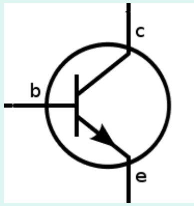
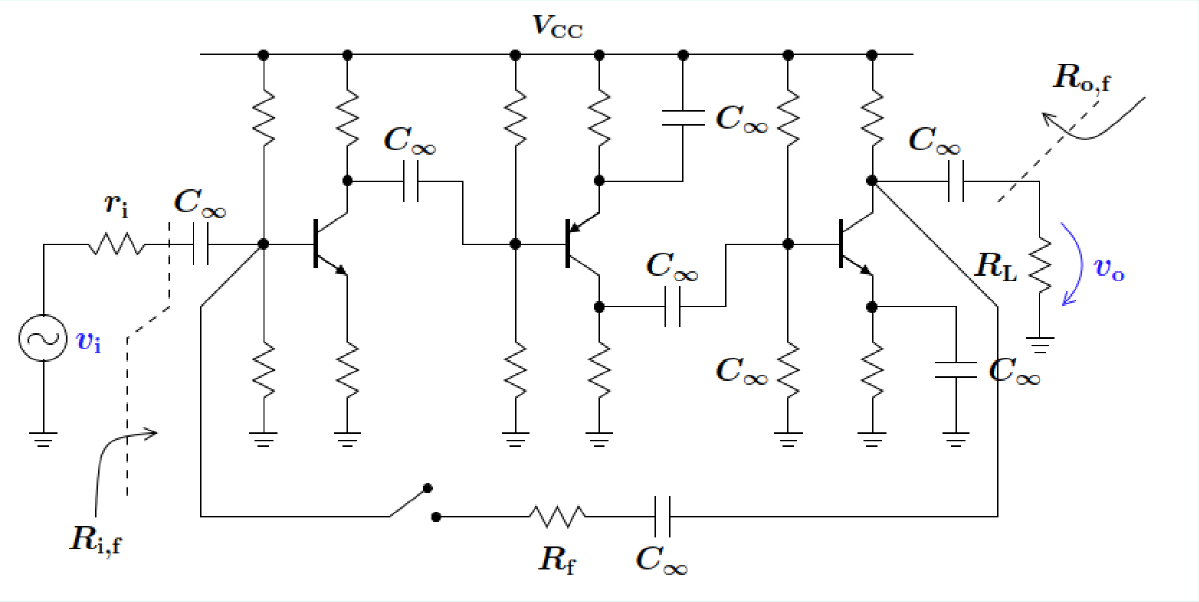
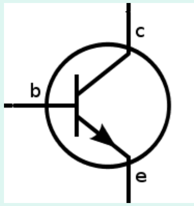


ELEC 301 - Oscillators

L31 - Nov 25

Instructor: Edmond Cretu

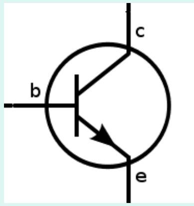




Last time

- Oscillators - linear (harmonic) vs nonlinear (relaxation) oscillators
- Barkhausen oscillation conditions
- Separate feedback loops: (a) control the frequency of oscillation + (b)control the amplitude of oscillation

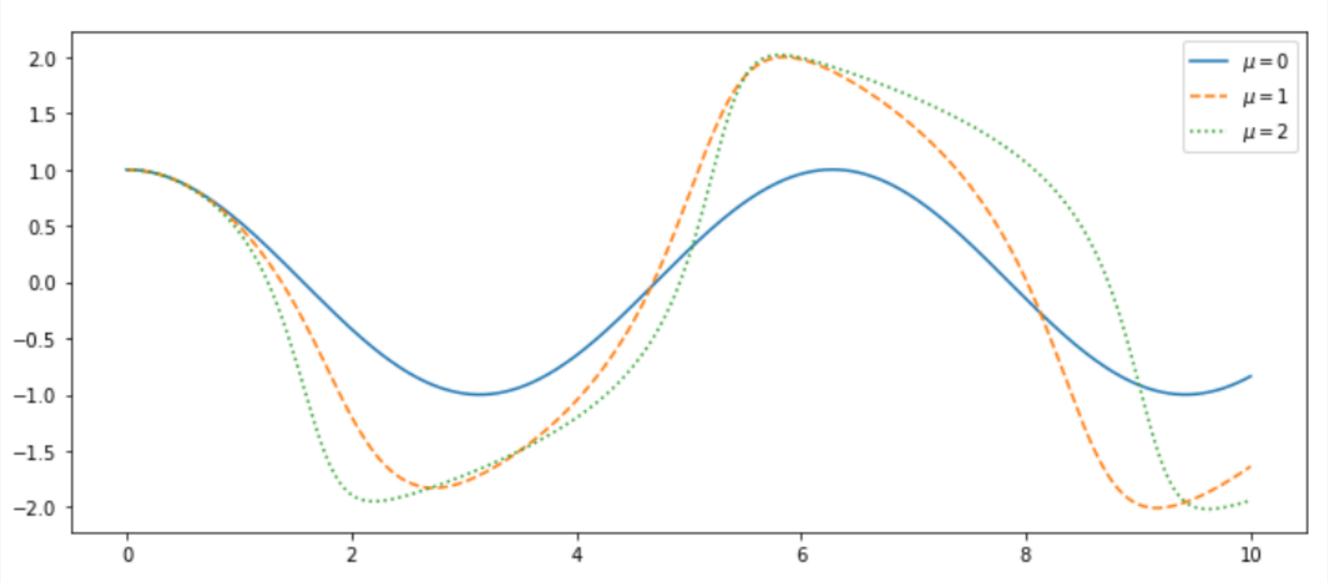
$$\text{The "Barkhausen criterion"}: \begin{cases} \left| A(j\omega_0)\beta(j\omega_0) \right| = 1 \\ \varphi\left(A(j\omega_0)\right) + \varphi\left(\beta(j\omega_0)\right) = 180^0 \end{cases}$$

