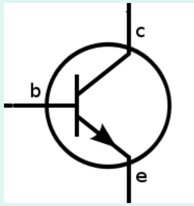
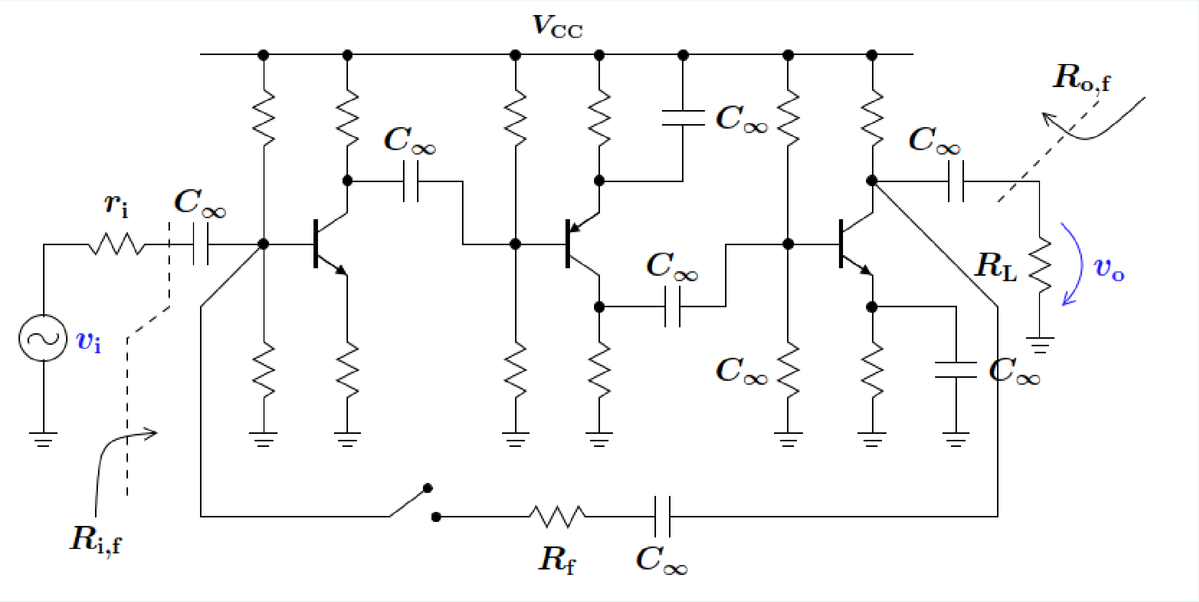
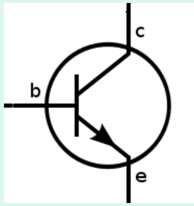


ELEC 301 - Switched- capacitor filters

L33 - Dec 1, 2025

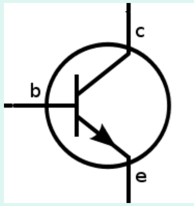
Instructor: Edmond Cretu





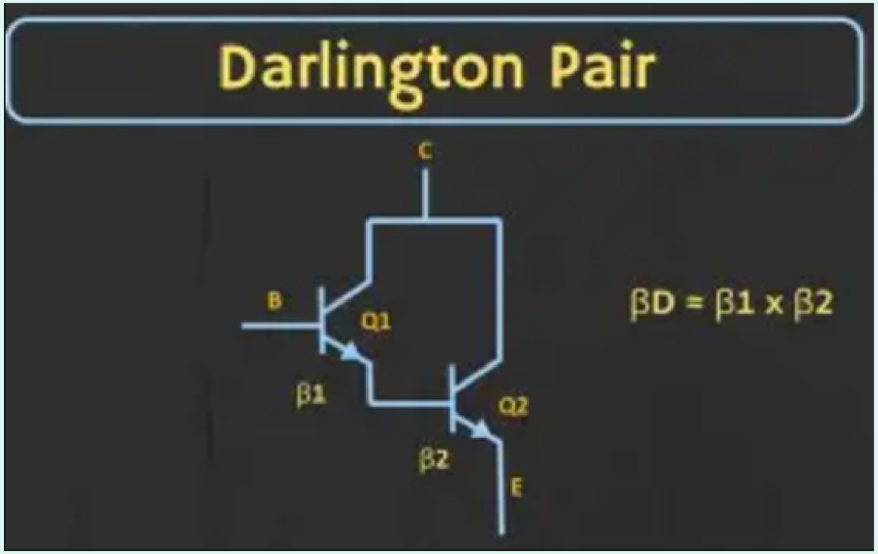
Last time

- Oscillators - design perspective, phase and gain conditions. frequency stability
- RLC oscillators
- Wien-bridge oscillators
- Amplitude control loop for oscillators



