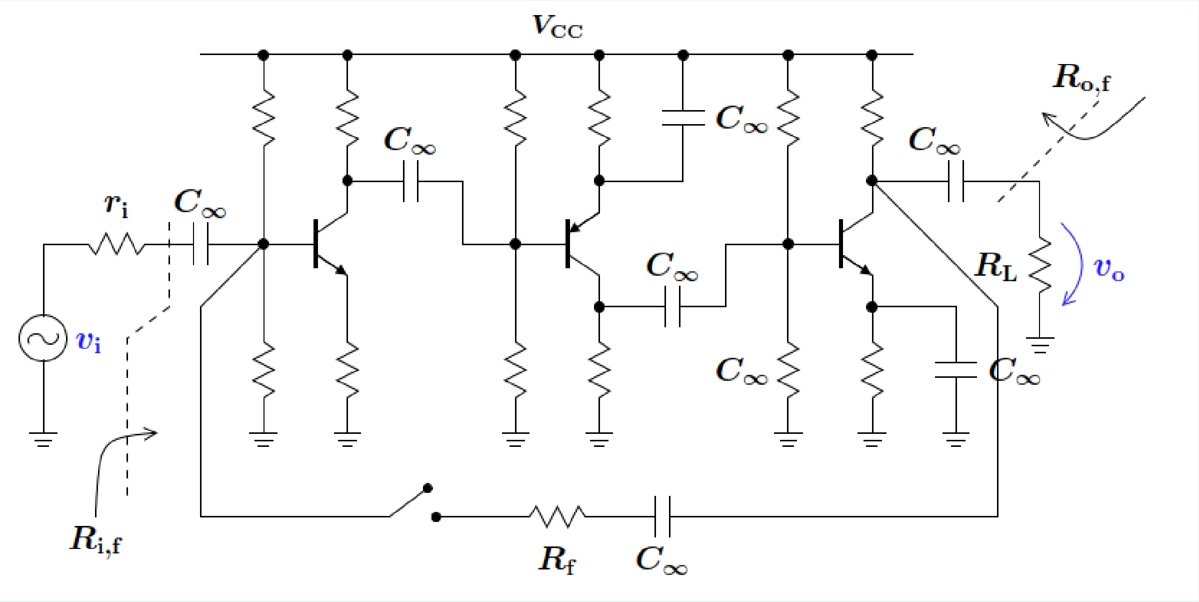
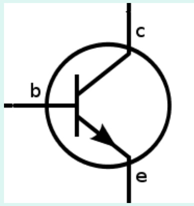


ELEC 301 - Filters - Butterworth filters

L29 - Nov 20

Instructor: Edmond Cretu



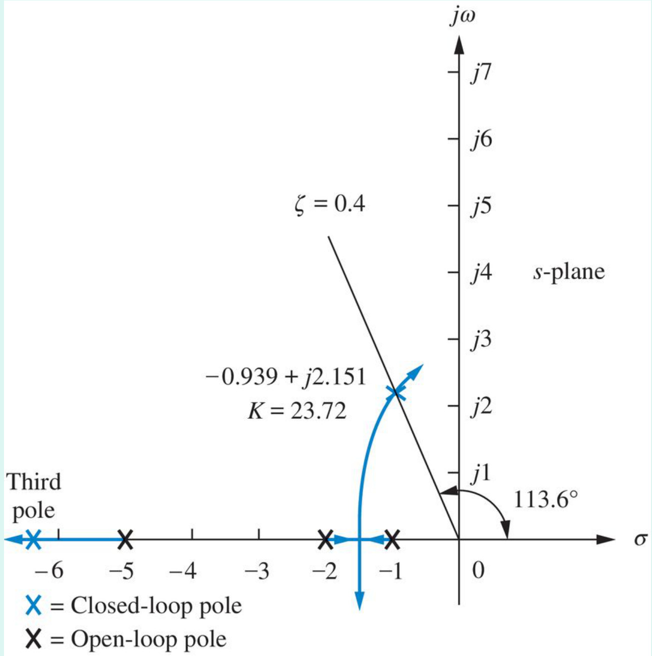


Last time

- Frequency compensation techniques
  - **dominant pole compensation** (a form of **lag compensation**) - a pole placed at an appropriate LF in the open-loop response, to reduce the gain of the closed-loop amplifier to 0dB for a frequency at or just below the location of the next pole.
  - **Miller compensation (feed-forward compensation)** and pole-splitting - capacitor action amplified by the feedback network
- Filters introduction

Other compensation methods

- **Lead compensation** - places a zero in the open-loop response to cancel one of the existing poles (**pole-zero cancellation**) (remember the PD controller)
- **Lead-lag compensation** - places both a zero and a pole in the open-loop response; usually the pole is at open-loop gain<1



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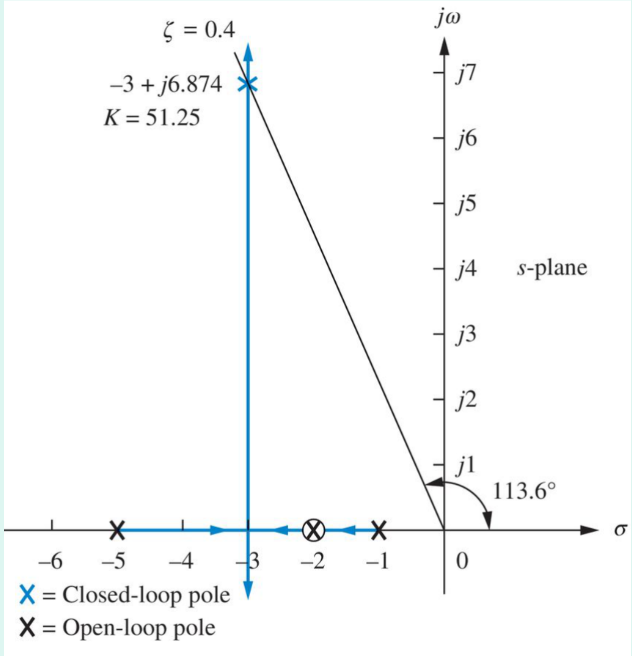