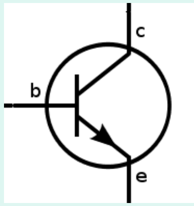


Mathematical model - van der Pol oscillator

- Balthasar van der Pol - proposed the equation while working at Philips (1926)
- Autonomous van der Pol equation (μ>0):

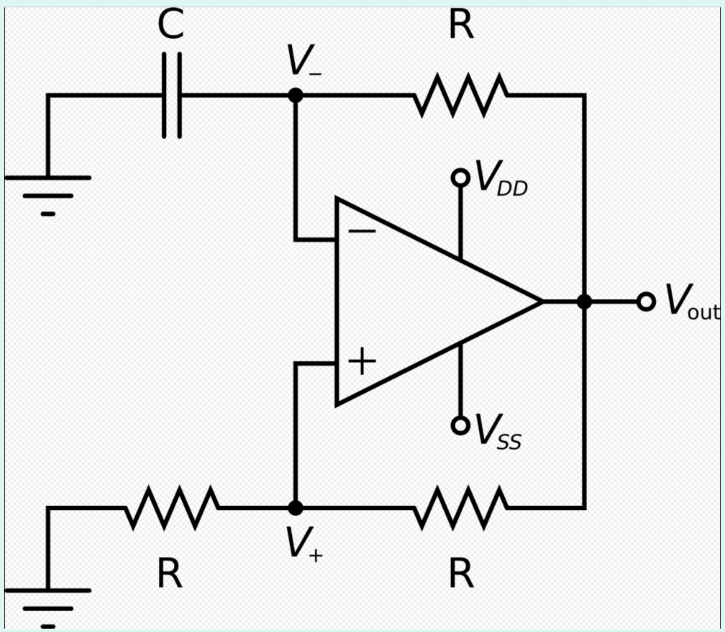
From: $$\frac{d^2x}{dt^2}+x=0\ (\omega_0=1, Q\to\infty)$$ to: $$\frac{d^2x}{dt^2}-\mu\left(1-x^2\right)\frac{dx}{dt}+x=0$$