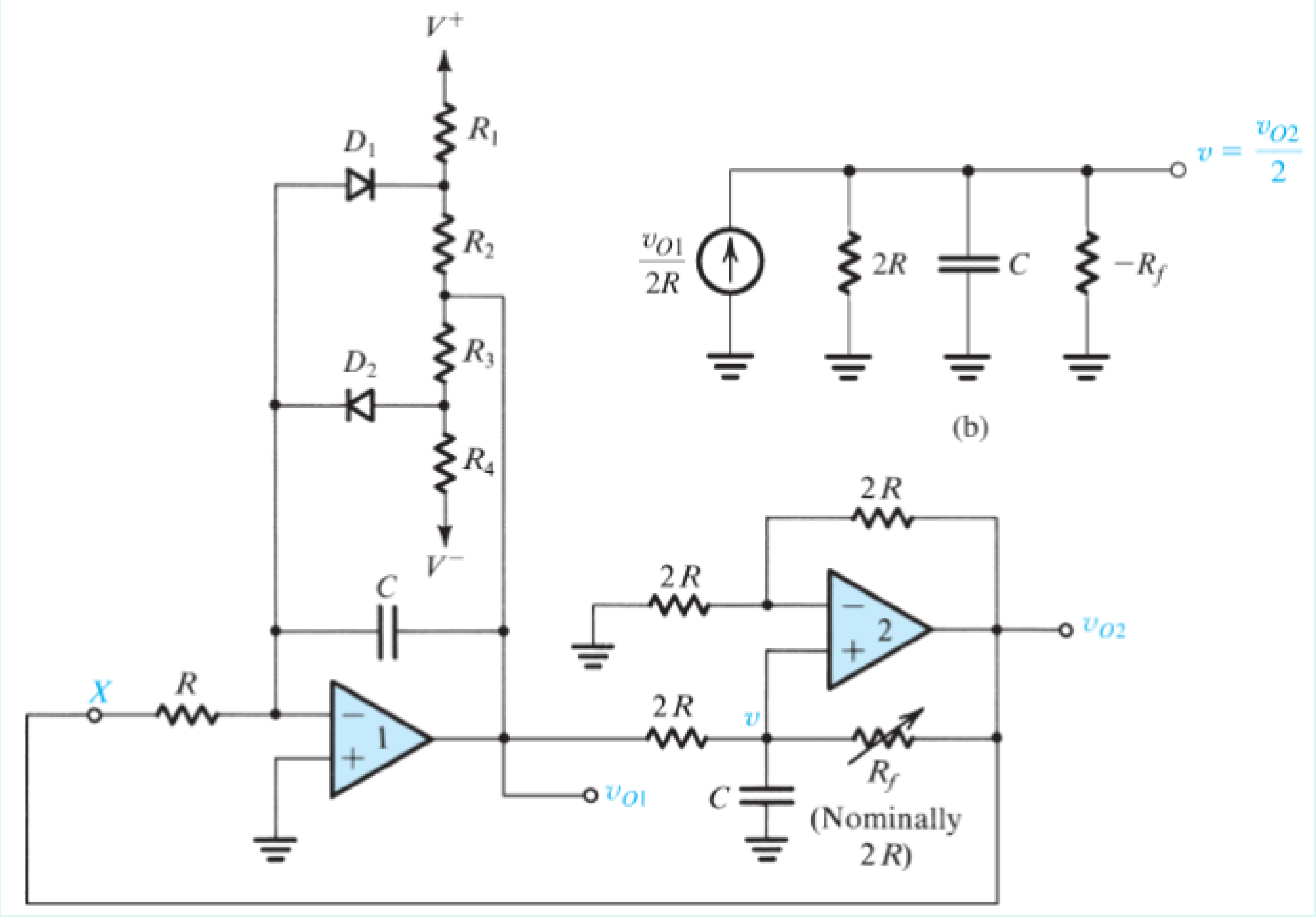
History bits

- Wilhelm Cauer (1900-1945) - used Chebyshev polynomials in an approach that unified the design of filters transfer functions + significant contributions to LC filter synthesis
- Sidney Darlington (1906-1997)- developed a complete design theory for LC filters, but better known for the transistor Darlington pair circuit



