ELEC 301 - Filters - Chebyshev

L30 - Nov 24

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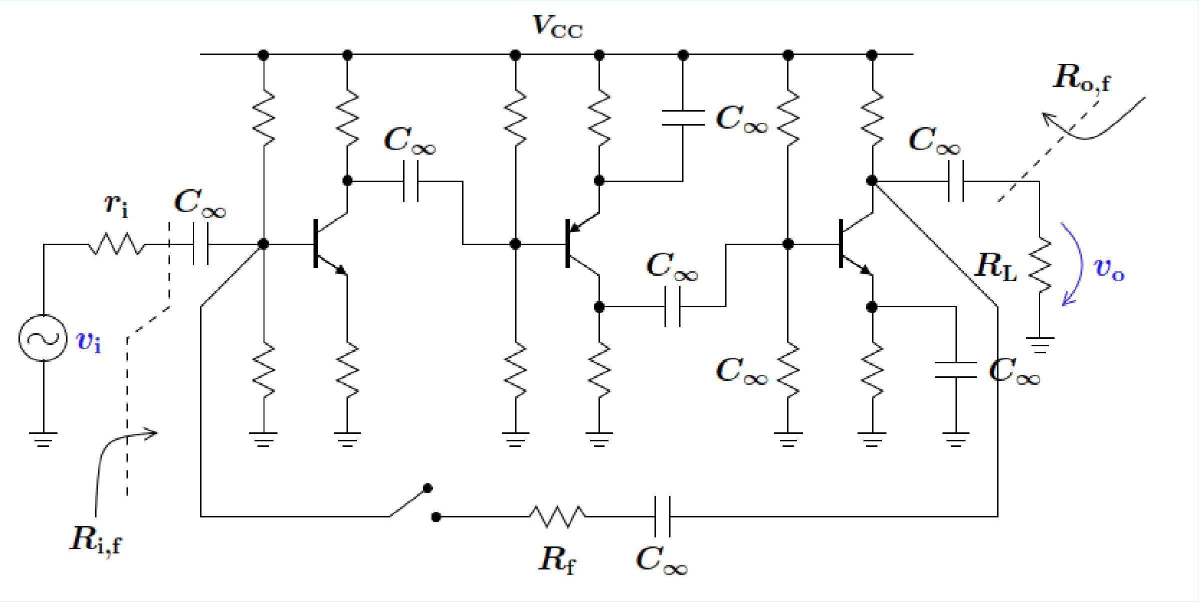
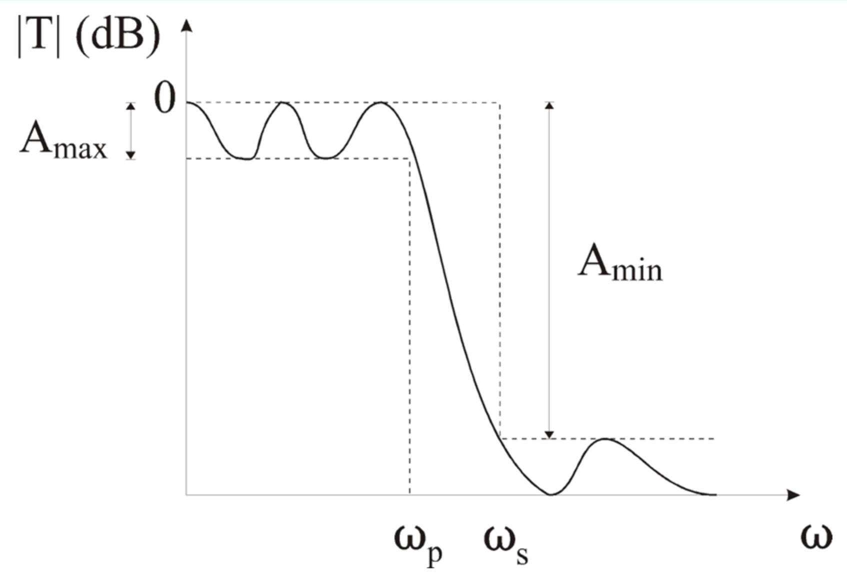
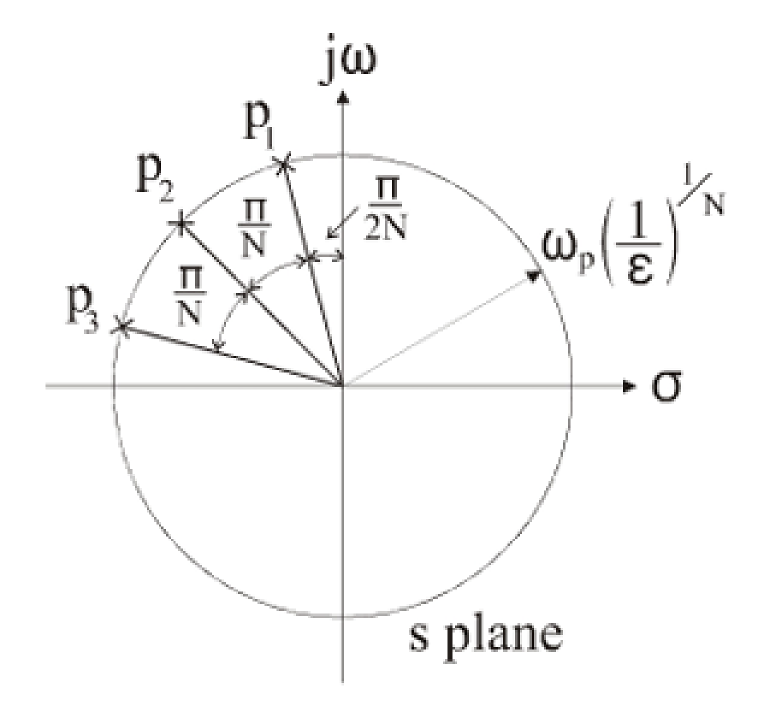