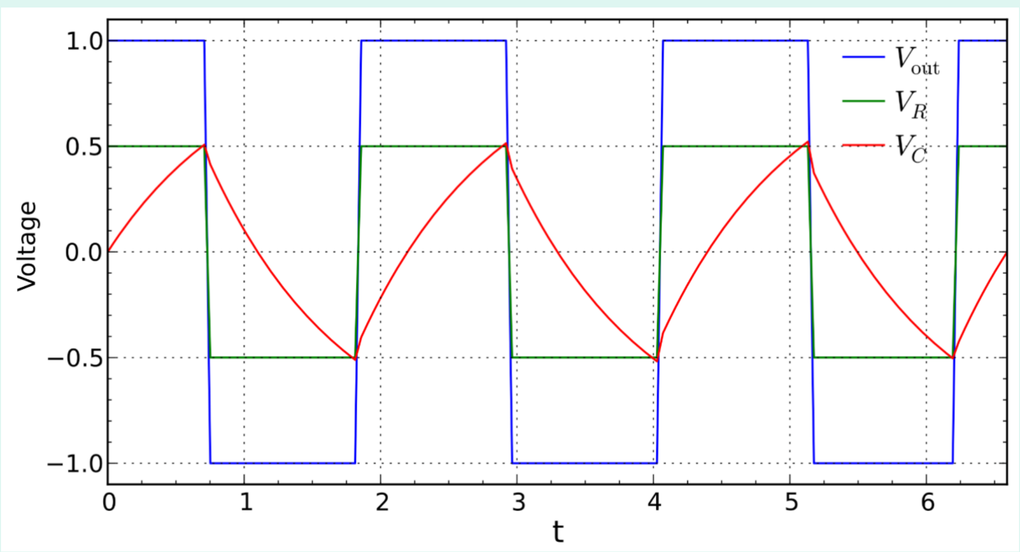




Exm: relaxation oscillator

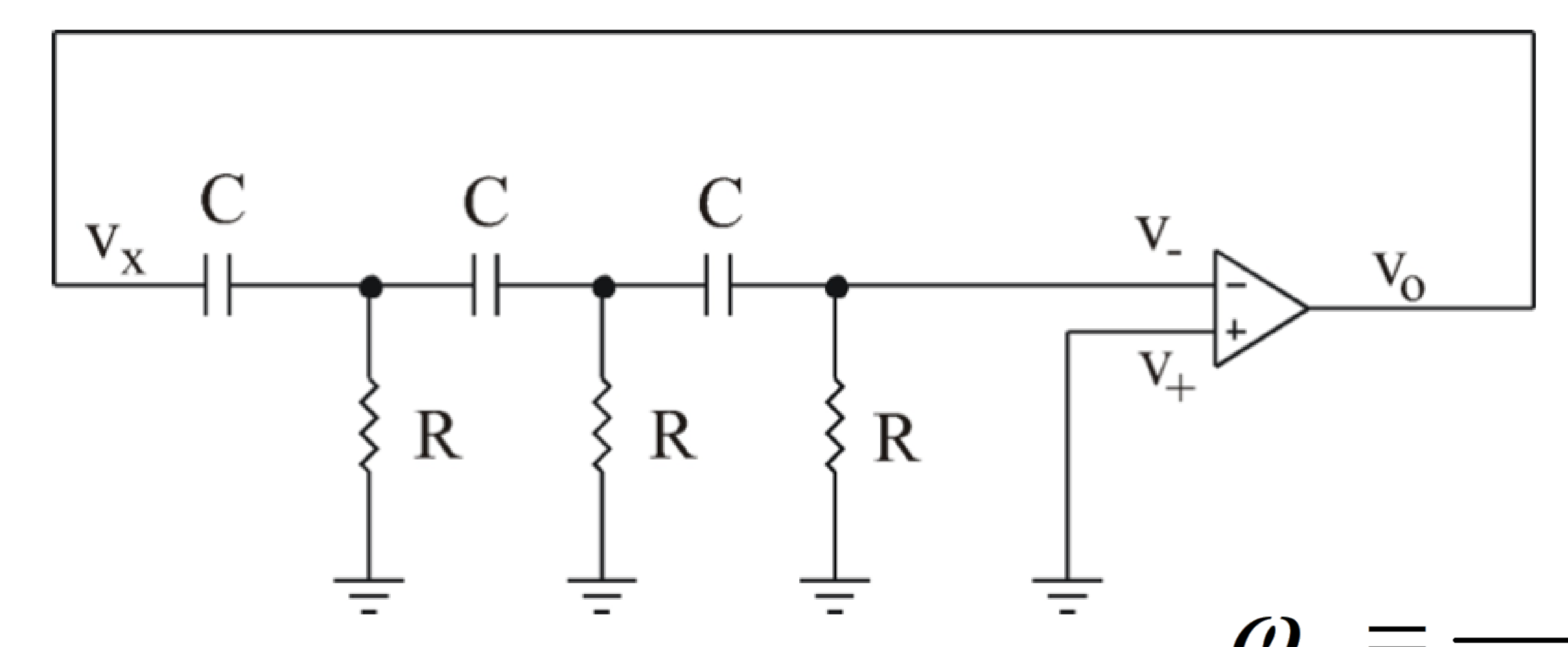
- Two feedback loops: a fast acting positive (regenerative) feedback loop + a slow acting but dominant negative feedback loop