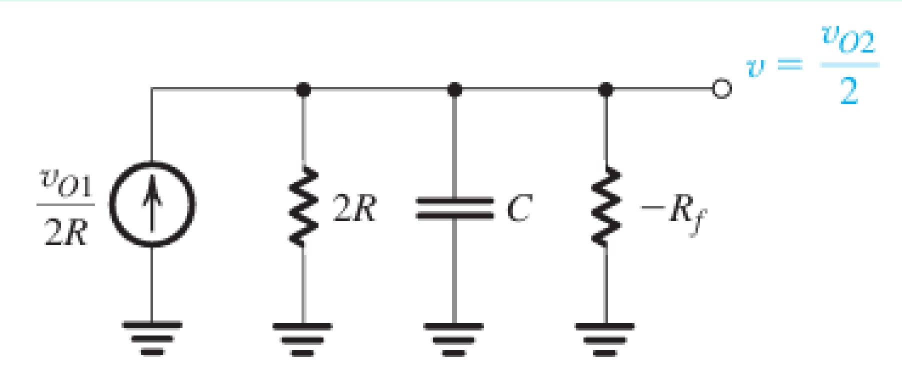
Quadrature oscillator

- Two op-amps - two series integrators: inverting integrator +non-inverting integrator



Quadrature oscillator analysis

- equivalent model (linear region, no limiter)



$$\frac{V_{o2}}{2} = \frac{V_{o1}}{2R}\frac{1}{sC} \Rightarrow V_{o2} = -\frac{V_x}{sRC}\frac{1}{sRC}$$

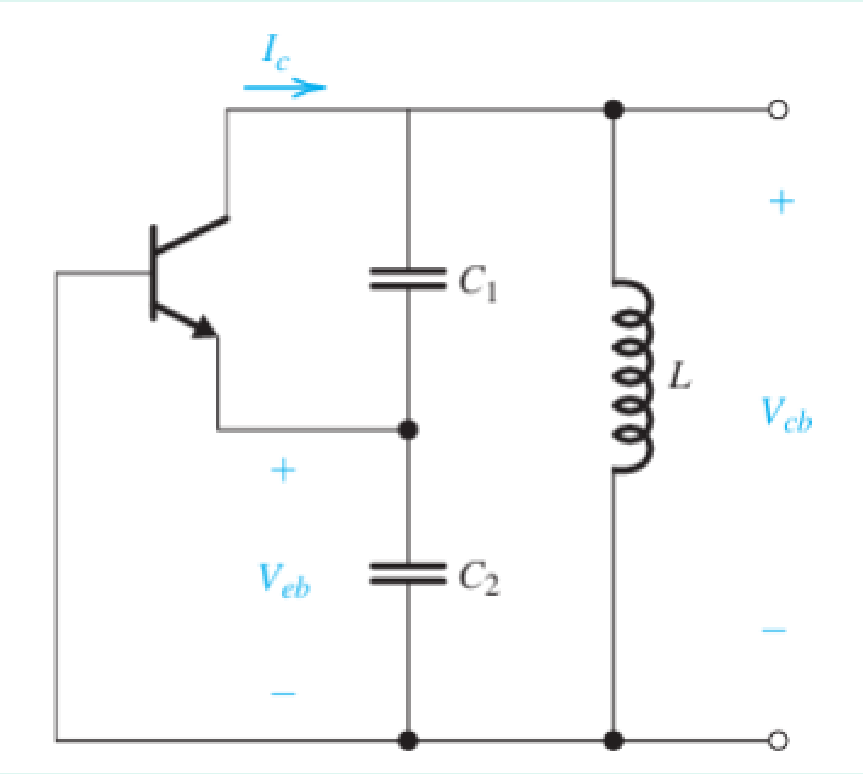
The loop gain: $$L(s) = \frac{V_{o2}}{V_x} = -\frac{1}{s^2(RC)^2}$$