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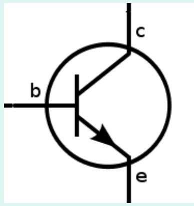
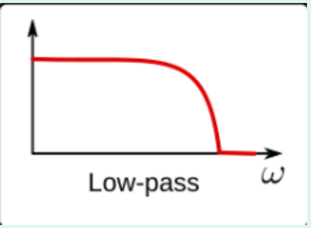
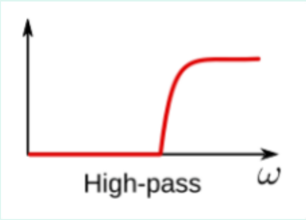
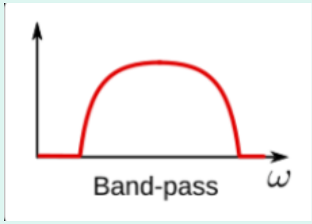


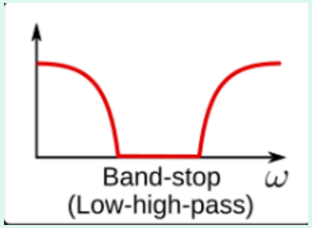
Back to signal filters

- Electronic filters
  - **low-pass filter** - allows frequencies below a certain point to pass while blocking higher frequencies
  - **high-pass filter** - allows frequencies above a certain point to pass and blocks lower frequencies
  - **band-pass filter** - allows only frequencies within a specific range to pass
  - **band-stop filter** - blocks frequencies within a specific frequency range
  - **notch filter** - a sub-type of band-stop filter, blocking only a narrow frequency band







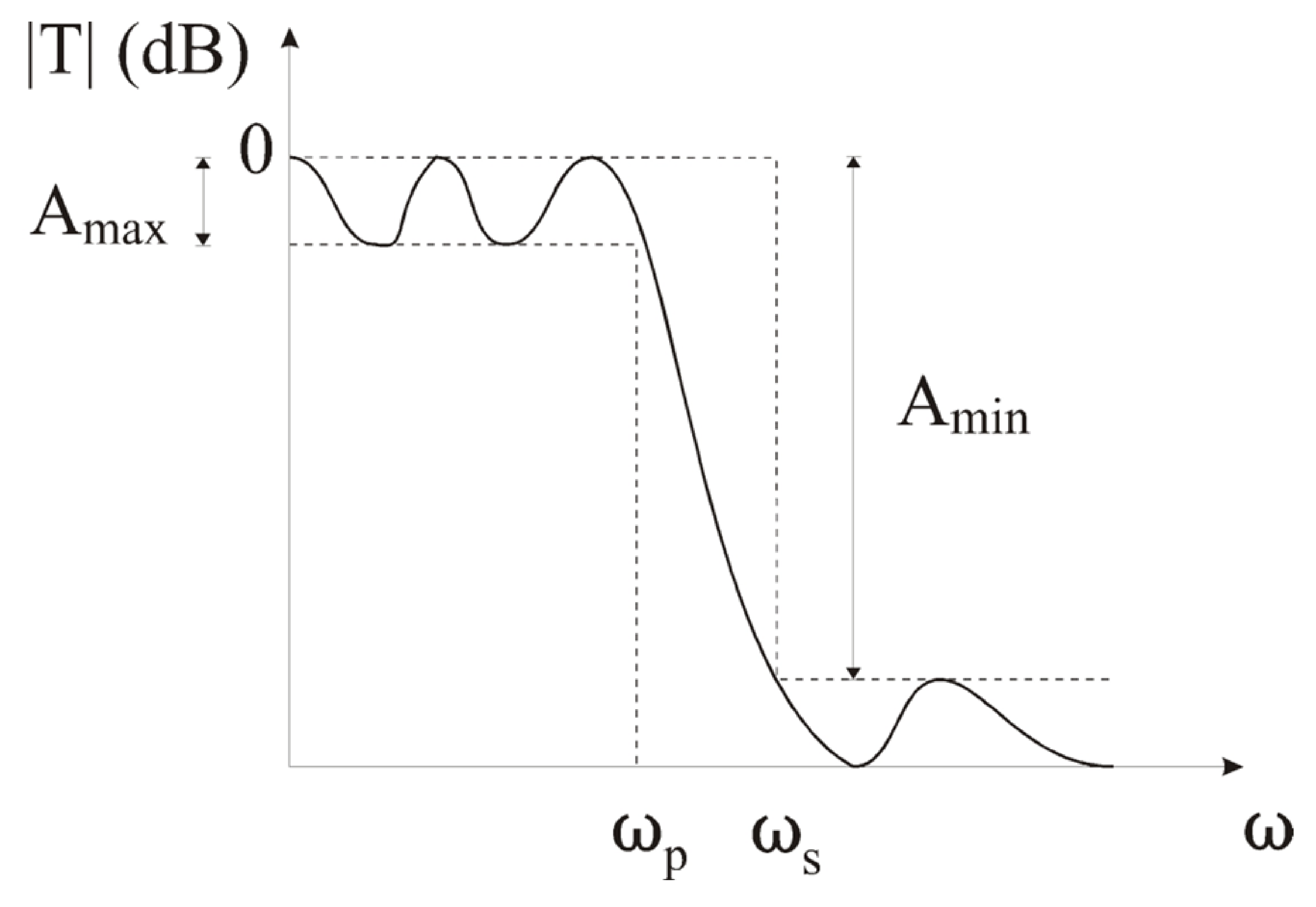


Network synthesis

- **Network synthesis** - design methodology for linear CT filters
- Technique: take an ideal filter frequency response and approximate it by a polynomial function
- Filter groups:
  - **Butterworth** filters - maximally flat frequency response in the pass-band
  - **Chebyschev** filters - best approximation to the ideal response of any filter for a specified order and ripple
  - **Elliptic** filters - steepest cutoff of any filter for a specified order and ripple
  - **Bessel** filters - maximally flat phase delay