Last time

- The design of Butterworth filters
- Use 1st and 2nd order Sallen-Key circuit blocks
- Butterworth polynomials - distribute the poles regularly on a half-circle in the HLP
- The Butterworth active filters minimize the ripples in the passband

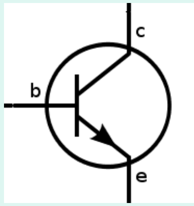




Recall: Network synthesis

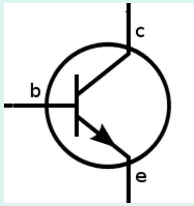
- **Network synthesis** - design methodology for linear CT filters
- Technique: take an ideal filter frequency response and approximate it by a polynomial function
- Filter groups:
  - **Butterworth** filters - maximally flat frequency response in the pass-band
  - **Chebyschev** filters - best approximation to the ideal response of any filter for a specified order and ripple
  - **Elliptic** filters - steepest cutoff of any filter for a specified order and ripple
  - **Bessel** filters - maximally flat phase delay

- Further information - see Ch 16 in Sedra and Smith - Microelectronic circuits



Chebyshev filter

- Active filter (opamps + Rs,Cs) - it can also be implemented using Sallen-Key circuit structure
- Chebyshev filter vs Butterworth filter:
  - the Chebyshev filter, for the same polynomial order, has a steeper roll-off (smaller gap between the pass-band and the stop-band
  - Chebyshev filter has more ripple in both the pass-band and the stop-band