The oscillation frequency: $$\omega_0 = \frac{1}{RC}$$

Remark: V_{o1}, V_{o2} have 90^0 phase shifts - used in many applications (e.g. Hilbert transform, quadrature modulation/demodulation

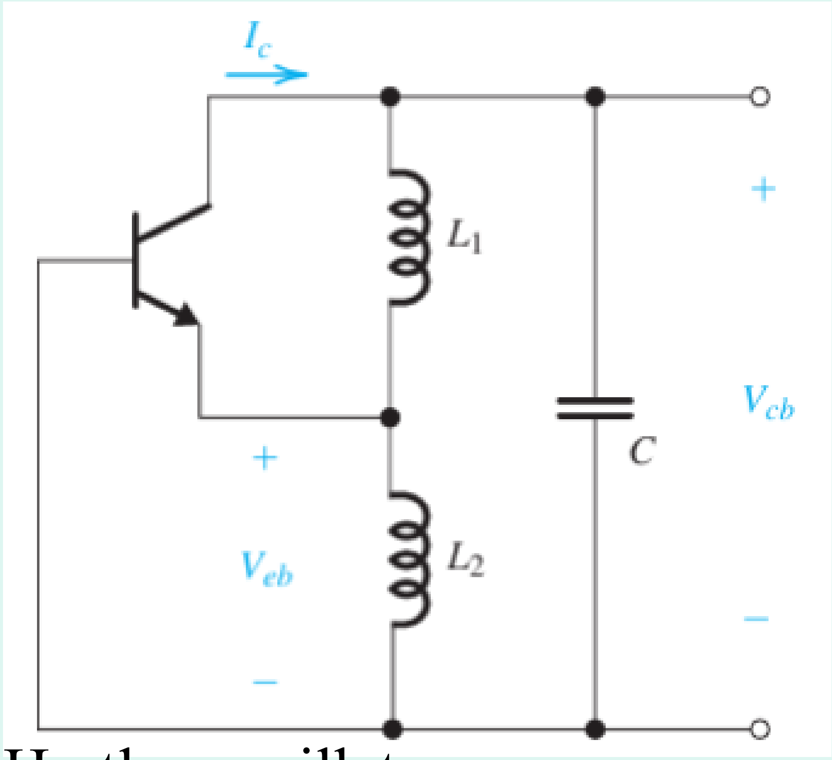
LC oscillators

- Difficult to tune over wide ranges
- see ch. 18.13 in Sedra&Smith



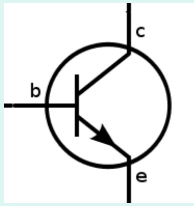
Colpitts oscillator

$$\omega_{0,Colpitts} = \frac{1}{\sqrt{L\frac{C_1 C_2}{C_1 + C_2}}}$$



Hartley oscillator

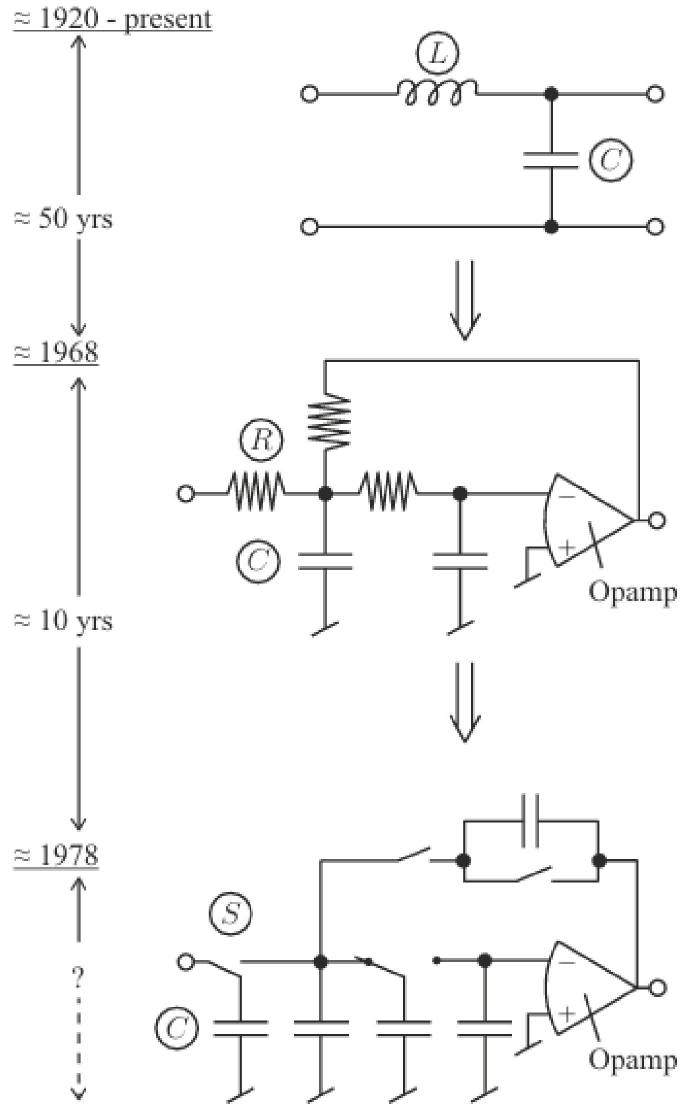
$$\omega_{0,Hartley} = \frac{1}{\sqrt{(L_1 + L_2)C}}$$