Exm: phase-shift oscillator

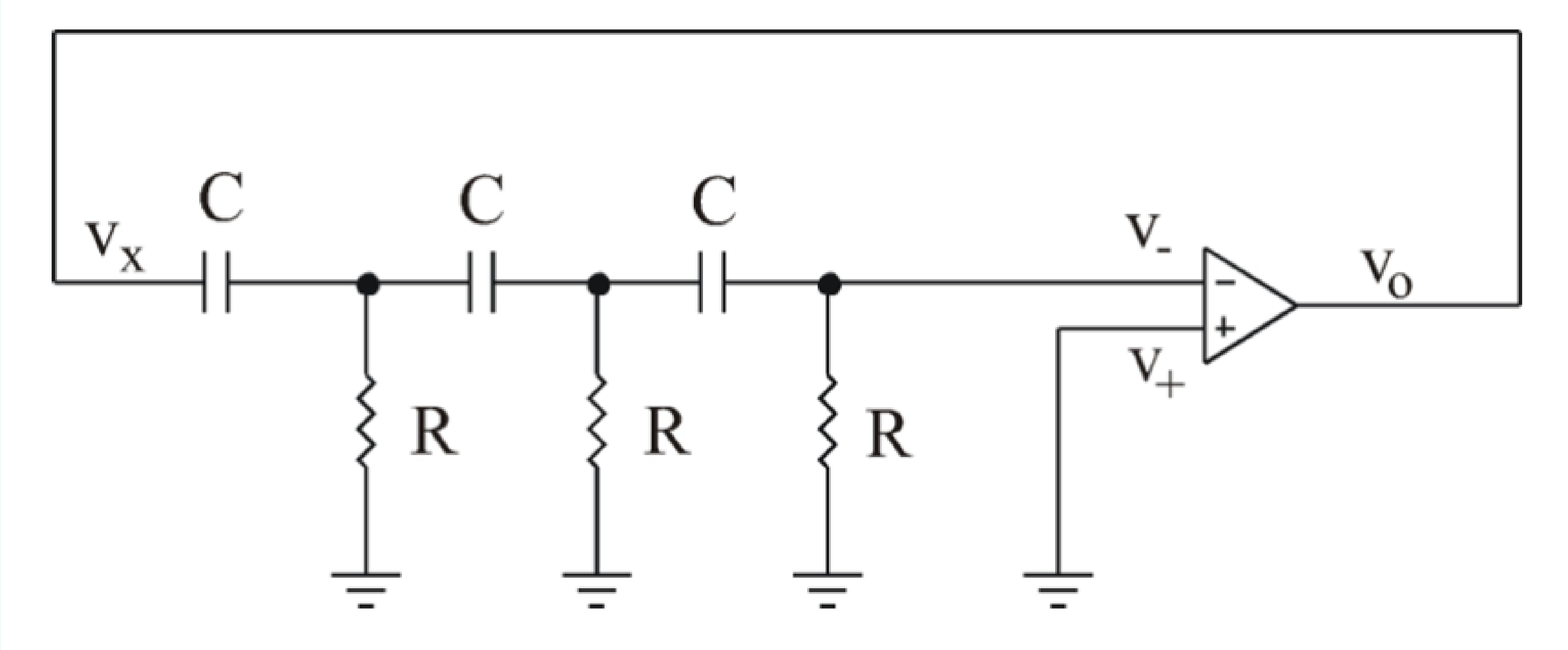
- Op-amp + Rs +Cs
- Concept: provide sequential phase shifts to sum up to 180^0



$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

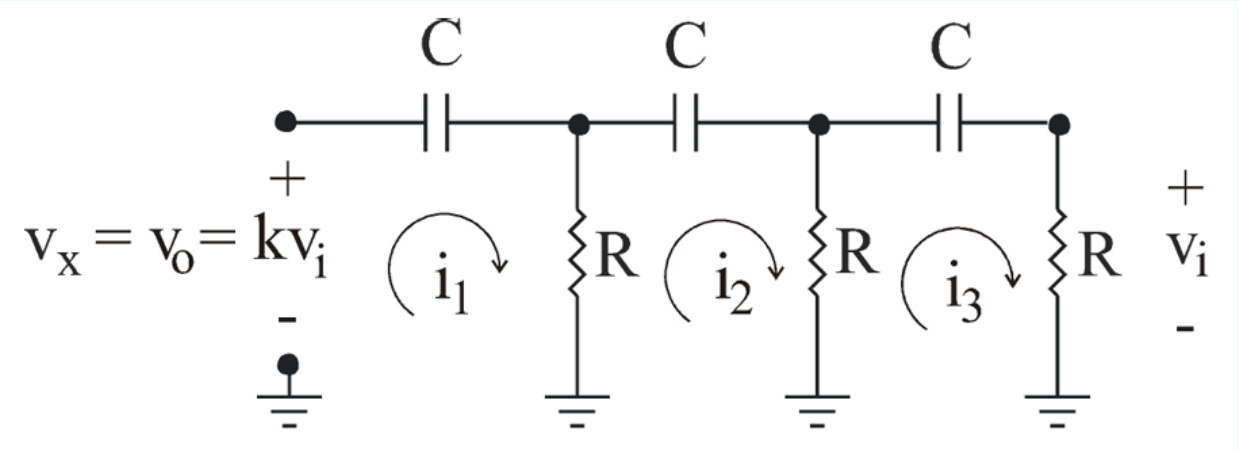
Phase-shift oscillator

- 2 poles would only asymptotically provide the required 180^0 phase-shift => need for 3 poles
- The 3rd pole makes it possible to set ω_0 (oscillation frequency) at any value by choosing the appropriate pole locations.



