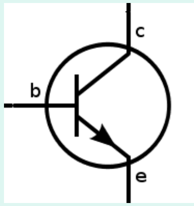
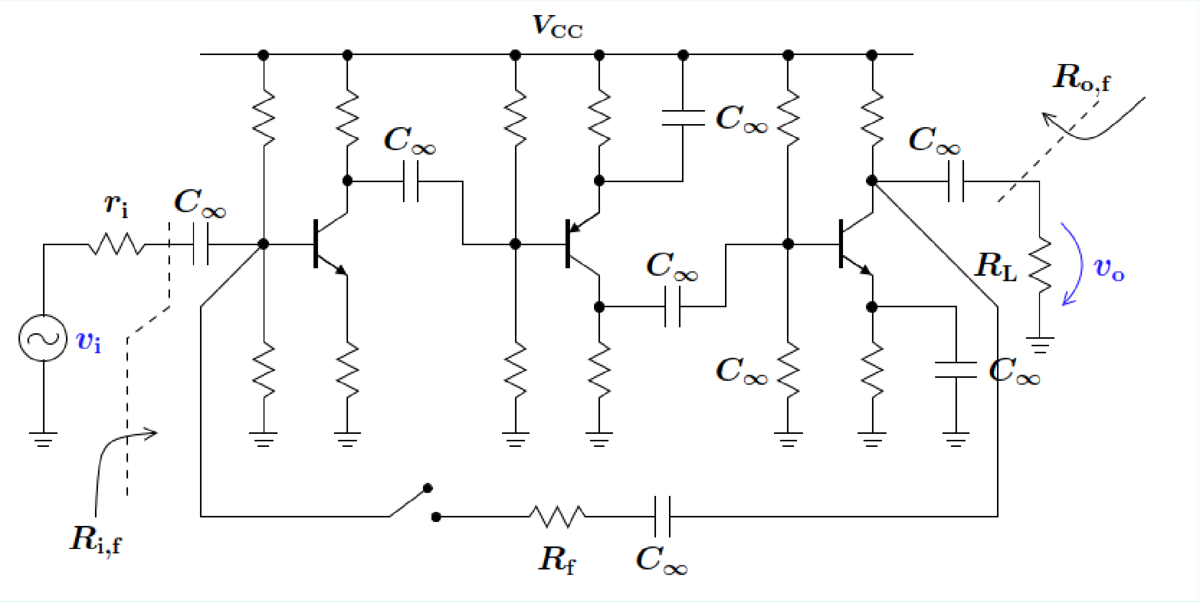
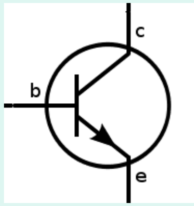


ELEC 301 - Oscillators (2)

L32 - Nov 27

Instructor: Edmond Cretu

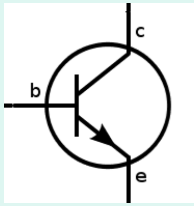




Last time

- Barkhausen oscillation conditions
- Modeling the (nonlinear) amplitude stabilization - van der Pol autonomous equation
- Separate feedback loops: (a) control the frequency of oscillation + (b)control the amplitude of oscillation
- Phase-shift harmonic oscillators

$$\text{The "Barkhausen criterion"}: \begin{cases} \left| A(j\omega_0)\beta(j\omega_0) \right| = 1 \\ \varphi\left(A(j\omega_0)\right) + \varphi\left(\beta(j\omega_0)\right) = 180^0 \end{cases}$$

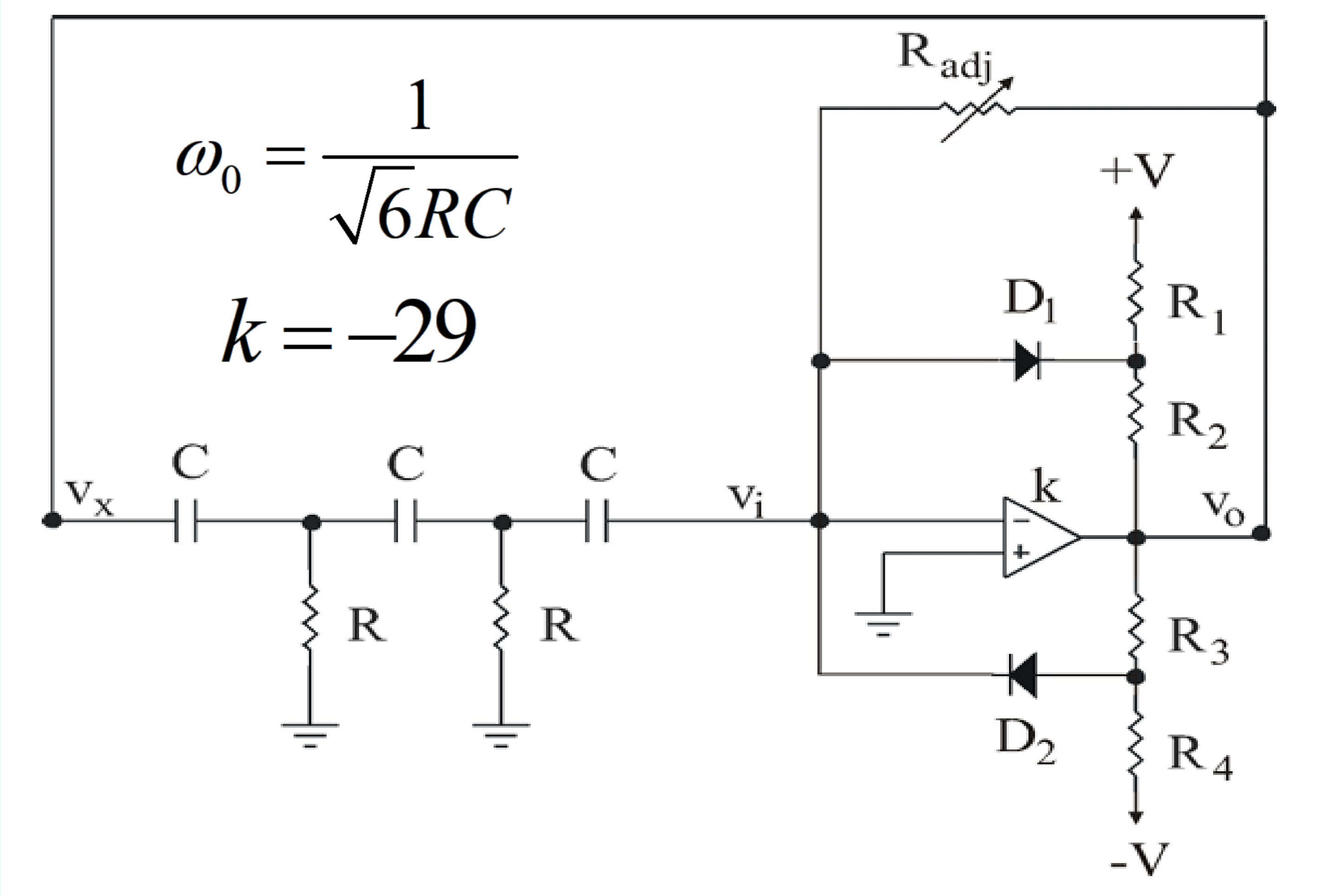


L32 Q01 - nonlinearity in oscillators

- Which feedback loop is responsible for nonlinearities in a harmonic oscillator?
- A. frequency-control loop
- B. amplitude-control loop
- C. both frequency and amplitude control circuits are linear
- D. both frequency and amplitude control are nonlinear

Phase-shift oscillator with amplitude control

- frequency of oscillation - controlled by the RC+ amplifier gain
- amplitude of oscillation - controlled by the diode soft-limiters