Switched-capacitor circuits

- Goal: reduce IC area by avoiding large resistance
- Concept: mimic a large value resistance at a lower frequency by fast switching a capacitor at high frequency
- Discrete-time circuits, not continuous time filters
- Applications: audio filters (low-frequency filters) in CMOS technology (CMOS switches, op amps and small capacitors)

Evolution of analog filters



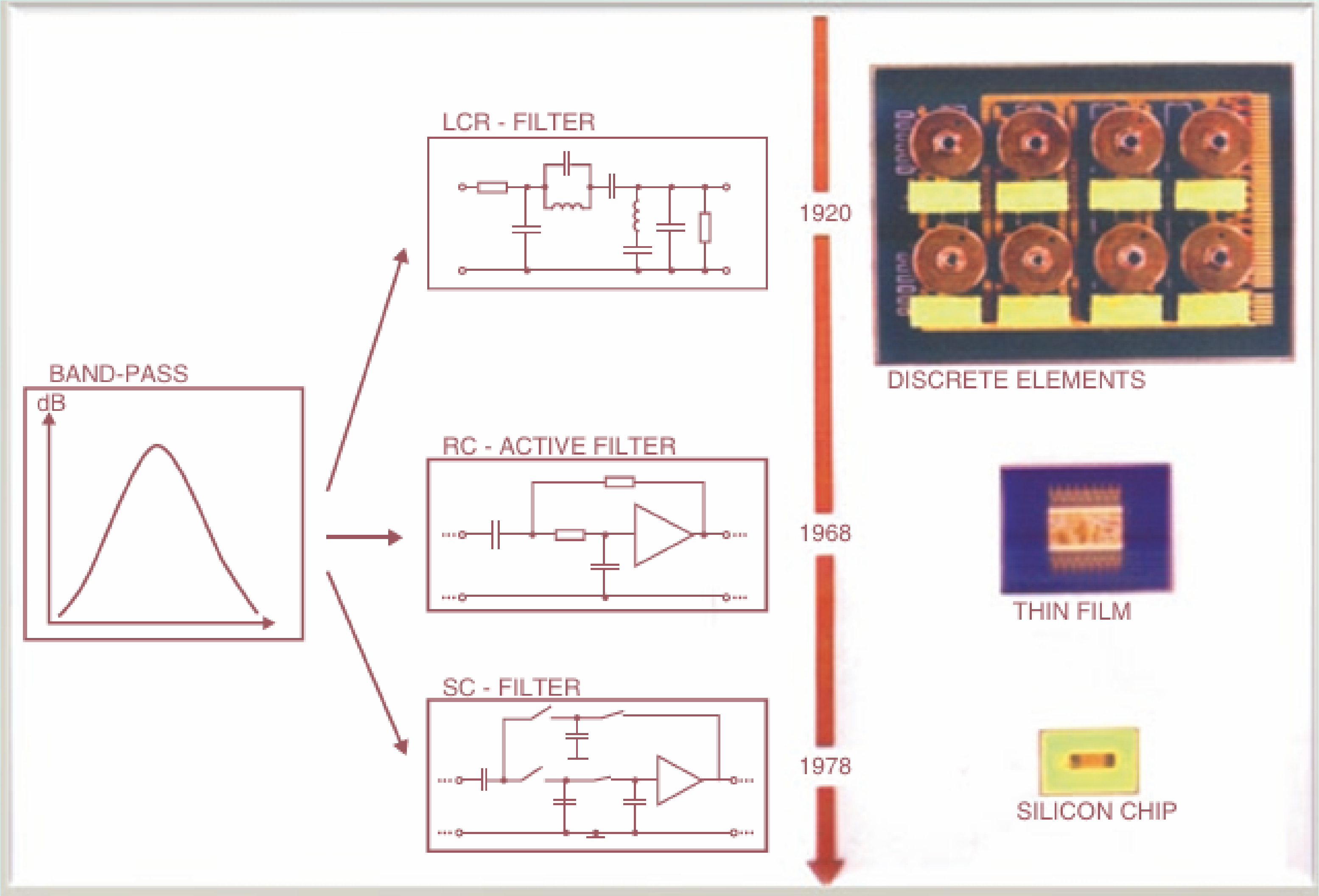
LC Filter
Discrete Components

Active RC Filter
Hybrid Integrated Circuit (Thin and Thick Film)