Phase-shift oscillator analysis

- Assume the op-amp stage gain=k

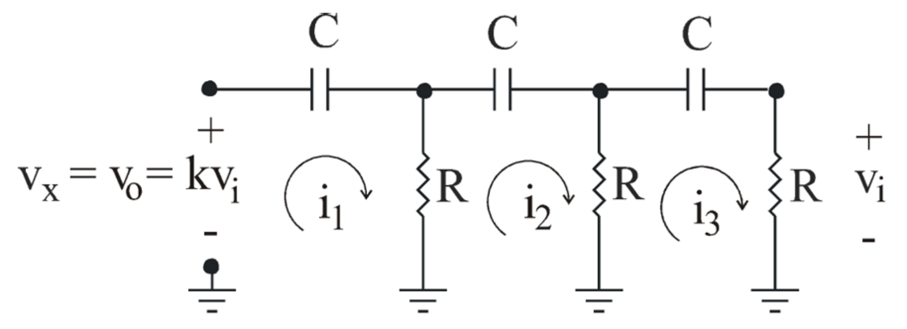


Loop equations:

$$\begin{cases} \left(R+\frac{1}{sC}\right)i_1 - Ri_2 = kv_i \\ -Ri_1 + \left(2R+\frac{1}{sC}\right)i_2 - Ri_3 = 0 \\ -Ri_2 + \left(2R+\frac{1}{sC}\right)i_3 = 0 \end{cases}$$

Eliminate the loop currents variables

- Eliminate variable i_1, i_2, i_3



$$i_3 = \frac{v_i}{R} \rightarrow i_2 = \frac{1}{R}\left(2R + \frac{1}{sC}\right) i_3 = \frac{v_i}{R^2}\left(2R + \frac{1}{sC}\right)$$

$$i_1 = \frac{v_i}{R^3}\left(2R + \frac{1}{sC}\right)^2 - \frac{v_i}{R}$$

$$\left(R + \frac{1}{sC}\right)\left[\frac{v_i}{R^3}\left(2R + \frac{1}{sC}\right)^2 - \frac{v_i}{R}\right] - \frac{v_i}{R}\left(2R + \frac{1}{sC}\right) = kv_i$$

Phase-shift oscillator (cont.)

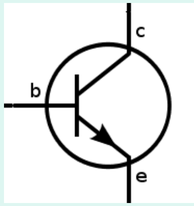
- Cancel v_i terms, expand and collect like terms:

$$\left(R + \frac{1}{sC}\right)\left[\frac{3}{R} + \frac{4}{sR^2C} + \frac{1}{s^2R^3C^2}\right] - 2 - \frac{1}{sRC} = k$$

$$3 + \frac{7}{sRC} + \frac{5}{s^2R^2C^2} + \frac{1}{s^3R^3C^3} - 2 - \frac{1}{sRC} = k$$

$$1 + \frac{6}{sRC} + \frac{5}{s^2R^2C^2} + \frac{1}{s^3R^3C^3} = k$$

$$\frac{s^3R^3C^3 + 6s^2R^2C^2 + 5sRC + 1}{s^3R^3C^3} = k$$



Oscillation condition

- set s=jω => the equation transforms into:

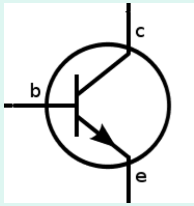
$$\frac{\omega^3 R^3 C^3 - 5\omega RC + j\left(6\omega^2 R^2 C^2 - 1\right)}{\omega^3 R^3 C^3} = k$$

The phase condition (assume k<0):

$$\tan^{-1}\left(\frac{6\omega^2 R^2 C^2 - 1}{\omega^3 R^3 C^3 - 5\omega RC}\right) = -180^0 \Rightarrow 6\omega^2 R^2 C^2 - 1 = 0 \Leftrightarrow \omega = \frac{1}{\sqrt{6}RC}$$