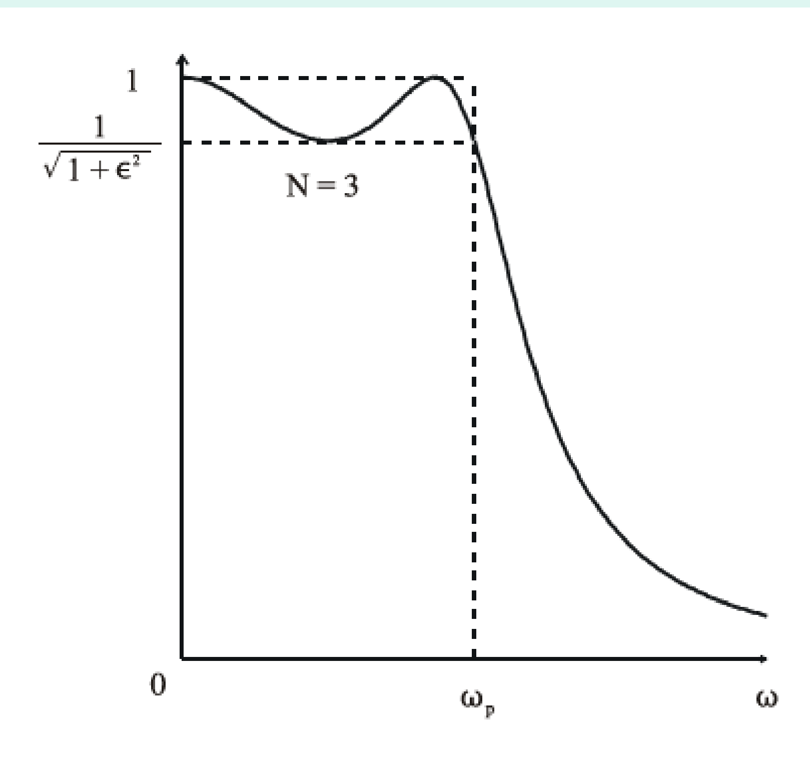


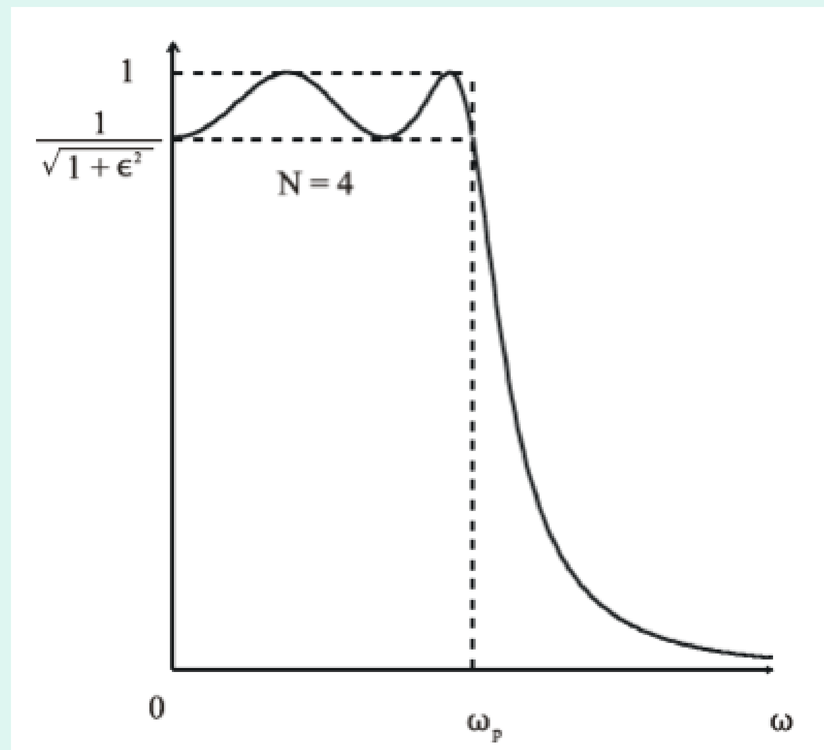
Chebyshev filters

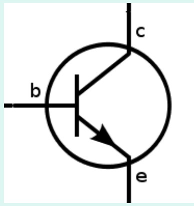
- Two common types:
  – Type 1 filter - “equiripple” response in the pass-band + monotonically decreasing transmission in the stop-band
  – Type 2 filter - maximally flat pass-band and equiripple response in the stop-band
- The number of passband maxima and minima combined is equal with the order of the filter

Odd- and even-order type I Chebyshev filters

- The even-order filter has a maximum magnitude deviation at ω=0
- The odd-order filter has a minimum deviation at ω=0
- Like the Butterworth filter, this is an all-pole filter







Chebyshev type I - T(s)

- Use same variables as for the Butterworth filter, in order to specify the key properties
- ε = pass-band deviation parameter - determines A_{max}
- Nth order Chebyshev filter, with a pass-band edge at ω_p:

$$
\left|T(j\omega)\right| = \begin{cases} \dfrac{1}{\sqrt{1+\varepsilon^2 \cos^2\left(N\cos^{-1}\left(\omega/\omega_p\right)\right)}} & \text{for } \omega \le \omega_p \\ \dfrac{1}{\sqrt{1+\varepsilon^2 \cosh^2\left(N\cosh^{-1}\left(\omega/\omega_p\right)\right)}} & \text{for } \omega \ge \omega_p \end{cases} \Rightarrow \left|T\left(j\omega_p\right)\right| = \frac{1}{\sqrt{1+\varepsilon^2}}
$$

The pass-band ripple A_{max}

- ε = pass-band deviation parameter - determines A_{max}

$$A_{max} = 20\log\sqrt{1+\varepsilon^2} = 10\log\left(1+\varepsilon^2\right) \Leftrightarrow \varepsilon = \sqrt{10^{A_{max}/10}-1}$$