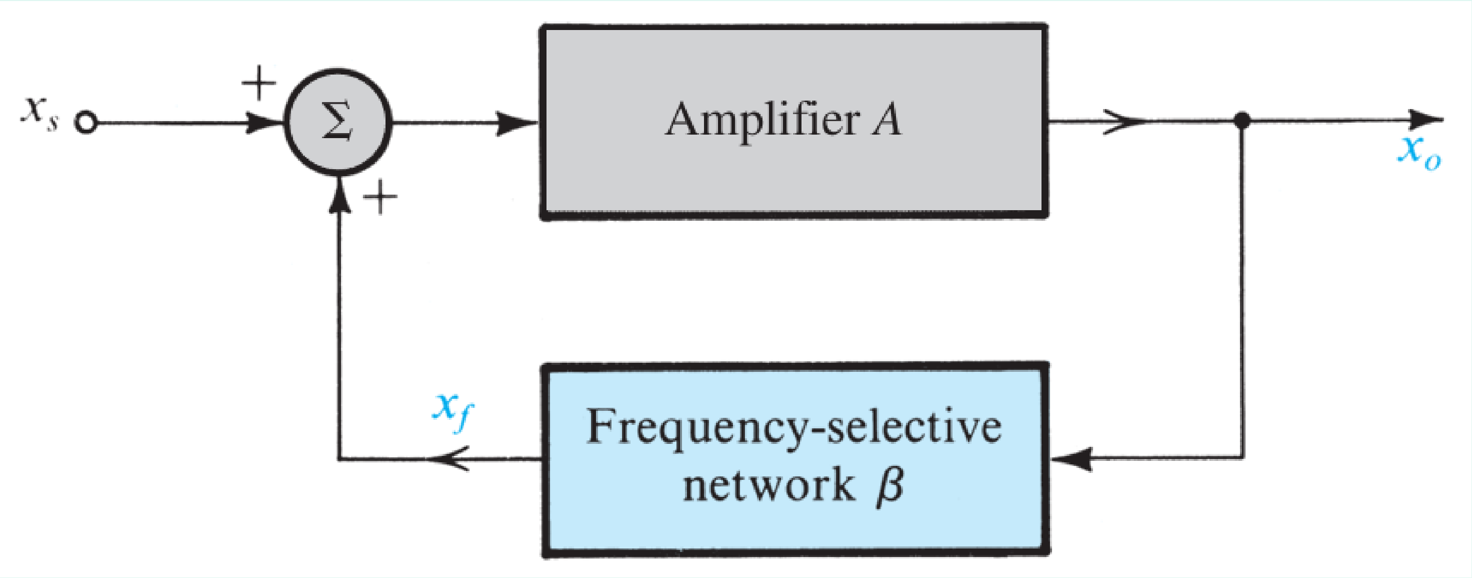


$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

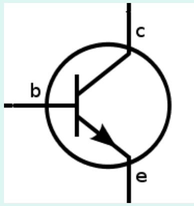
$$k = -29$$

Further remarks on the oscillation criterion

- $L(j\omega_0)=A(j\omega_0)\beta(j\omega_0)=-1$ (negative feedback scheme)
- In many textbooks, a different summation sign (positive feedback scheme) is used in oscillator theory

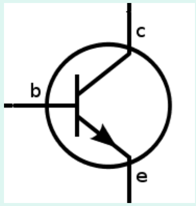


$$1-A(j\omega_0)\beta(j\omega_0)=0 \Leftrightarrow \begin{cases} |A(j\omega_0)\beta(j\omega_0)|=1 \\ \varphi(A(j\omega_0))+\varphi(\beta(j\omega_0))=0^0 \end{cases}$$



Oscillation criterion (2)

- For the circuit to oscillate at a specific frequency ω_0, the oscillation (Barkhausen) criteria should be only satisfied for $\omega=\omega_0$, or otherwise the resulting waveform will not be a simple harmonic signal
- The frequency of oscillation is determined solely by the phase characteristics of the feedback loop
- The stability of the oscillation frequency - determined by the how the phase $\varphi(\omega)$ of the loop gain varies around ω_0

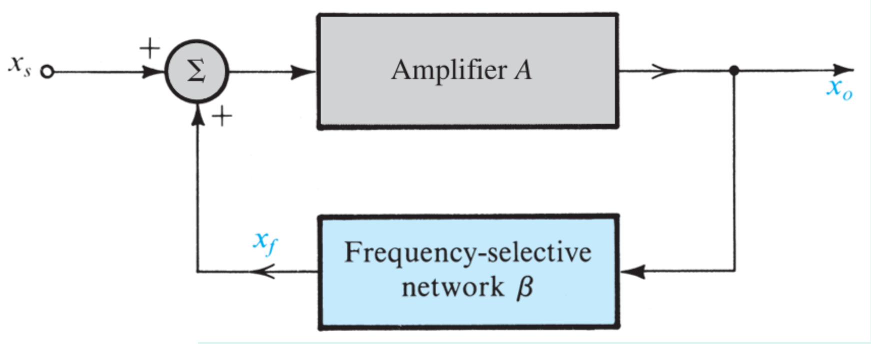


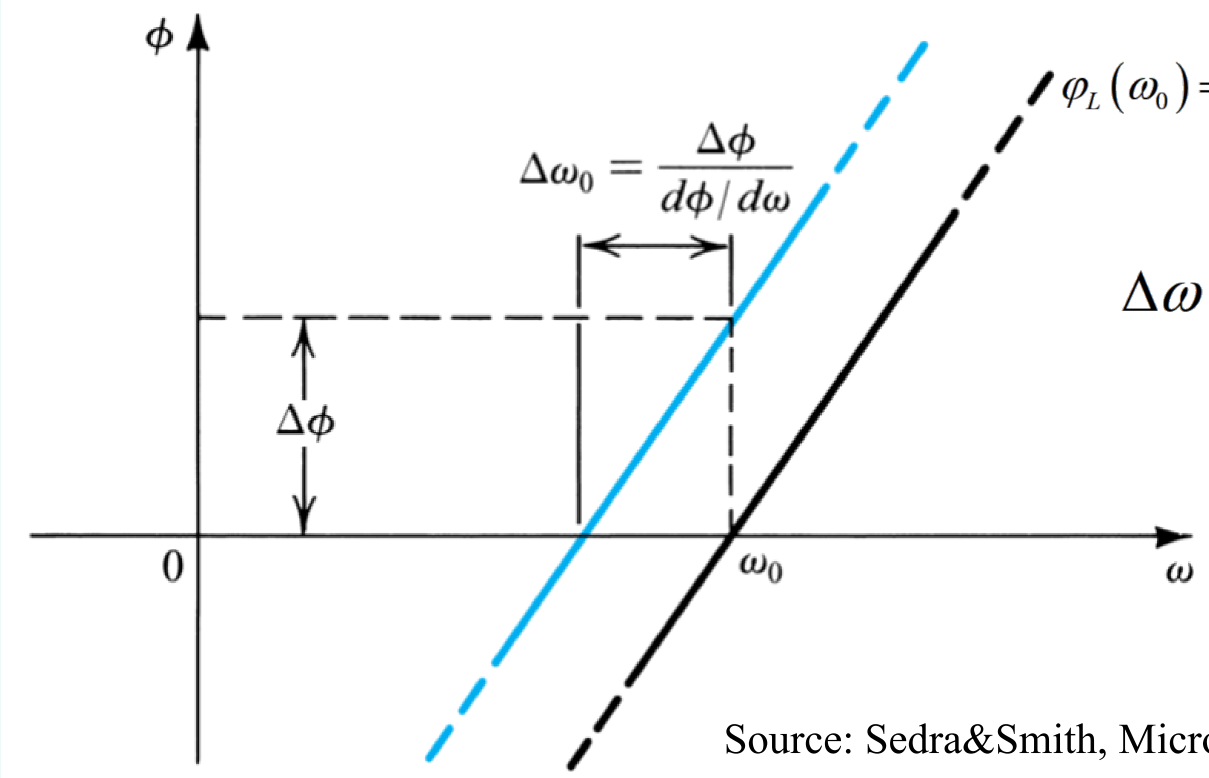
L32 Q02 - ω_0 stability

- What type of dependence of the phase on frequency will ensure a more stable frequency of oscillation?
- A. a smooth, slow-slope dependence $\varphi(\omega)$ around ω_0
- B. a steep variation of $\varphi(\omega)$ around ω_0
- C. The stability of the oscillation frequency is not dependent of the slope of $\varphi(\omega)$ around ω_0