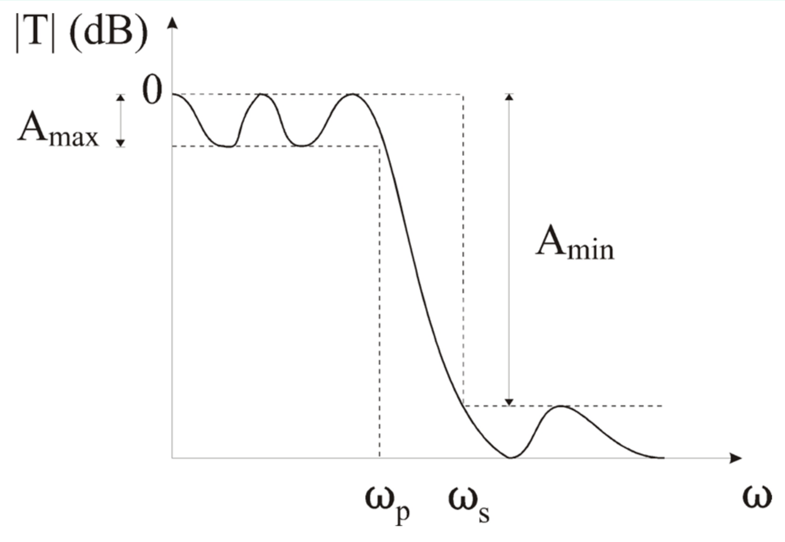
Back to filters - terminology

- A generic LP filter transfer function



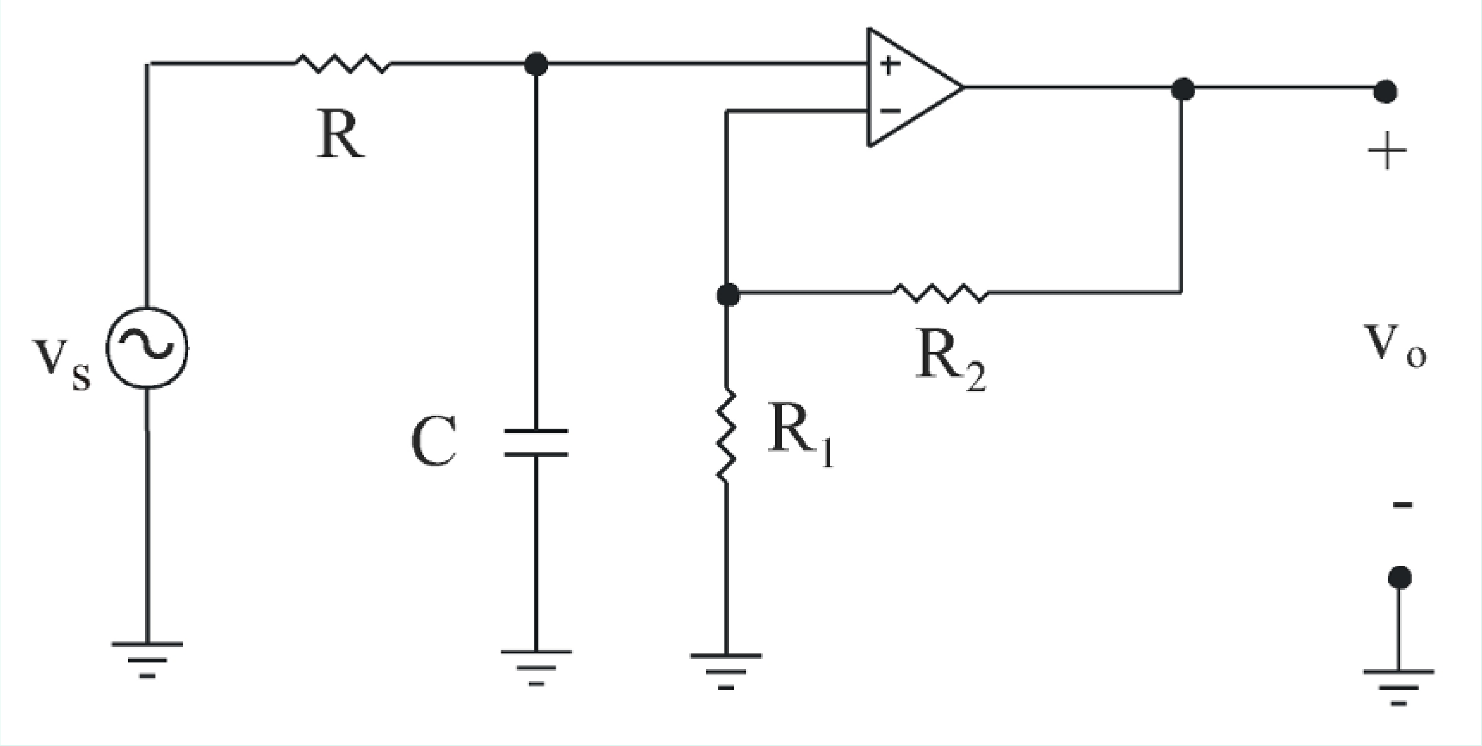
Filter specification and terminology

- Standard properties of a filter - the example of LP filter
- Pass-band = range of frequencies allowed to pass ($<\omega_p$)
- Stop-band = frequency range that is attenuated ($>\omega_s$)
- Transition-band ($\omega_p<\omega<\omega_s$)
- Selectivity factor = ω_p/ω_s
- Maximum ripple level tolerated in the pass-band (A_{max}[dB])
- A_{min}[dB]=min amount of attenuation between the stop-band and the transmission peak in the pass-band