The attenuation of the Chebyshev filter at the stop-band edge ($\omega=\omega_s$):

$$\left|T(j\omega)\right|_{\omega\geq\omega_p} = \frac{1}{\sqrt{1+\varepsilon^2\cosh^2\left(N\cosh^{-1}\left(\omega/\omega_p\right)\right)}}$$

$$A(\omega_s) = 10\log\left(1+\varepsilon^2\cosh\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right)$$

Design - determine N

- For a desired minimum attenuation Amin of the stop-band, we can determine the minimum number N of poles needed to achieve the requirement $A(\omega_s) > A_{min}$
- Increasing N => the magnitude response of the filter will approach the ideal case

$$A(\omega_s) = 10\log\left(1+\varepsilon^2\cosh\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right)$$

Chebyshev filter - poles

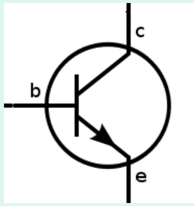
- The poles of the Chebyshev filter, for n=1,2 .. N

$$p_n = -\omega_p \sin\left(\frac{2n-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_p \cos\left(\frac{2n-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right)$$

The transfer function:

$$T(s) = K\frac{\omega_p^N / \varepsilon}{2^{N-1}(s-p_1)(s-p_2)\ldots(s-p_N)}$$

Remark: poles are spaced along an elliptical path in the LHP of the s-plane. The closer proximity of the poles to the jω-axis (compared with the Butterworth filter) is the source of both the increased ripple in the transfer function and the steeper roll-off beyond the 3dB point

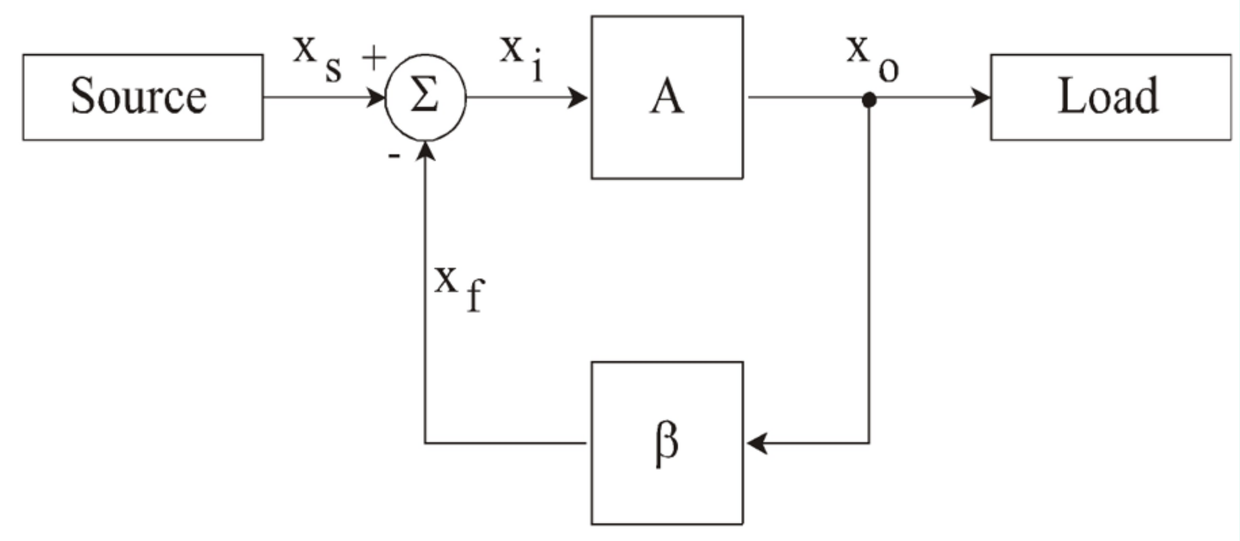


Oscillators

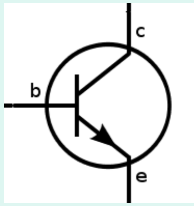
- **Linear oscillator** = a circuit that generates a stable sinusoidal waveform at a specific frequency, with uniform amplitude
- **Nonlinear oscillator** = a circuit that generates a function different than an harmonic signal, e.g. a square or triangular waveform - Exm: use as a timing device (provide clock pulses to be used as timing reference for digital systems)
- Oscillators can generate an output signal **without** an input signal