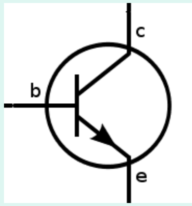
Switched-Capacitor (SC) Filter
"Filter on a chip!"

Source: Moschytz[2019]Analog circuit theory and filter design

Evolution of frequency-selective filters

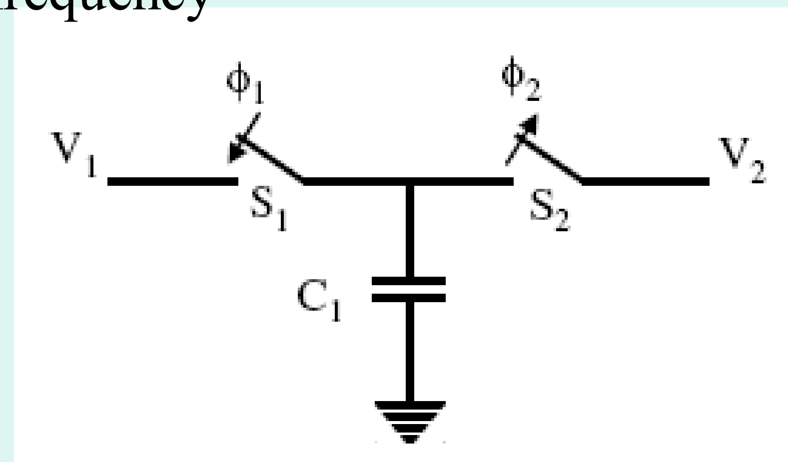


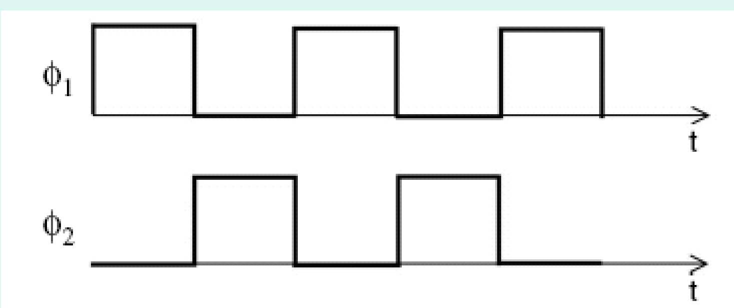
Source: Moschytz[2019]Analog circuit theory and filter design



Switched-capacitor resistor

- Important: non-overlapping clock phases
- small C => large $R_{eq,LF}$ + Equivalent resistance value changed by clock frequency