Stability of the oscillation frequency

- Positive feedback convention
- The slope of φ(ω) gives the stability: a steep variation results in a more stable frequency

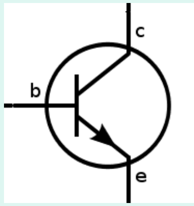




$$\varphi_L(\omega_0) = 0 \Rightarrow \varphi_L(\omega_0 + \Delta\omega) \approx 0 + \frac{d\varphi_L}{d\omega}(\omega_0)\Delta\omega$$

$$\Delta\omega = \frac{\varphi_L(\omega_0 + \Delta\omega) - \varphi_L(\omega_0)}{\frac{d\varphi_L}{d\omega}(\omega_0)}$$

Source: Sedra&Smith, Microelectronic circuits

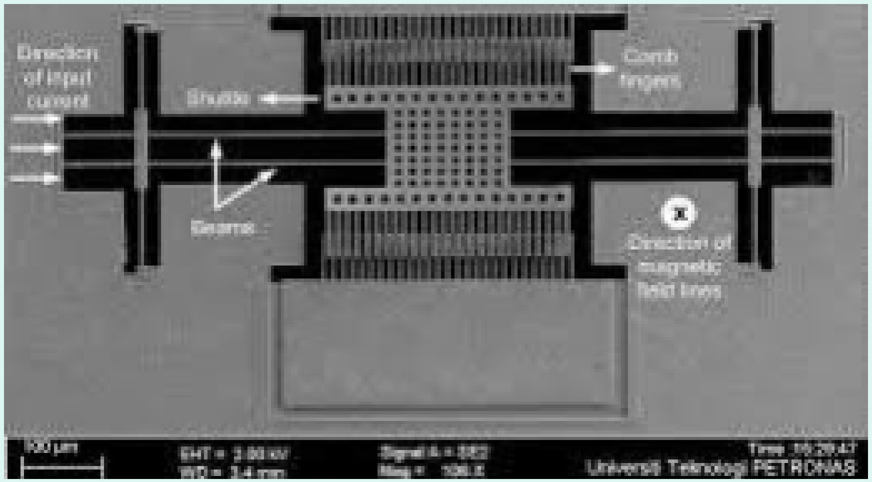


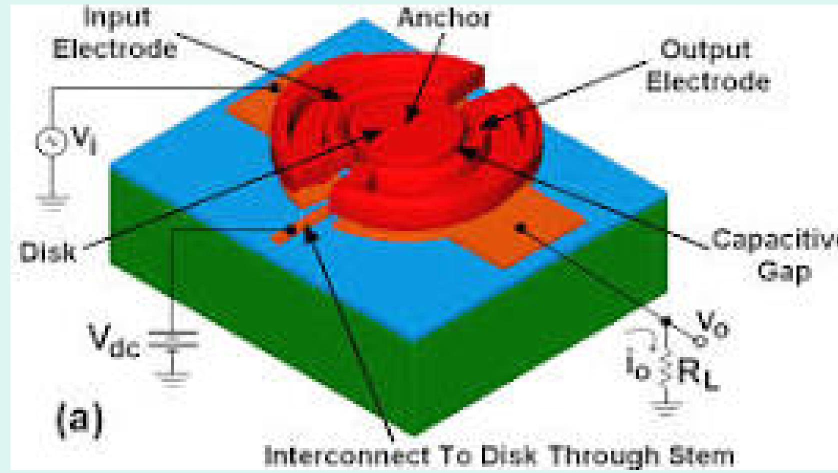
RLC oscillator example

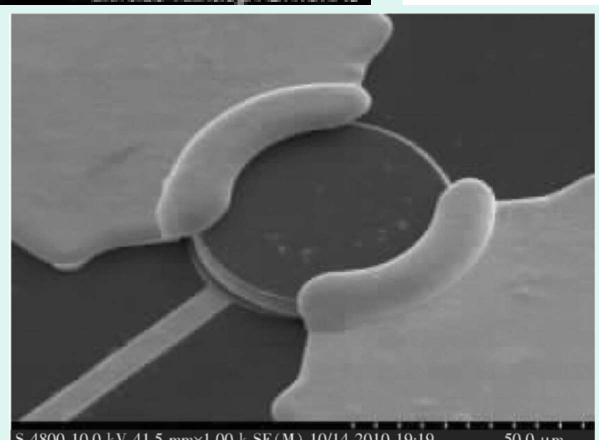
- Exploit the selectivity of the LC resonant circuit (or equivalent resonant structure, e.g. MEMS resonator)
- General analysis steps:
  1. Break the feedback loop and determine the loop gain $A(s)\beta(S)$
  2. Find oscillation frequency from the phase condition (180^0 or 360^0, depending on the feedback model used)
  3. Find the condition for the oscillations to start $|A(j\omega_0)\beta(j\omega_0)|\geq 1$
  4. Add an additional amplitude control feedback to set the desired amplitude level - force $|A\beta|$ to remain unity at the desired output amplitude

MEMS resonators

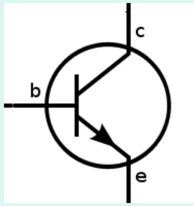
- Used in microelectronics, instead of physical inductors - high Q at high frequencies