Getting back to the oscillation condition eqn:

$$k = -29$$

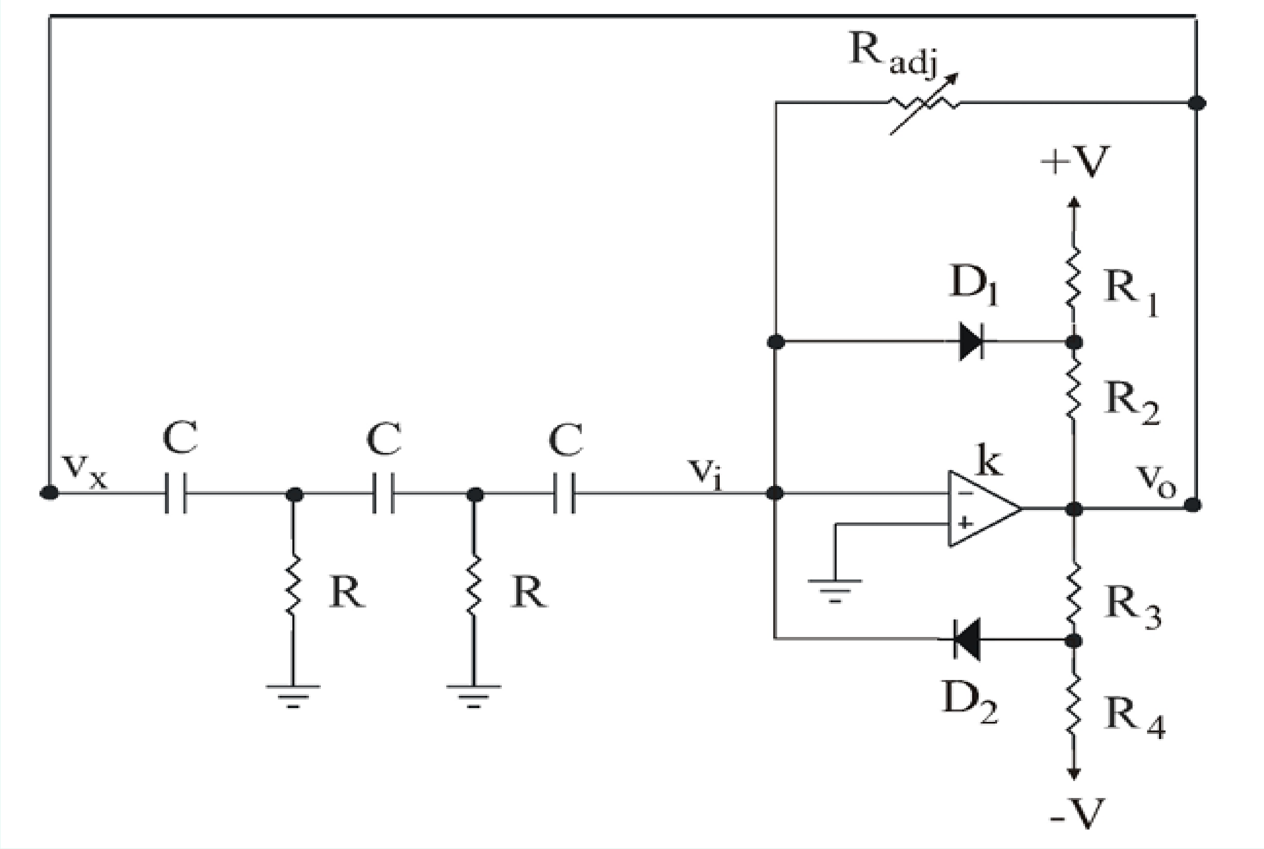


The amplitude control

- Goal: set and maintain a desired oscillation amplitude, despite potential change (e.g. dur to temperature) in Aβ
- Need: a control circuit to force Aβ=1 at the desired output amplitude
  - Transient regime: to make sure oscillations can and will begin, the circuit is designed so that |Ab|>1 (only slightly higher) → poles initially in the RHP (exponential amplitude growth)
  - Damping regime: when the desired amplitude is reached, the control network must reduce |Aβ| to unity → poles pulled onto the jω-axis
  - Maintenance: if the loop gain |Aβ| varies in either direction, then the control must act to readjust the loop gain to unity

Amplitude control - soft limiter

- A simple way: use a limiter circuit to prevent amplitude from exceeding a preset level
- Soft limiter - usually used to minimize nonlinear distortions