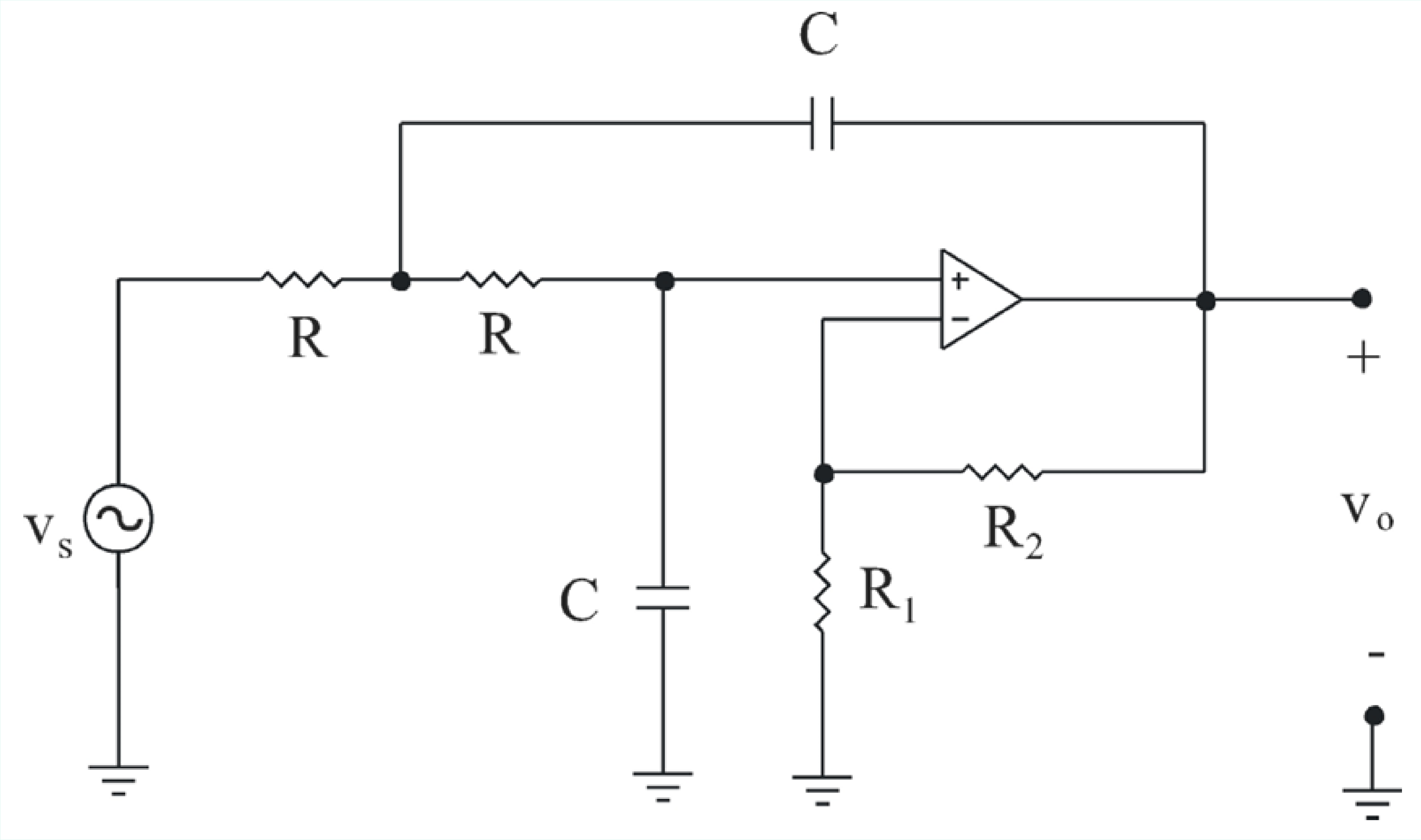
The Butterworth filter

- LP Butterworth filter = all-pole filter (all its zeros are at ω=∞)
- 1st order LP active filter (**Sallen-Key** configuration)