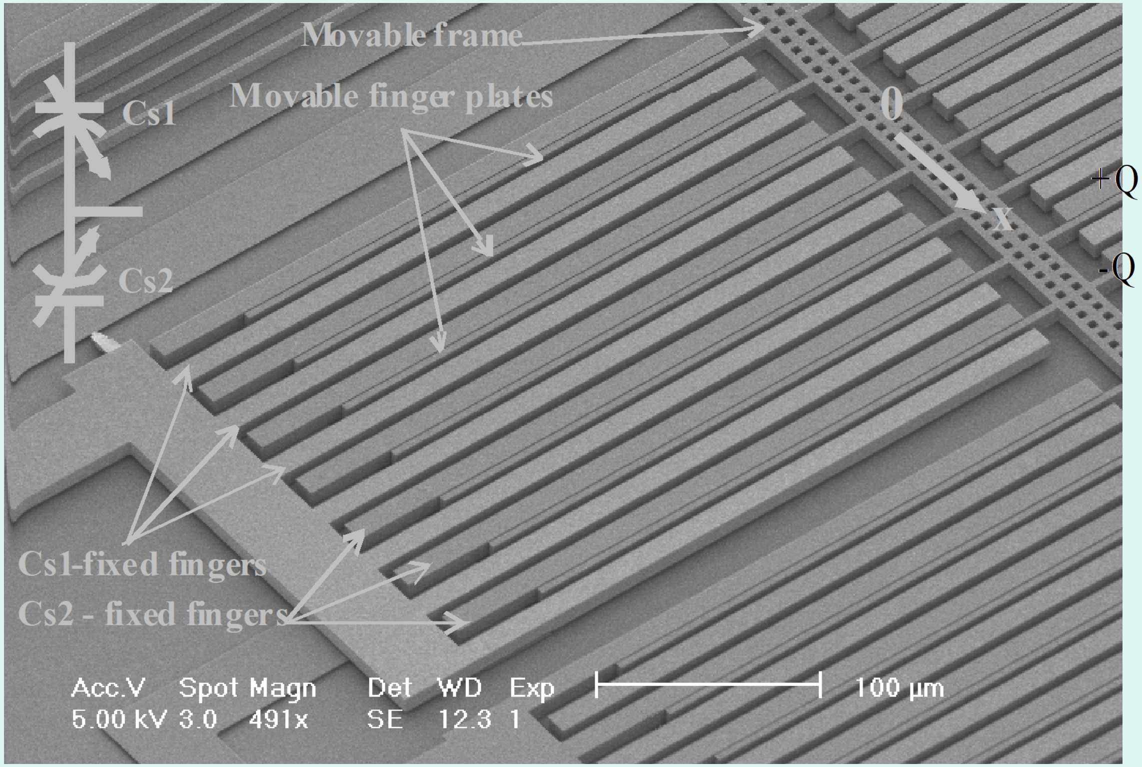


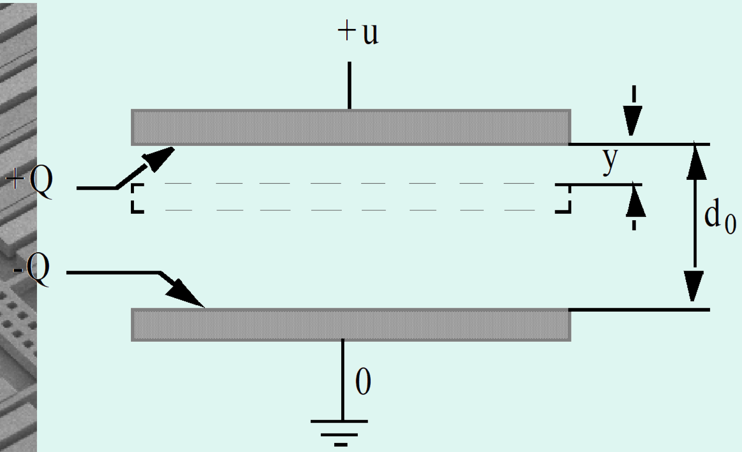
Source: H Xu et al[2025]Stability of capacitive MEMS oscillators, Microsystems Technologies



Electromechanical coupling in MEMS capacitive structures

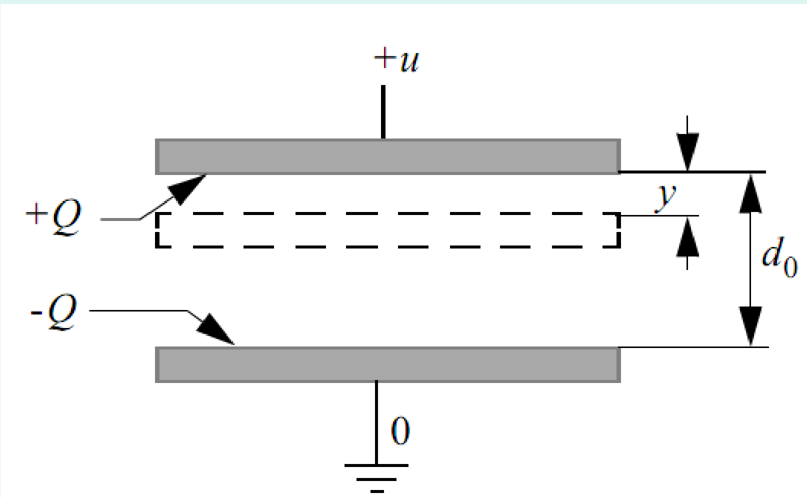
- MEMS capacitor - movable plate

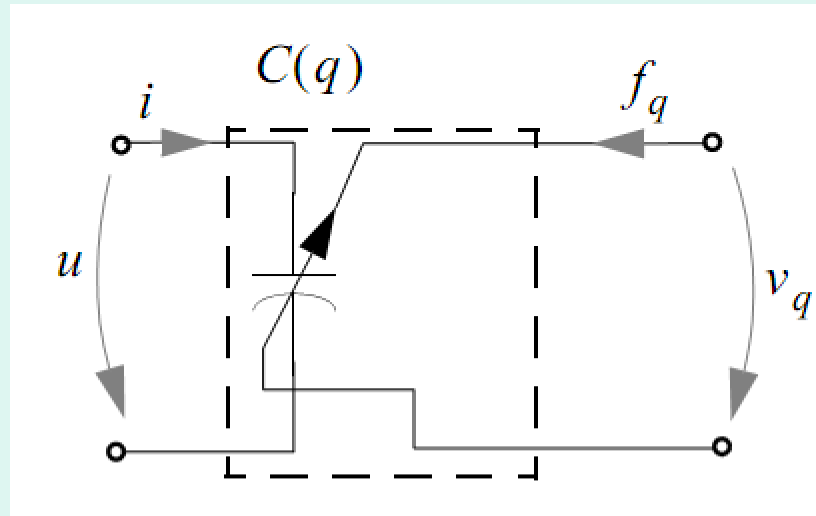




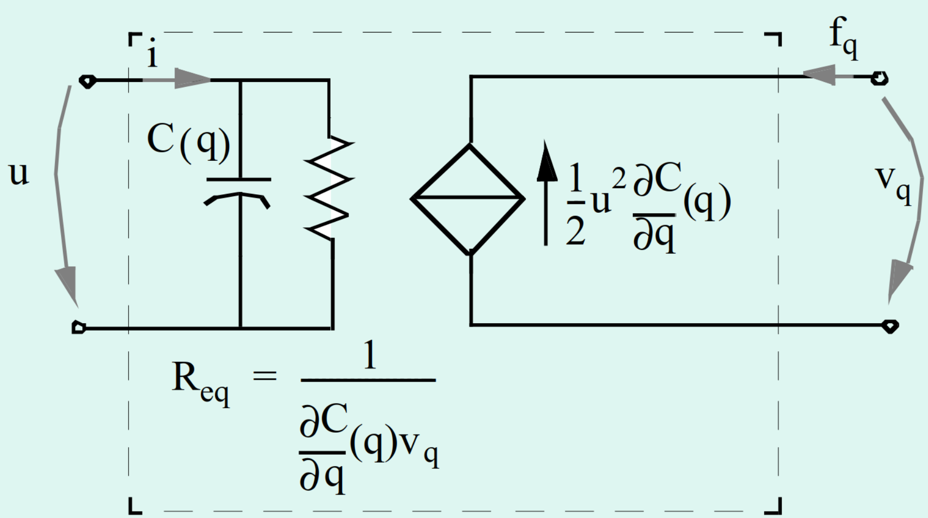
MEMS Capacitor model

- Assume single mechanical degree of freedom





$$C(y)=\varepsilon\frac{A}{d_0-y}=\frac{C}{1-}$$



$$ui+f_q v_q=\frac{dW_c}{dt}$$

L32 Q03 electrostatic forces

- How are the electrostatic forces between the movable and fixed plates of a MEMS capacitor?

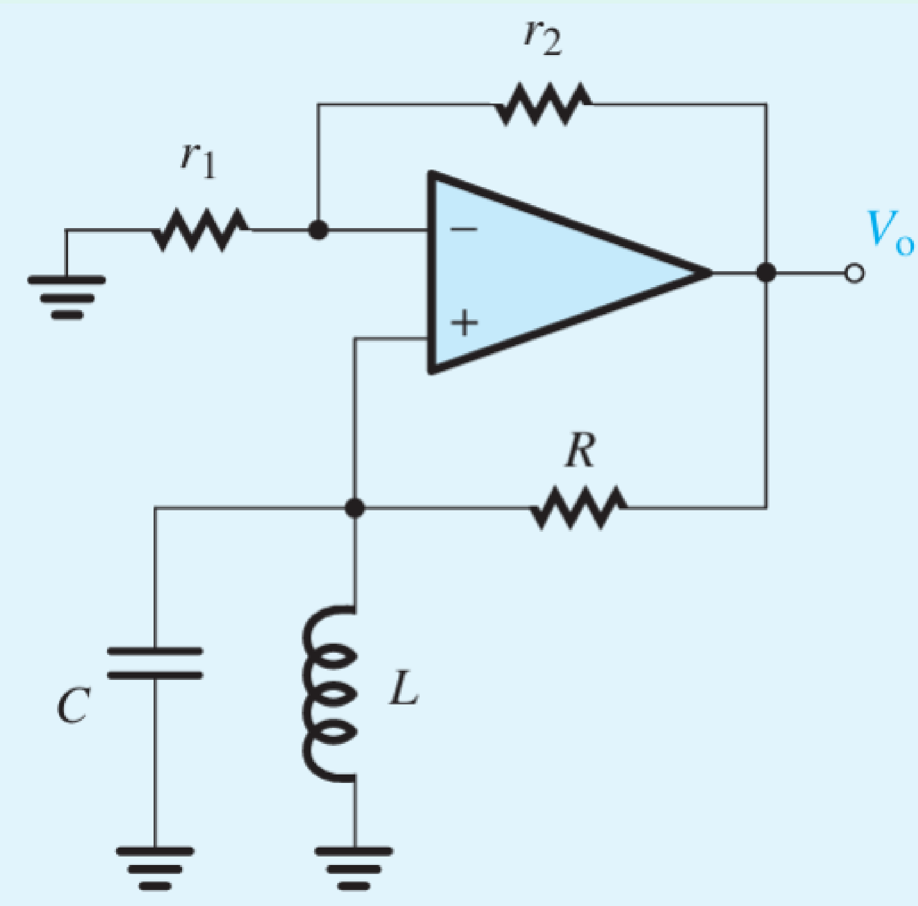
A. Always repulsive, regardless of the sign of the applied voltage between plates