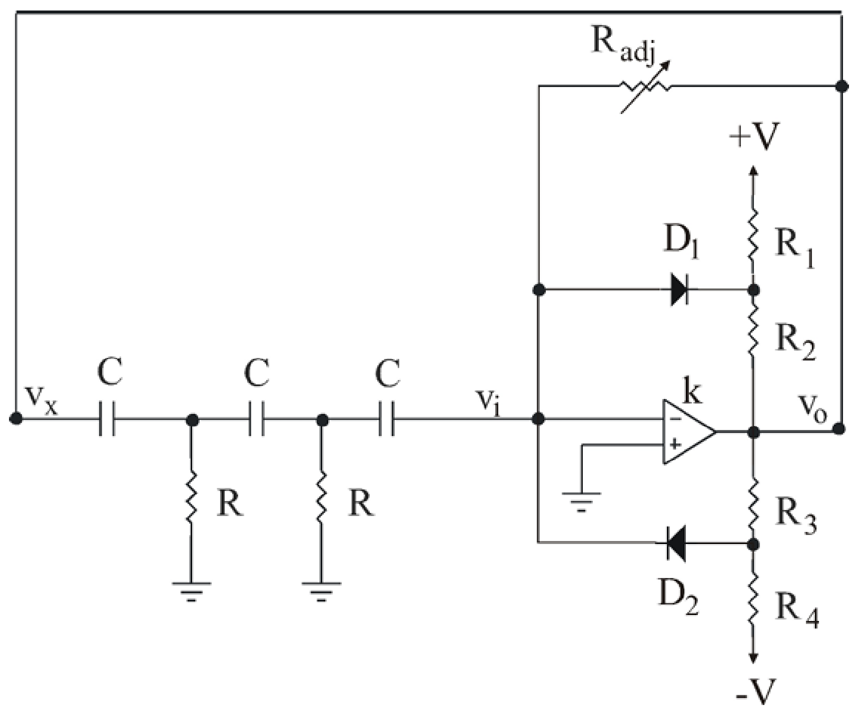


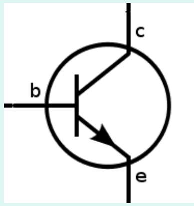
Soft limiter

- Resistor network + diodes

$$\text{For } u_{D2} < 0.6V : u_{D2} = \frac{G_3 v_O + G_4(-V)}{G_3 + G_4} = \frac{R_4}{R_3 + R_4} v_O - \frac{R_3}{R_3 + R_4} V$$

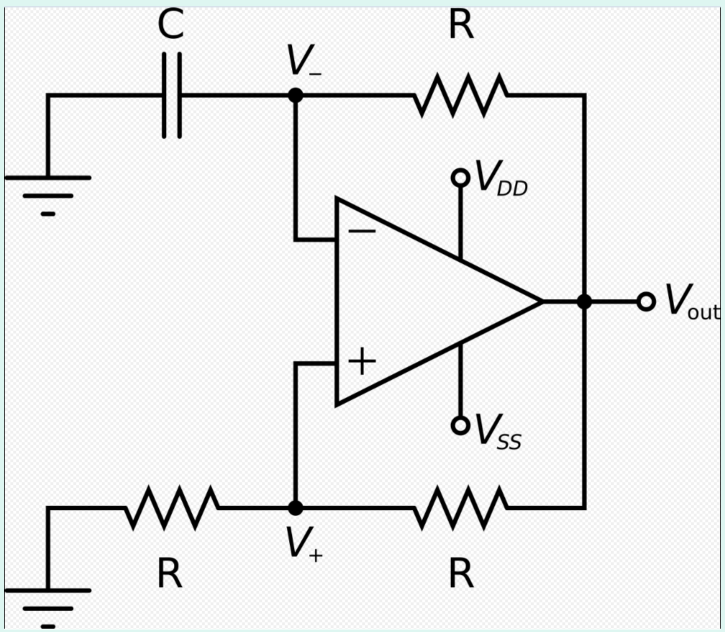
$$\text{For } u_{D1} > -0.6V : u_{D1} = \frac{R_1}{R_1 + R_2} v_O + \frac{R_2}{R_1 + R_2} V$$