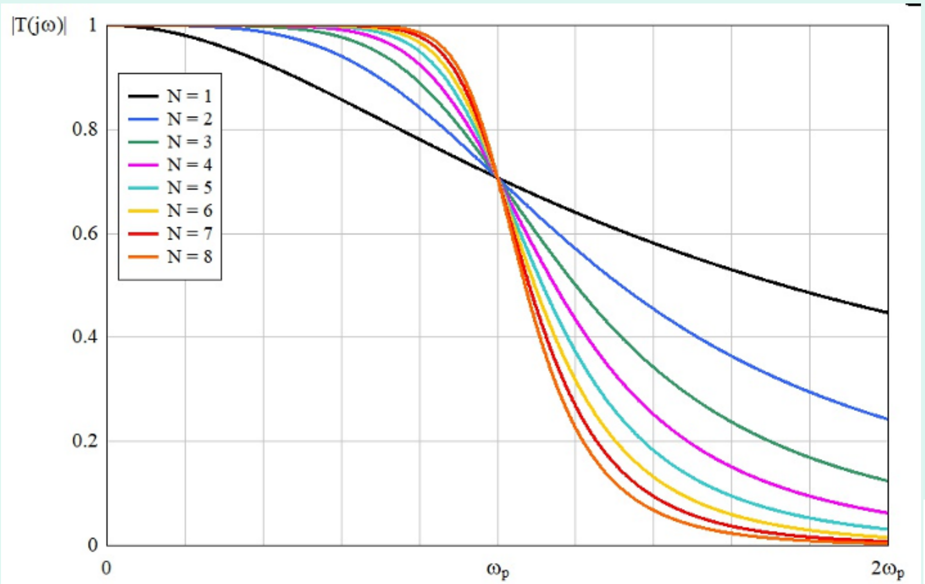
2nd order LP Butterworth filter

- Active filter, Sallen-Key configuration

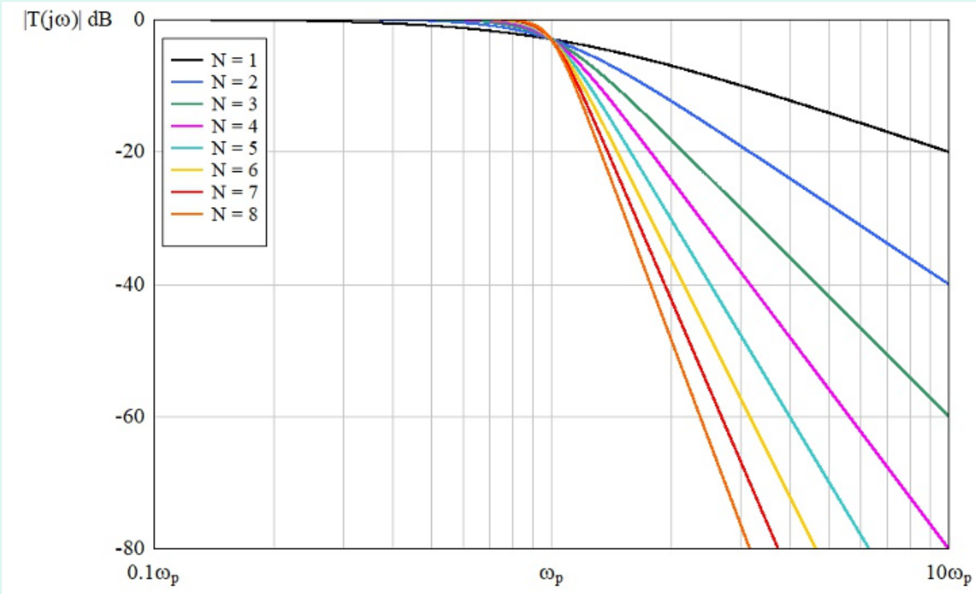


Magnitude response and filter order (N)

- Selectivity increases with N



Linear scales



Logarithmic scales

LP Butterworth - TF

- For an Nth order Butterworth filter, the magnitude response:

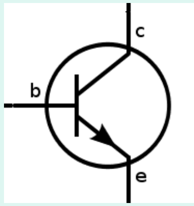
$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} \Rightarrow |T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}}$$

ε = pass-band deviation parameter - determines A_{max}

$$A_{max} = 20\log\sqrt{1+\varepsilon^2} = 10\log\left(1+\varepsilon^2\right) \Leftrightarrow \varepsilon = \sqrt{10^{A_{max}/10}-1}$$

For a Butterworth filter, the response is very flat near ω=0, and A_{max} is maximum when $\omega=\omega_p$ - this is known as **maximally flat response** (one of the features of Butterworth filters)

$$\text{For } A_{max} = 3dB = 10\log_{10}2 \Rightarrow \varepsilon = \sqrt{10^{10\log_{10}2/10}-1} = 1$$



Filter order influence

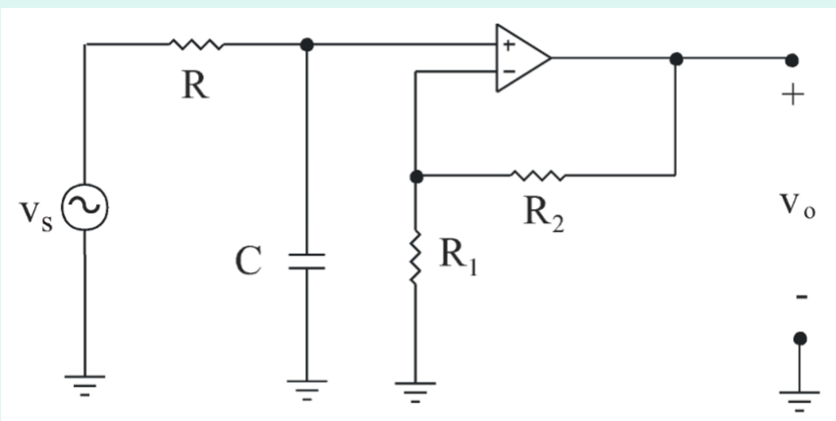
- As N increases, the pass-band region approaches a maximally flat response throughout the pass-band, the transition region decreases, and the filter response approaches the ideal case
- Attenuation at ω_s=edge of the stop-band