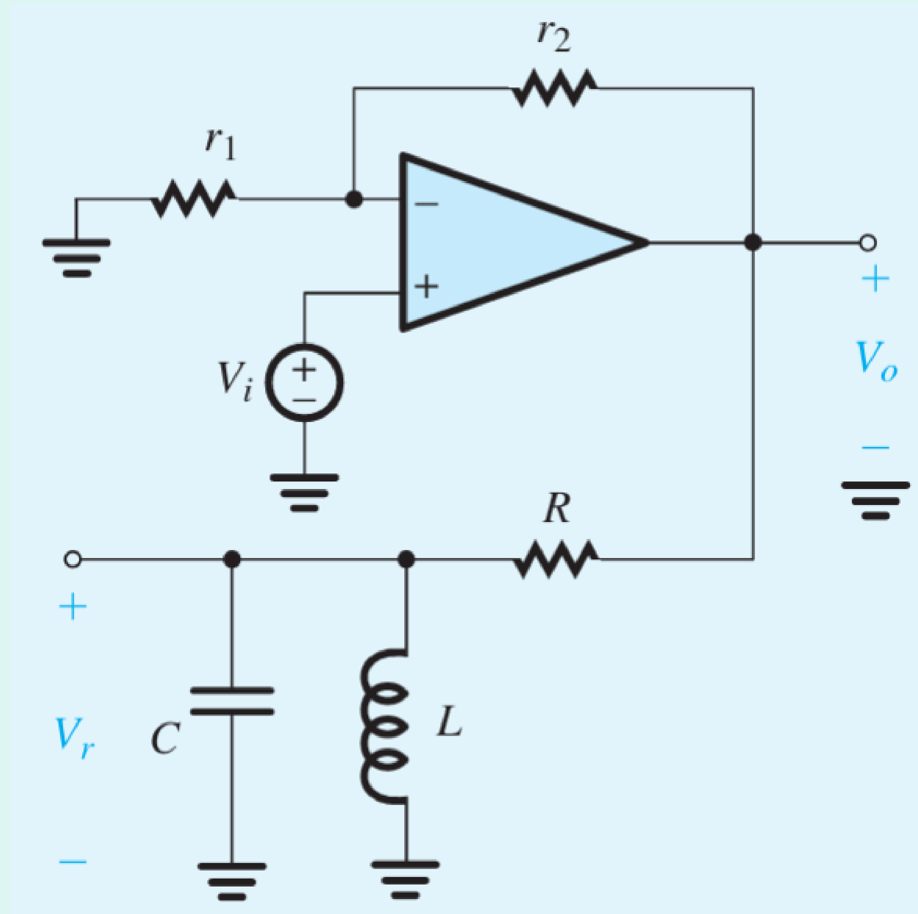
B. Attractive for positive voltages, and repulsive for negative voltages

C. Always attractive, regardless of the sign of the applied voltage between plates

RLC oscillator analysis

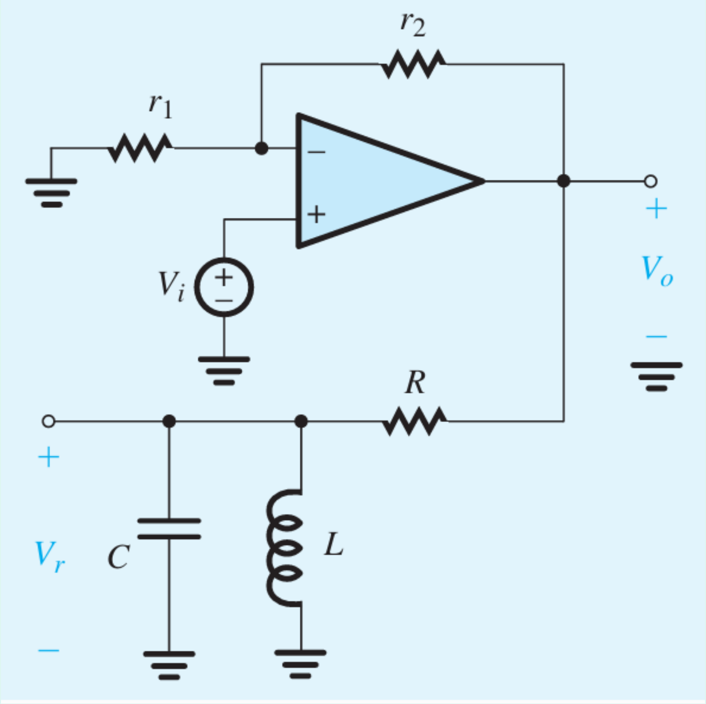
- LC resonator can be even a MEMS resonator device





RLC oscillator analysis (2)

- analysis after loop breaking



$$A(s) = \frac{V_o}{V_i} = 1 + \frac{r_2}{r_1}$$

$$\beta(s) = \frac{V_r}{V_o} = \frac{s\frac{1}{RC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$L(s) = A(s)\beta(s) = \frac{s\frac{1}{RC}\left(1 + \frac{r_2}{r_1}\right)}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

L32 Q04 Non-ideal opamp

- How would a finite Rin of a non-ideal op-amp influence the oscillator behavior?

A. $Rin<\infty$ does not influence the oscillation conditions in this configuration

B. $Rin<\infty$ has a stronger influence on ω_0

C. $Rin<\infty$ has a stronger influence on the minimum loop gain to onset the oscillation

RLC oscillator - oscillation condition

- Set s=jω and impose oscillation condition

$$L(j\omega)=A(j\omega)\beta(j\omega)=\frac{j\dfrac{\omega}{RC}\left(1+\dfrac{r_2}{r_1}\right)}{\left(\dfrac{1}{LC}-\omega^2\right)+j\dfrac{\omega}{RC}}$$

Positive feedback: $\varphi\left(L\left(j\omega_0\right)\right)=0\Leftrightarrow\omega_0=\dfrac{1}{\sqrt{LC}}$

Loop gain magnitude at ω_0:

$$\left|A\left(j\omega_0\right)\beta\left(j\omega_0\right)\right|=1+\frac{r_2}{r_1}\Rightarrow \text{ oscillation starts for: } \frac{r_2}{r_1}\geq 1$$