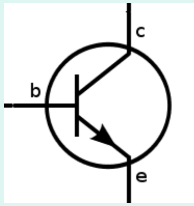




- Charge transfer from V1 to V2 as dictated by clock

$$i = \frac{\Delta Q}{\Delta t} = \frac{N \cdot C_1 \left(V_1 - V_2 \right)}{\Delta t}$$

$$\Rightarrow R_{eq,LF} = \frac{V_1 - V_2}{i_{LF}} = \frac{1}{C_1 f_{clk}}$$

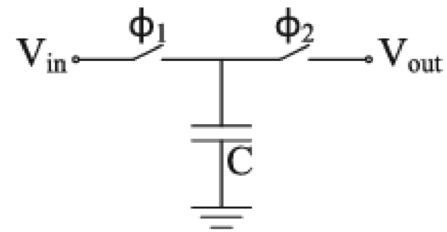
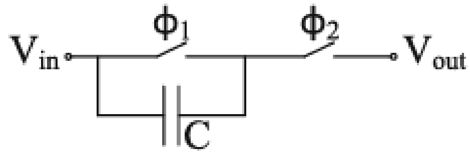
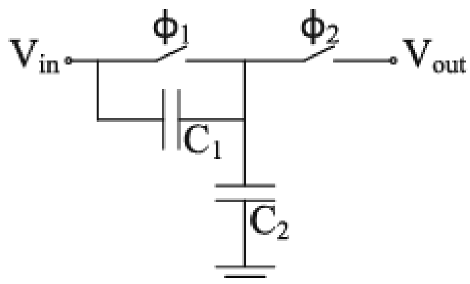
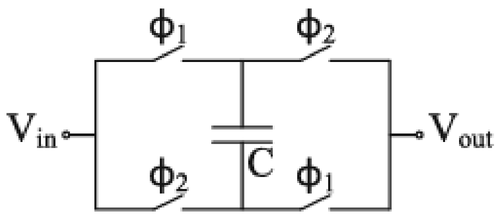


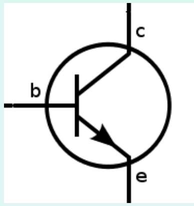
Switched-capacitor (SC) circuits features

- Discrete-time interface - easier to interface with digital electronics
- Very suited to CMOS signal processing chain
- Transfer function depends on ratios of capacitors (not individual values)
- Filter frequency is tuned by changing the clock frequency of the SC circuit
- High-value resistors avoided
- Switches implemented with MOS transistors

SC resistor emulation circuits

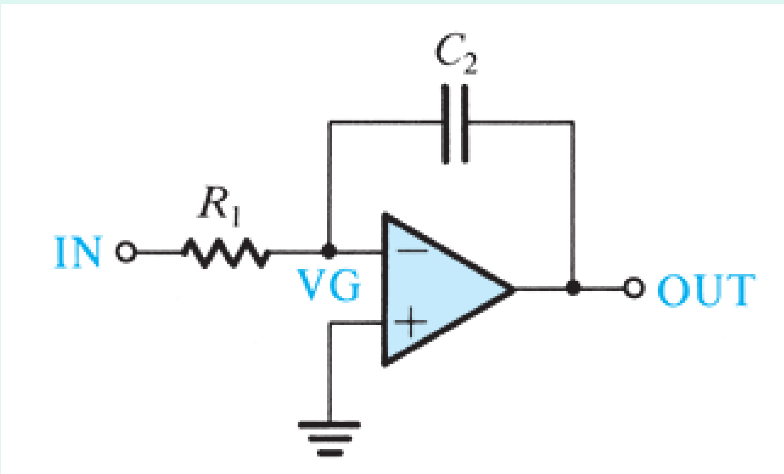
- Alternative ways to mimic a LF resistor through charge transfer

Circuit	Schematic	R_{eq}	$Q(\varphi_1)$	$Q(\varphi_2)$
Parallel		$\frac{T}{C}$	$V_{in}C$	$V_{out}C$
Series		$\frac{T}{C}$	0	$(V_{in} - V_{out})C$
Series-Parallel		$\frac{T}{C_1 + C_2}$	0 $V_{in}C_2$	$(V_{in} - V_{out})C_1$ $V_{out}C_2$
Bilinear		$\frac{1}{4}\frac{T}{C}$	$(V_{in} - V_{out})C$	$(V_{out} - V_{in})C$