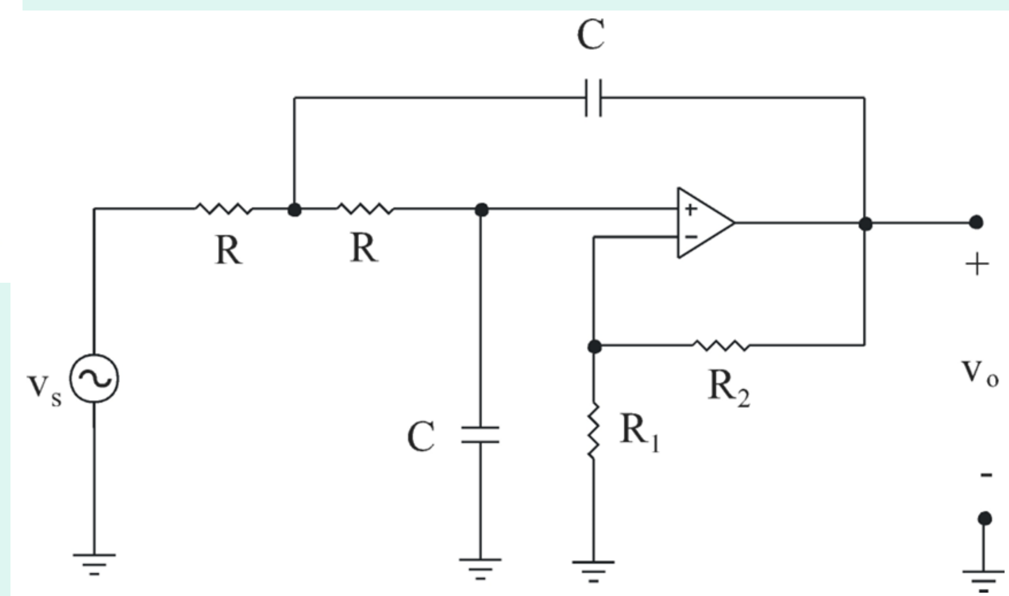
$$A(\omega_s) = -20\log\frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}}} = 10\log\left(1+\varepsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$$

Design perspective: find the lowest integer value of N that satisfies $A(\omega_s)$>Amin

Building higher order filters

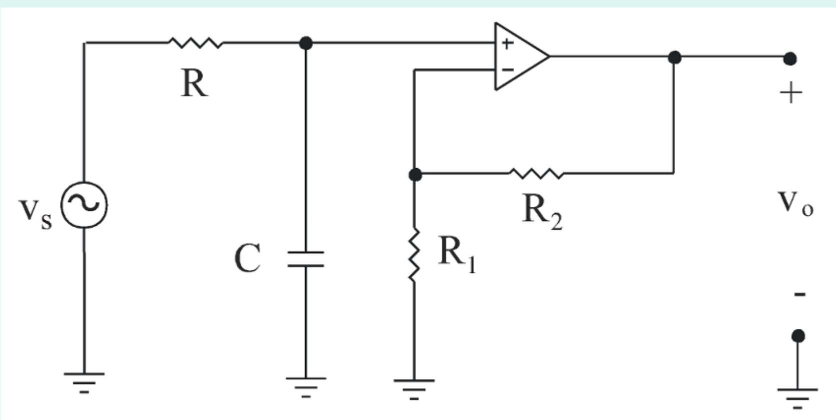
- Analog filter of order N - can be achieved by cascading 1st and 2nd order filters until the desired order is attained
- Exm: 5th order filter = 2x (2nd order filter)+ 1x (1st order filter)

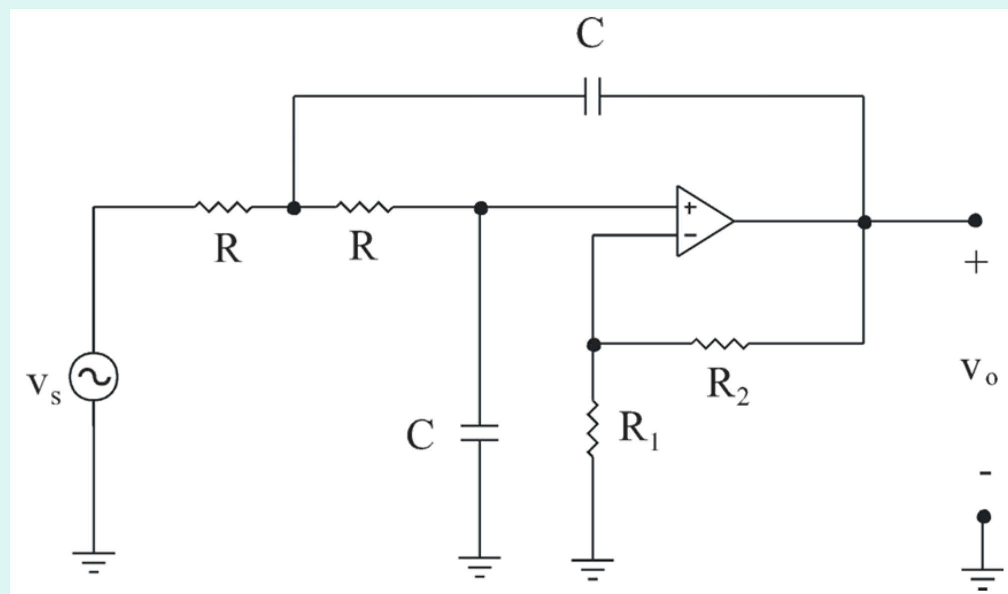




Filter transformation

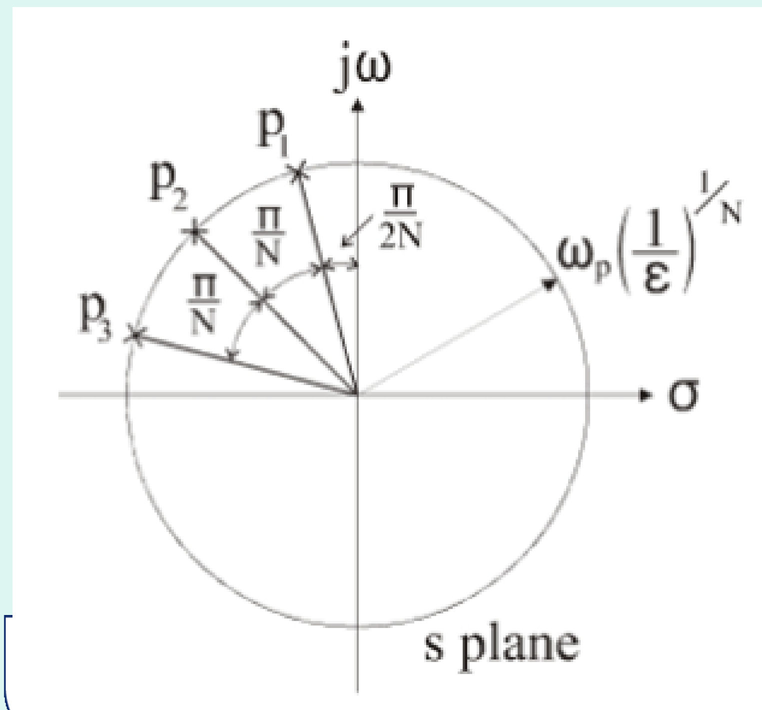
- HP Butterworth filter - exchange Rs and Cs (leaving R_1 and R_2 unchanged)
- for band-pass or band-stop filters - combine LP and HP filter stages