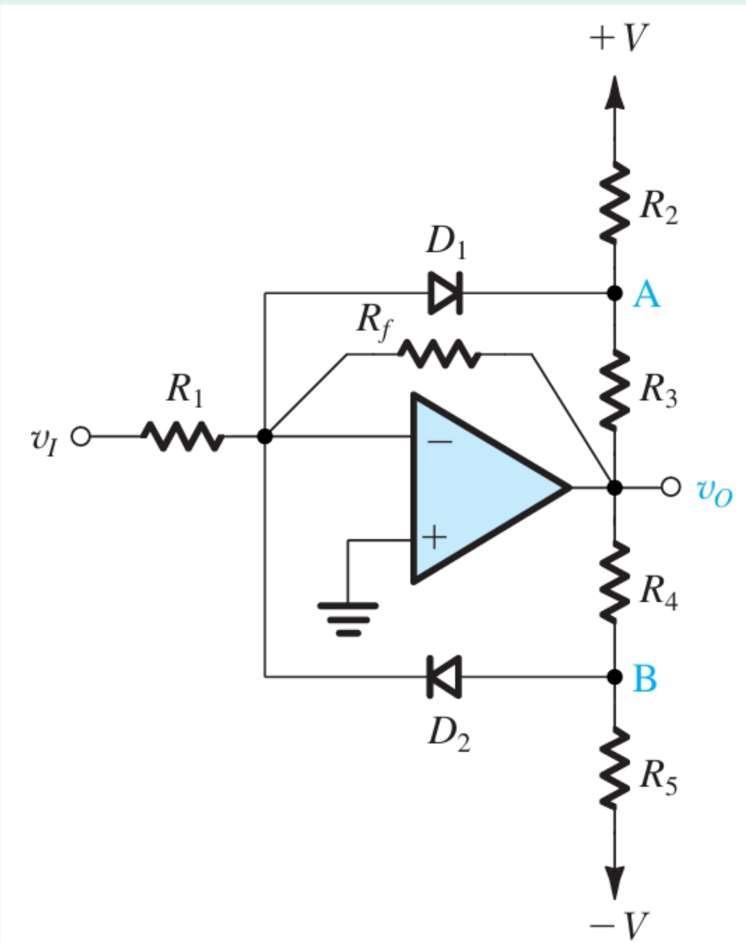
Alternative analysis

- Used already for the phase-shift oscillator

1. We assume the circuit is oscillating => $V(j\omega_0), I(j\omega_0)$
2. Analyze the circuit - reduce to a single eqn. in terms of a voltage or current variable
3. The circuit oscillates => non-zero $V(j\omega_0)$(or $I(j\omega_0)$) => eliminate variable by dividing in the eqn.
4. Obtain the oscillation condition from the remaining eqn: $D(s)=0$ ($D(s)$ = polynomial in s)=> $Re(D(j\omega_0))=0$, $Im(D(j\omega_0))=0$
5. The resulting equations yield ω_0 value and the condition for sustained oscillations

Amplitude control - limiter circuit

- Soft limiter => avoid distortions



When both D_1 and D_2 are off:

$$v_O = -\frac{R_f}{R_1} v_I$$

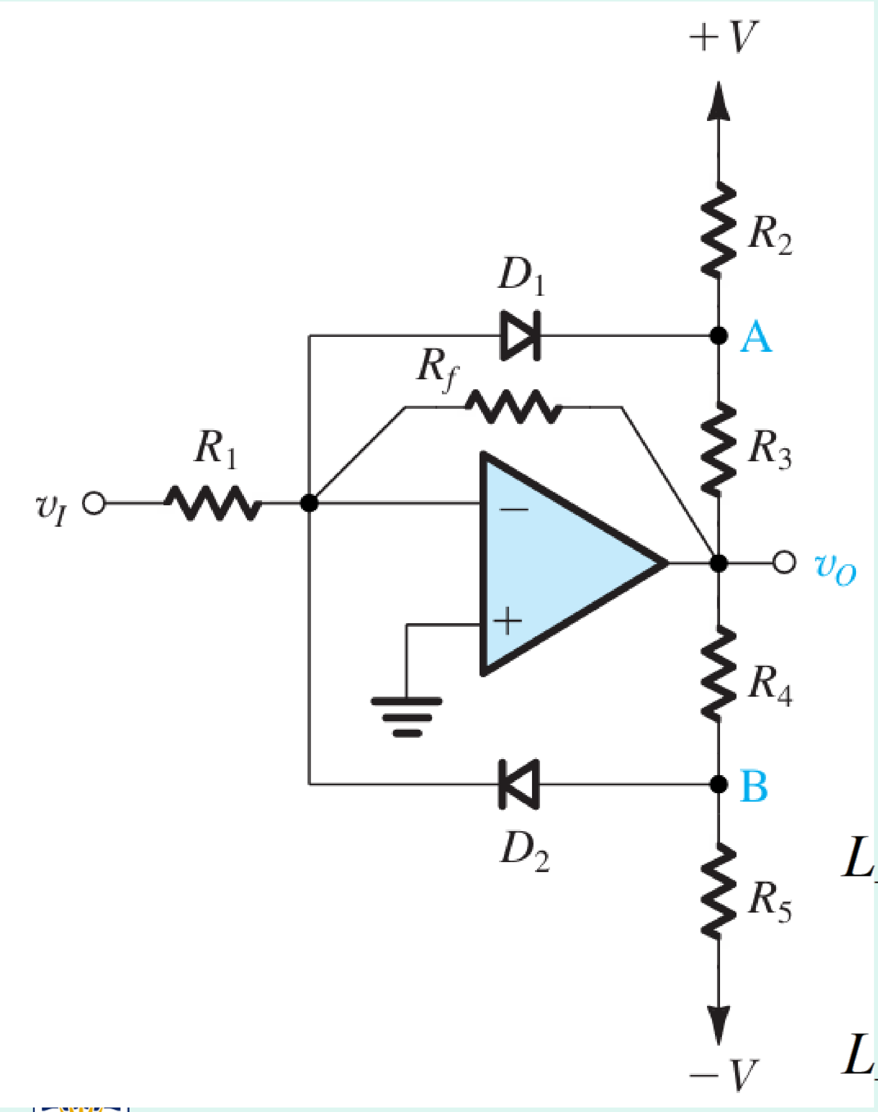
$$v_A = V\frac{R_3}{R_2 + R_3} + v_O\frac{R_2}{R_2 + R_3}$$

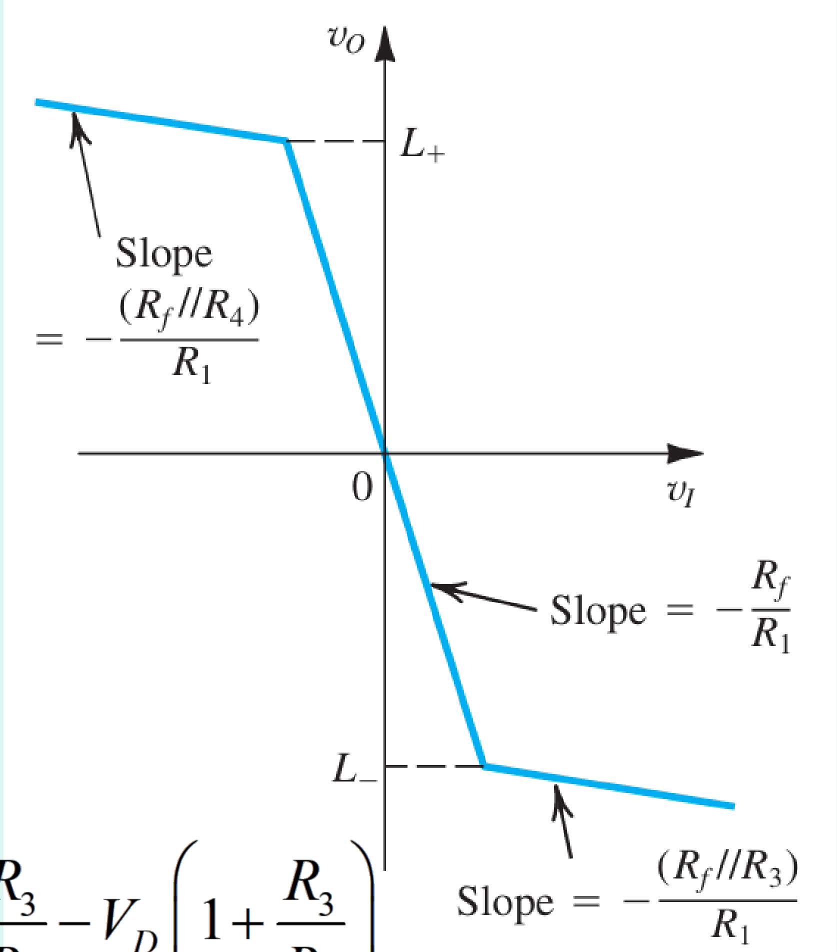
$$v_B = -V\frac{R_4}{R_4 + R_5} + v_O\frac{R_5}{R_4 + R_5}$$