Op-amp sine-wave oscillators

- Remark: even for a linear oscillator, some nonlinearity is essential in order to provide an amplitude control of the output sine wave



$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} \xrightarrow{A(j\omega)\beta(j\omega)\to -1} \left(A_f(j\omega) \to \infty\right)$$



S-plane behavior

- An oscillator circuit will maintain poles on the jω-axis

$$\text{Roots at } \pm j\omega_0 \Rightarrow 1 + A(j\omega)\beta(j\omega) = s^2 + \omega_0^2$$

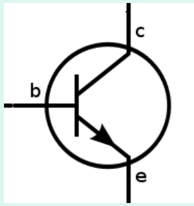
$$\sin(\omega_0 t) \xleftrightarrow{L} \frac{\omega_0}{s^2+\omega_0^2}, \quad \cos(\omega_0 t) \xleftrightarrow{L} \frac{s}{s^2+\omega_0^2}$$

•The oscillation condition needs to be satisfied for only one frequency in order to get harmonic output signal!

$$1 + A(j\omega_0)\beta(j\omega_0) = 0$$

$$\Updownarrow$$

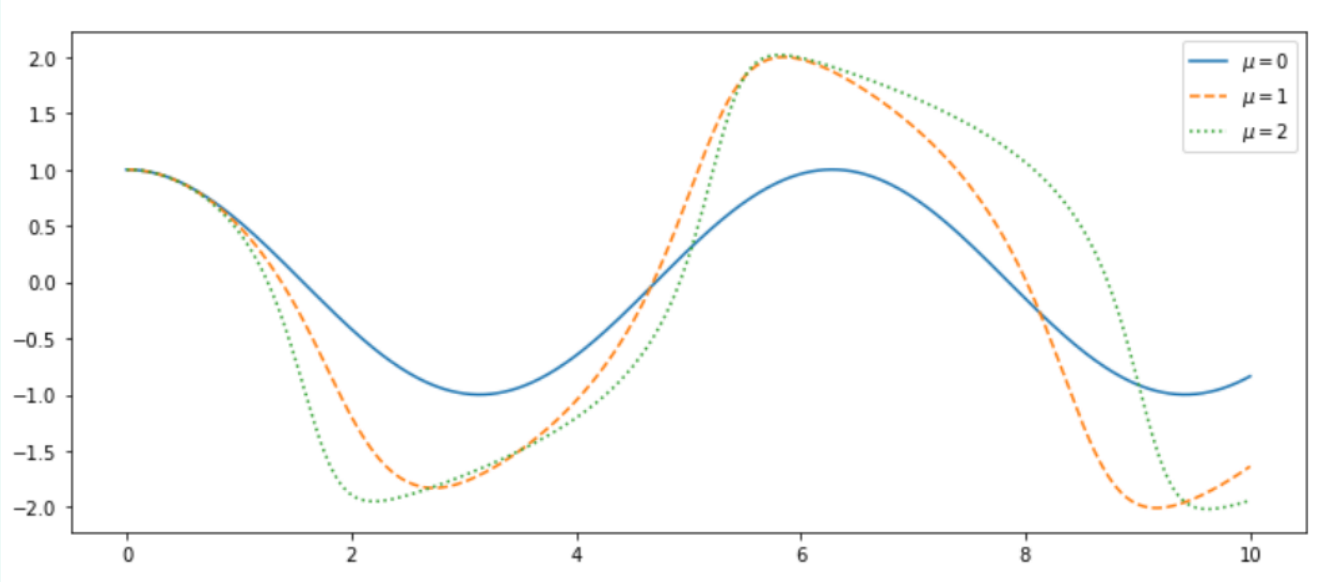
$$\text{The "Barkhausen criterion"}: \begin{cases} \left|A(j\omega_0)\beta(j\omega_0)\right| = 1 \\ \varphi\left(A(j\omega_0)\right) + \varphi\left(\beta(j\omega_0)\right) = 180^0 \end{cases}$$

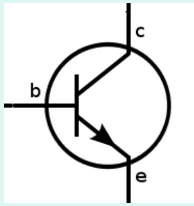


Mathematical model - van der Pol oscillator

- Balthasar van der Pol - proposed the equation while working at Philips (1926)
- Autonomous van der Pol equation (μ>0):

$$\frac{d^2x}{dt^2} - \mu\left(1 - x^2\right)\frac{dx}{dt} + x = 0$$

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