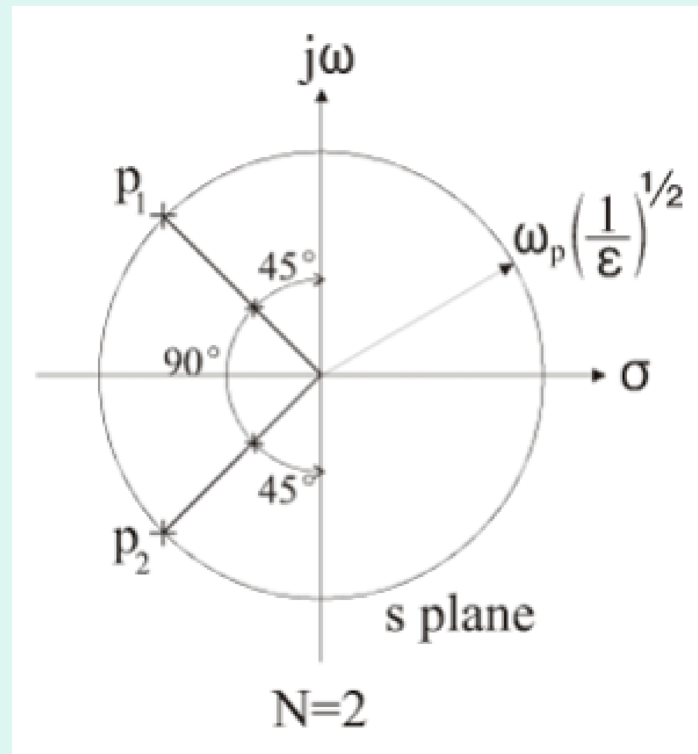




Nth order Butterworth filter

- Pole location - through a graphical approach
- All poles are in the LHP, and lie on a circle of radius $\omega_p(1/\varepsilon)^{1/N}$, equally spaced π/N apart.
- First pole located at an angle of $\pi/2N$ from the $+j\omega$ axis





Nth order Butterworth filter

- Once the pole locations are determined, the filter transfer function T(s):

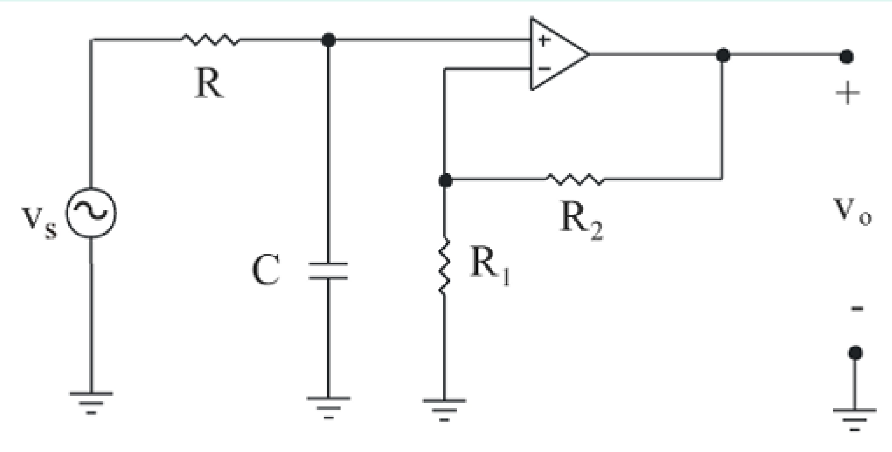
$$T(s) = K \frac{\frac{\omega_p^N}{\varepsilon}}{(s-p_1)(s-p_2)..(s-p_N)} = K \frac{\omega_0^N}{(s-p_1)(s-p_2)..(s-p_N)},$$

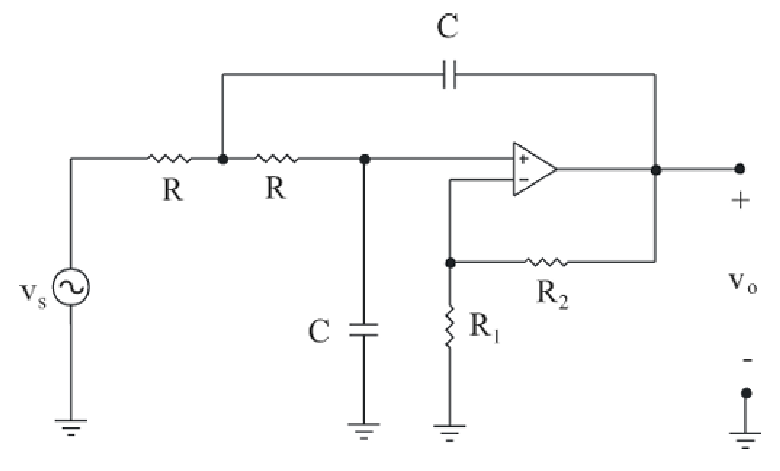
$$\omega_0 = \omega_p \left(\frac{1}{\varepsilon}\right)^{1/N}$$