Negative limiting level v_A=-0.7V

Positive limiting level v_B=+0.7V

Amplitude limiter action



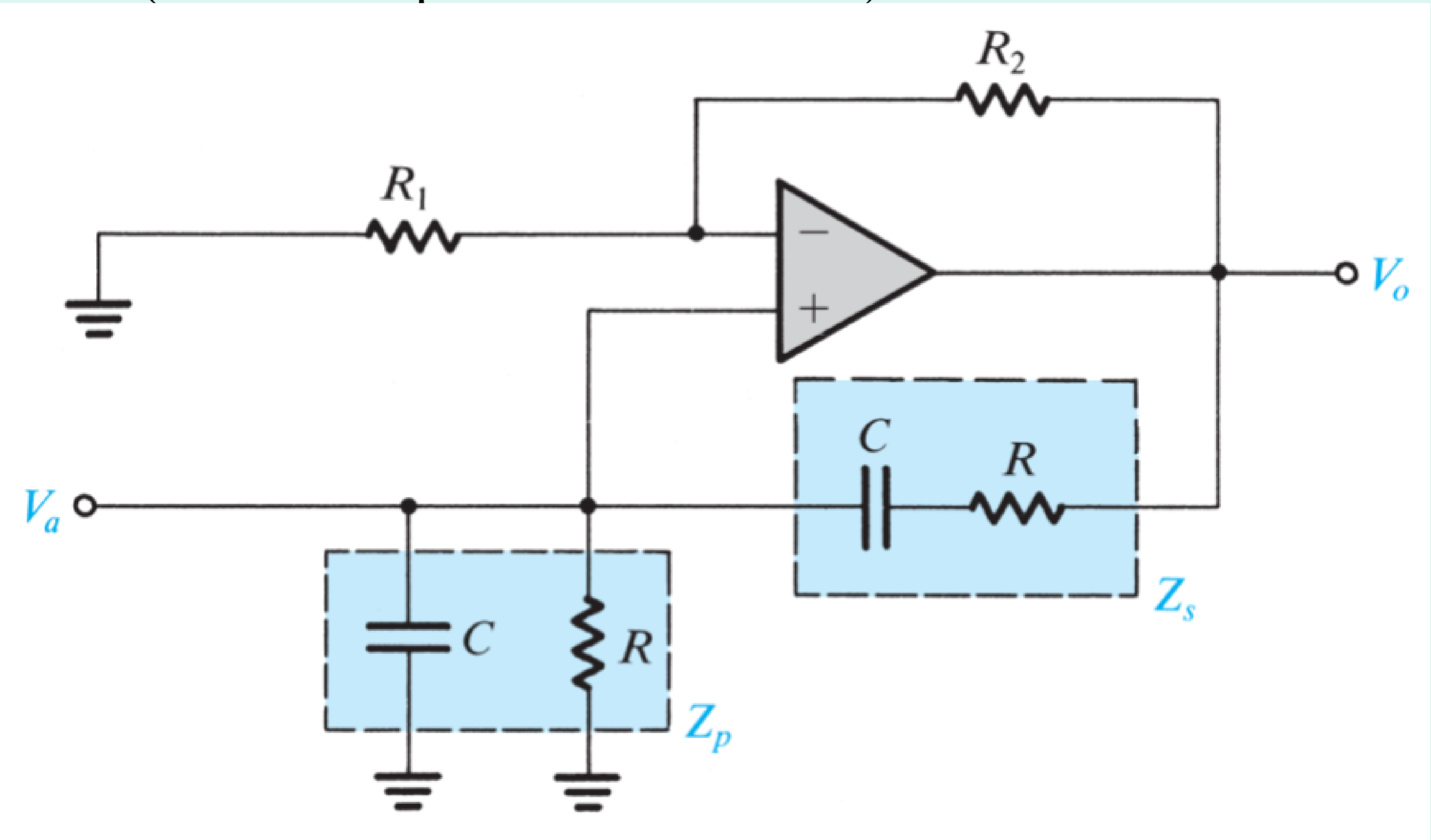


$$L_- = -V\frac{R_3}{R_2} - V_D\left(1+\frac{R_3}{R_2}\right)$$

$$L_+ = V\frac{R_4}{R_5} + V_D\left(1+\frac{R_4}{R_5}\right)$$

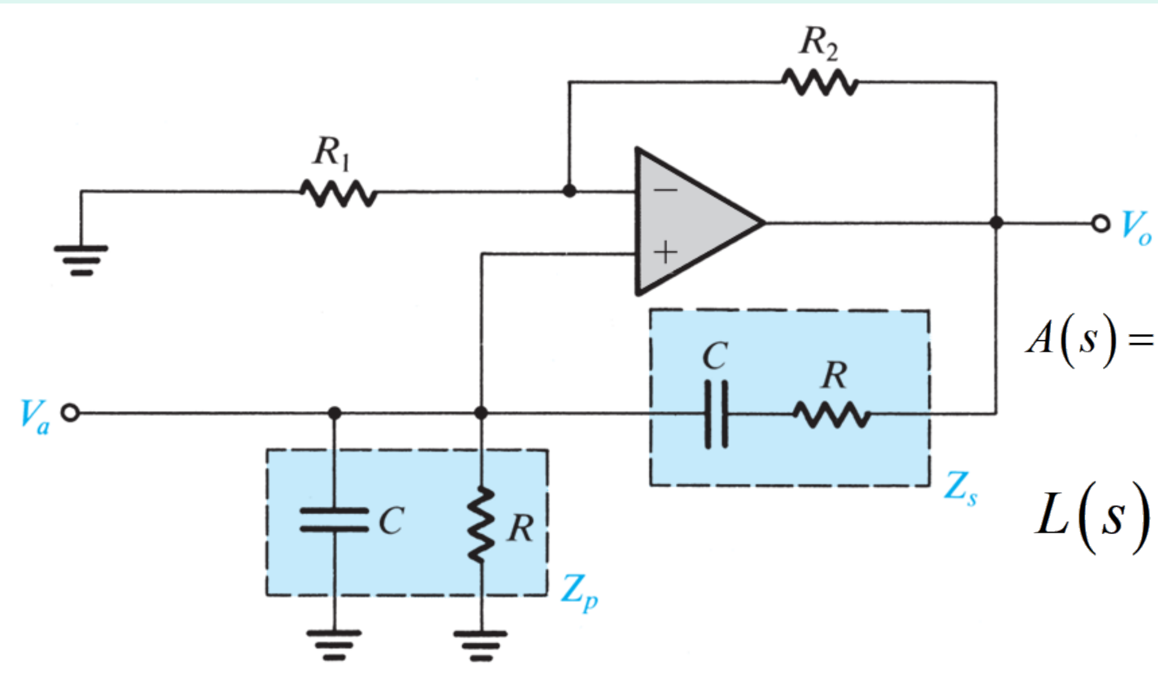
Wien-bridge oscillator

- Active RC oscillator type - Max Wien (1891) for L measurement, then W. Hewlett (1939) - HP200A sine-wave generator
- Circuit (without amplitude stabilization)



Wien-bridge oscillator analysis

- Compute the loop gain



$$A(s) = 1 + \frac{R_2}{R_1}, \beta(s) = \frac{Z_p(s)}{Z_p(s) + Z_s(s)} = \frac{1}{1 + Z_s Y_p}$$

$$L(s) = \frac{1 + R_2 / R_1}{1 + Z_s Y_p} = \frac{1 + R_2 / R_1}{3 + sCR + \frac{1}{sCR}}$$