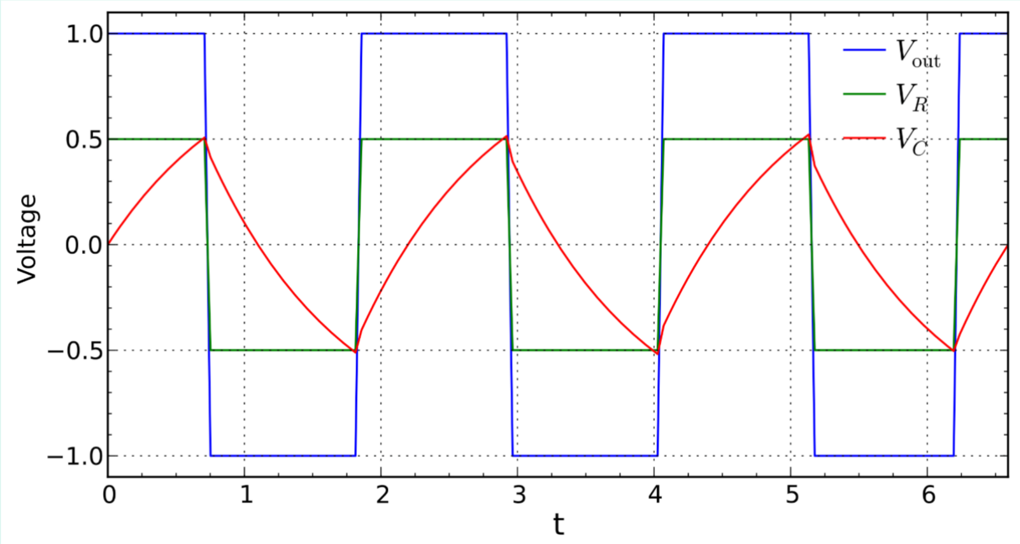




Relaxation oscillator

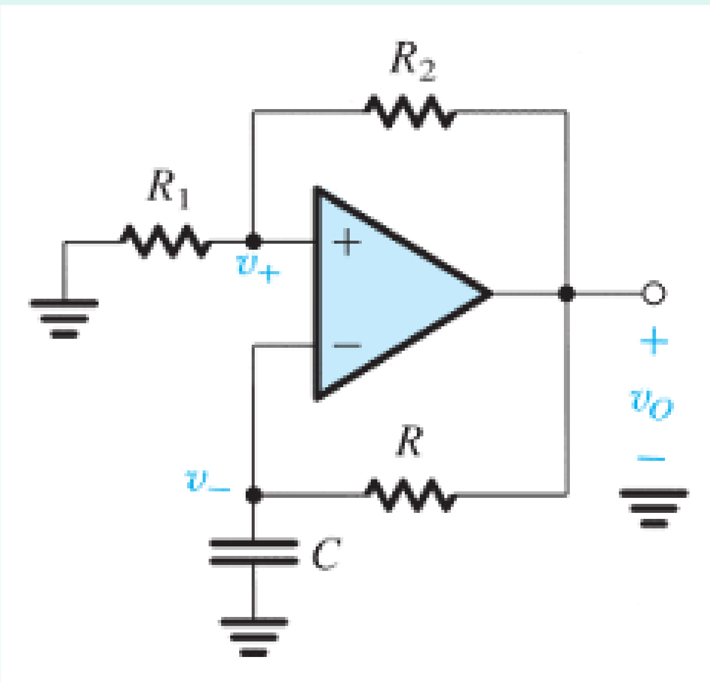
- Two feedback loops: a fast acting positive (regenerative) feedback loop + a slow acting but dominant negative feedback loop

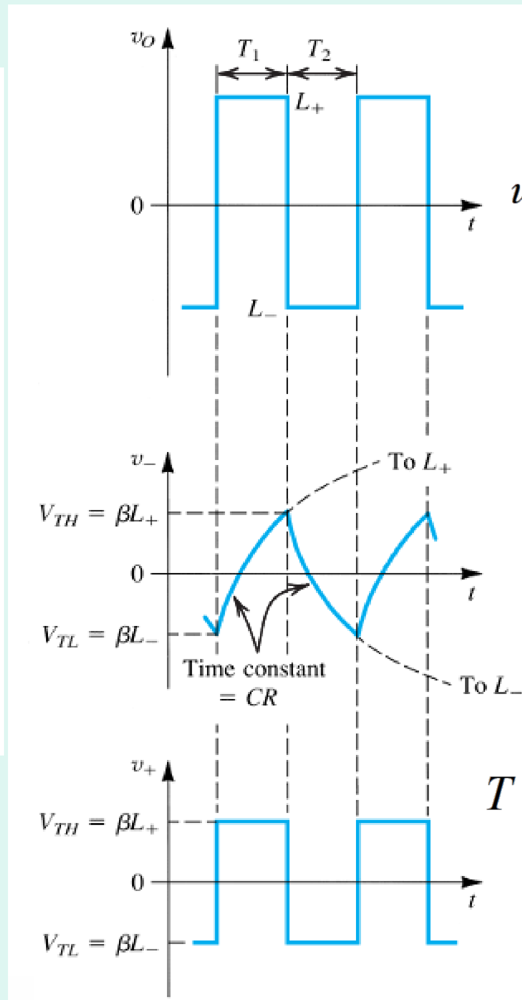




Relaxation oscillator analysis

- Astable multivibrator circuit





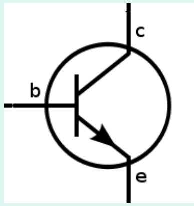
$$\beta = \frac{R_1}{R_1 + R_2}, \tau = RC$$

$$u_C(t) = u_C(\infty) - (u_C(\infty) - u_C(0+))e^{-t/\tau}$$

$$T_1 = \tau \ln \frac{1 - \beta \frac{L_-}{L_+}}{1 - \beta}$$

$$T_2 = \tau \ln \frac{1 - \beta \frac{L_+}{L_-}}{1 - \beta}$$

$$T = T_1 + T_2 \xrightarrow{L_+ = L_-} T = 2\tau \ln \frac{1 + \beta}{1 - \beta}$$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