For a Butterworth filter we use $\varepsilon=1$=> for 1st order filter $|\omega_{p1}|=\omega_0$, for a 2nd order filter $|\omega_{p1}|=|\omega_{p2}|=\omega_0$

Normalized look-up tables

- The design can be accomplished using normalized look-up tables
- Simplified circuits with equal values components(R, C) → simplify the process selecting component values
- Normalized Butterworth polynomials - to determine the denominator of T(s) for a given filter order

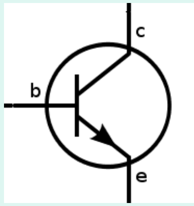




Normalized Butterworth polynomials

Normalized Butterworth Polynomials	
Order	Factors of Polynomial
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.932s + 1)$

- The table is used to determine T(s) for a given order



Butterworth polynomials

- Factorization into 1st and 2nd order terms <=> cascading 1st and 2nd order filter stages
- Second order polynomials have the form:

$$B_2\left(s\right) = s^2 + 2\zeta s + 1, \quad \zeta = \text{damping factor}$$

- The gain for each stage/section is given by:

$K=3-2\zeta=1+R_2/R_1$

- The cut-off frequency is $\omega_c=1/(RC)$

Design example

- Problem: Use the normalized tables to design a 4th order LP Butterworth filter with a cut-off frequency f_c=10kHz
- Solution: N=4 → look into the table

Normalized Butterworth Polynomials	
Order	Factors of Polynomial
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
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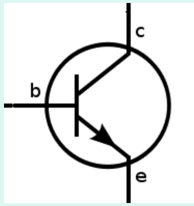


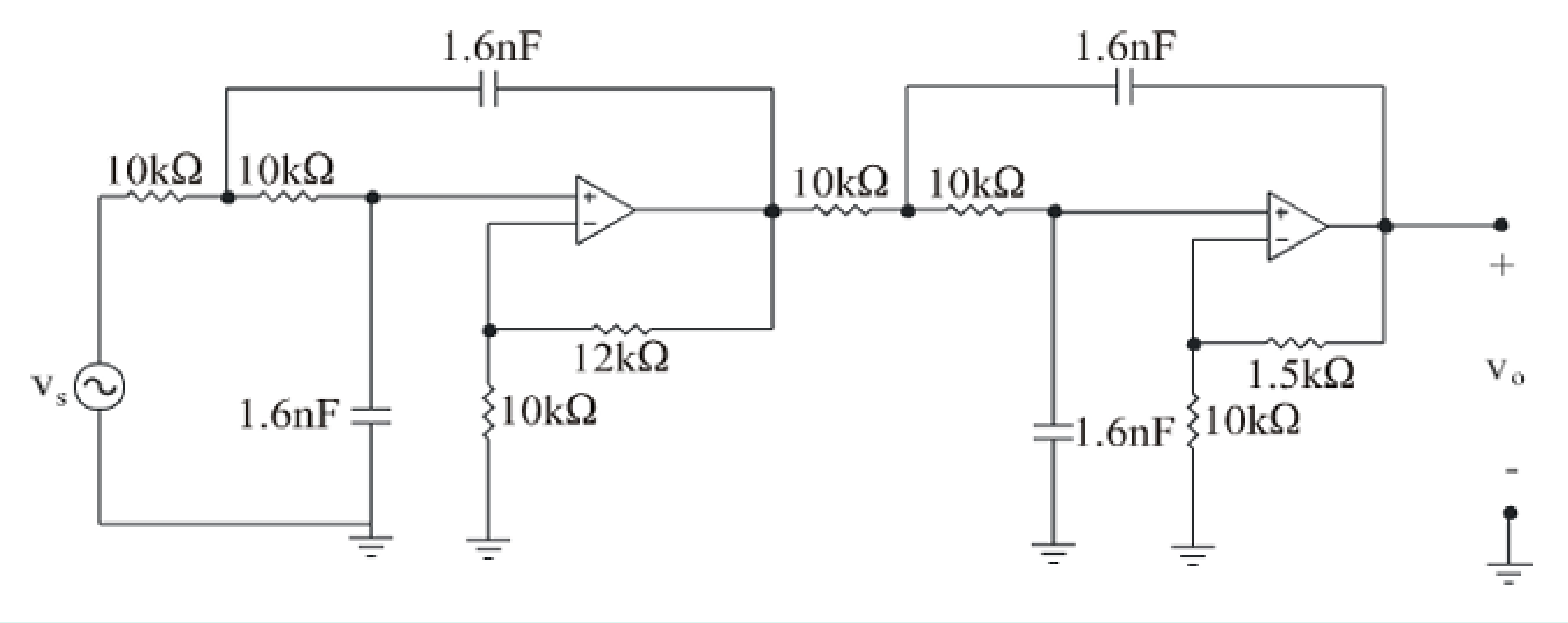
Design example (2)

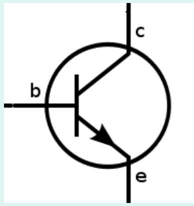
$$D(s) = \left(s^2 + 0.765s + 1\right)\left(s^2 + 1.848s + 1\right)$$

- For stage 1: 2nd order filter, gain K_1=3-0.765=2.235
- For stage 2:2nd order filter, with gain K_2=3-1.848=1.152
- use $K=1+R_2/R_1$, arbitrarily choose a common value (choose standard resistor values) R_1=10kΩ => R_2=12.35kΩ for stage 1
- Stage 2 - choose R_1=10kΩ => R_2=1.52kΩ
- For R, C - we wish small values for C, so that the physical component is not bulky => for R=10kΩ, we get C=1.6nF

Design example (3)

- Resulting circuit - gain~2.5

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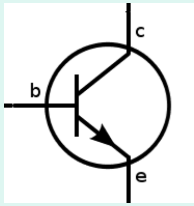
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