Find the oscillation condition for $\omega = \omega_0$:

$$L(j\omega) = \frac{1 + R_2 / R_1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

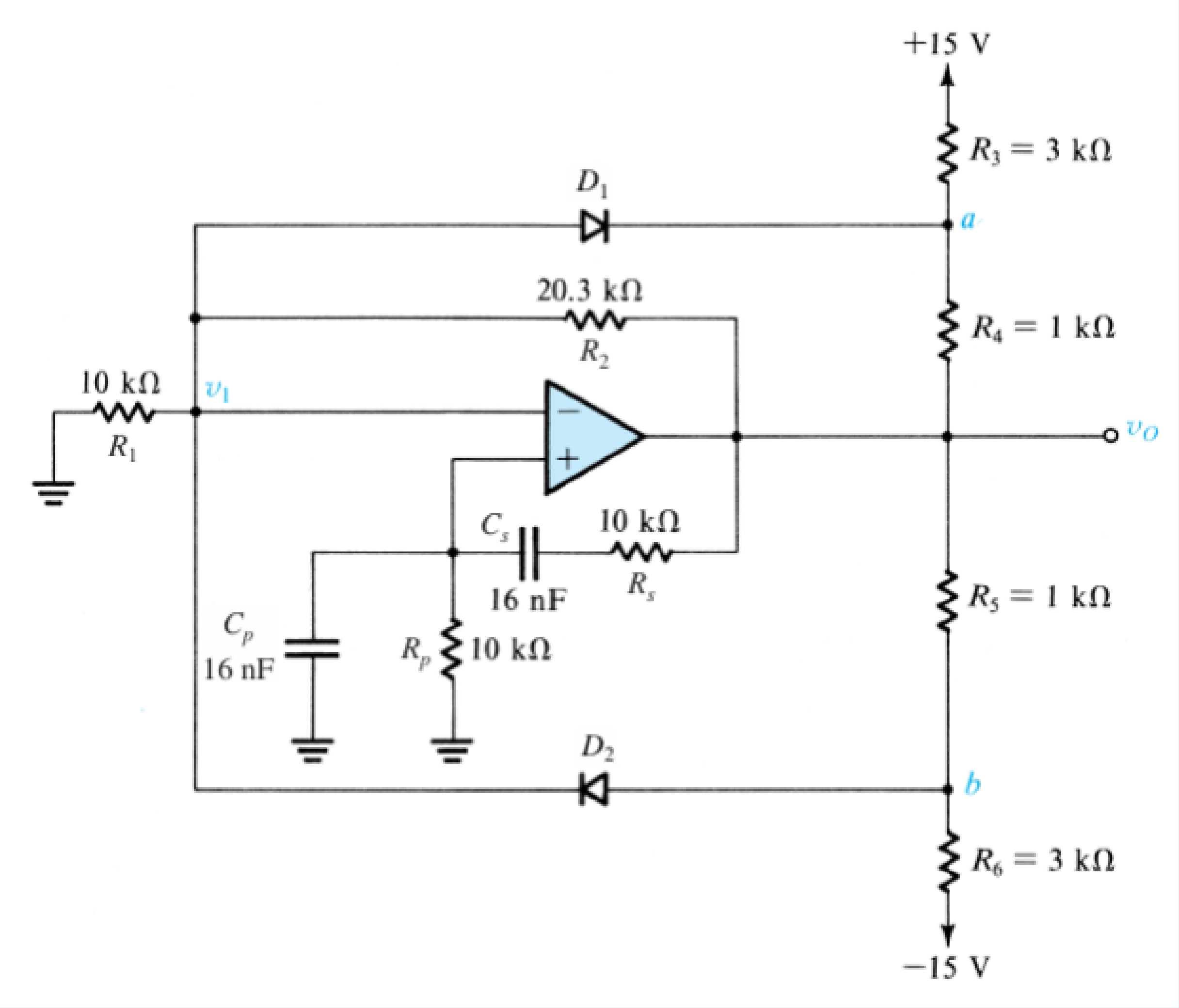
Phase condition => ω_0: $\omega_0 = \frac{1}{RC}$

Oscillation start condition:

$$L(j\omega_0) = \frac{1 + R_2 / R_1}{3} \geq 1 \Leftrightarrow \frac{R_2}{R_1} \geq 2$$

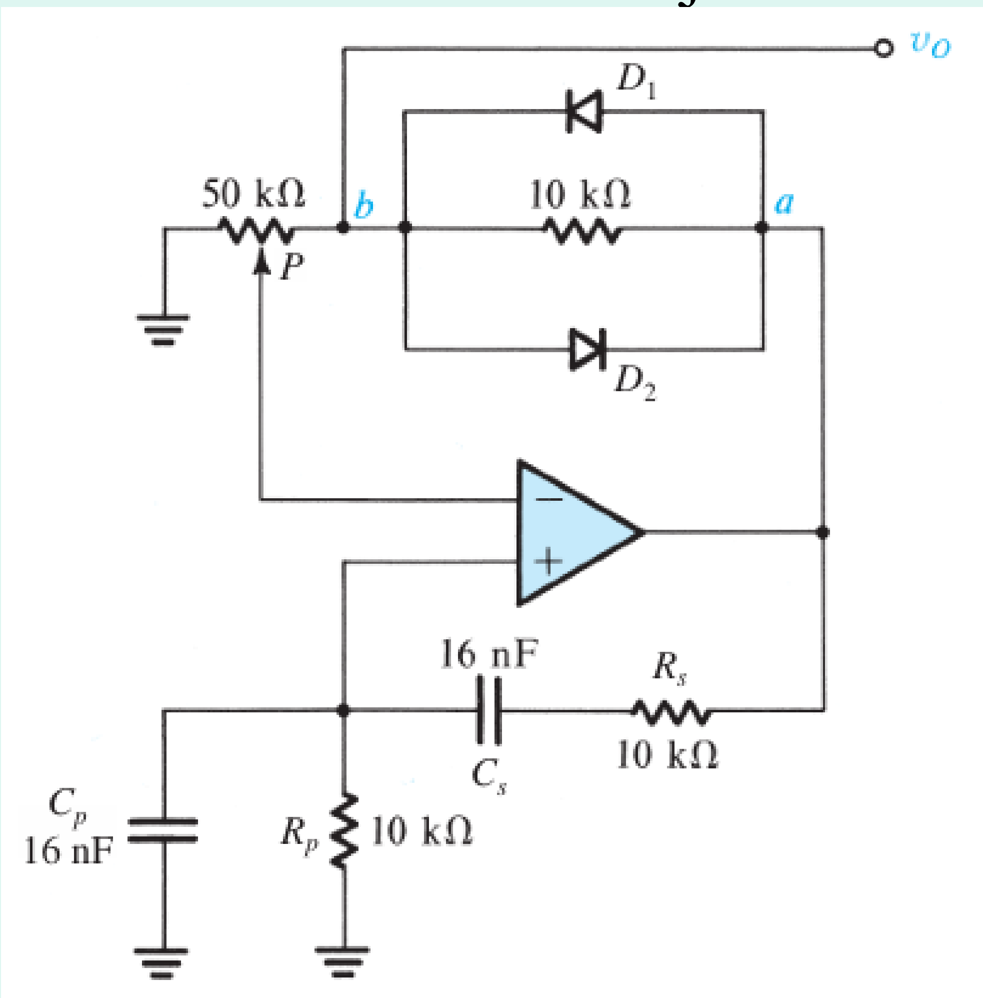
Wien-bridge oscillator - Amplitude control

- Version (a)

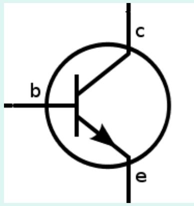


Wien-bridge oscillator amplitude control

- Version (b) - reduced no of components
- Potentiometer P adjusts the the output amplitude



Node (b) output:
- lower distortion than (a)
- higher impedance than (a)


c
b
e