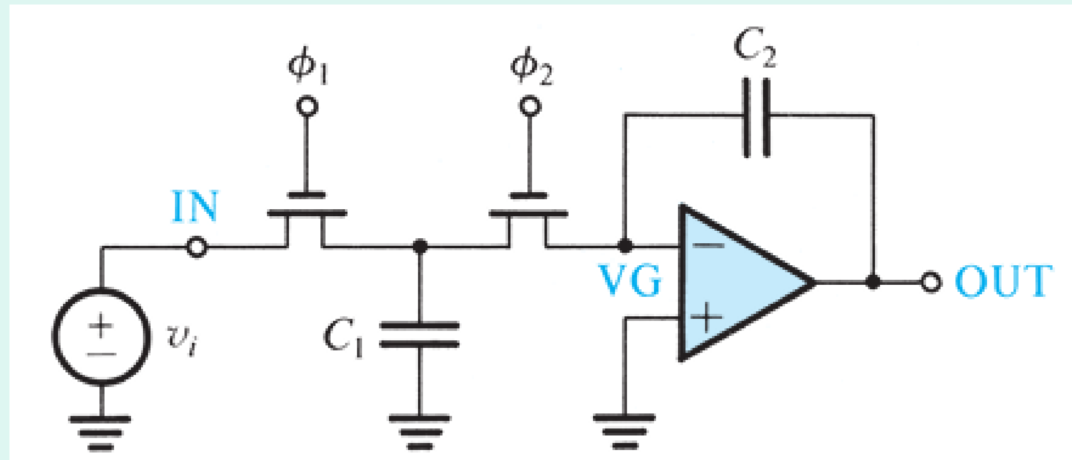


Exm: integrator circuit

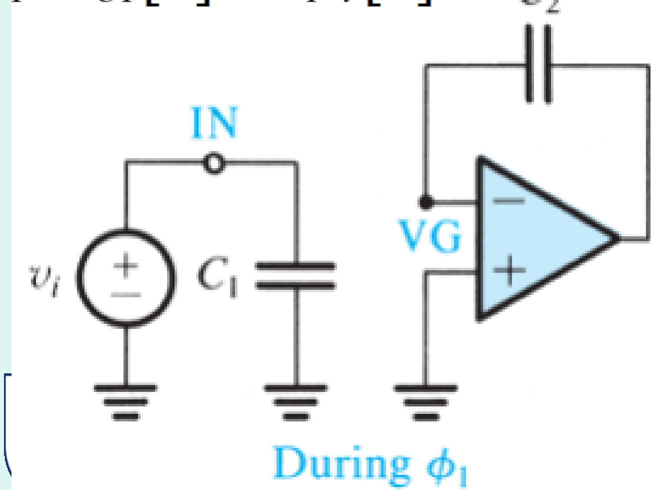
Continuous-time active integrator:

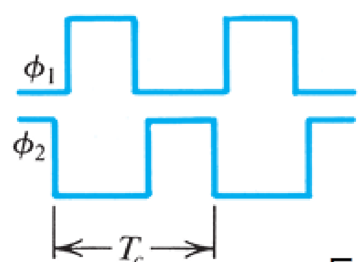


Switched-capacitor integrator (f_C>>f_H):



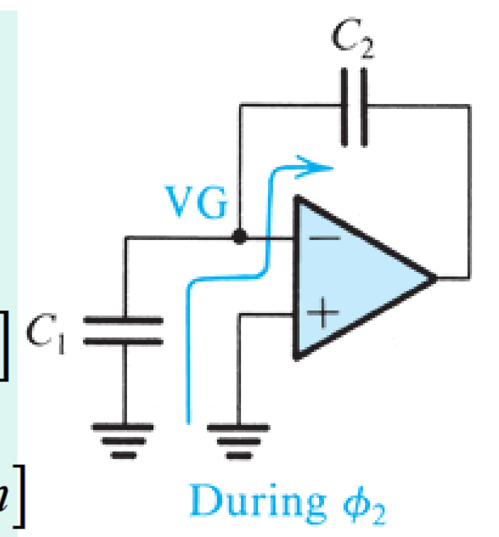
$$\varphi_1 : q_{C1}[n] = C_1 v_i[n]$$

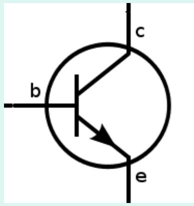




$$\varphi_2 : q_{C2}\left[n+\frac{1}{2}\right] = q_{C1}[n]$$

$$v_o\left[n+\frac{1}{2}\right] = v_o[n] - \frac{C_1}{C_2} v_1[n]$$





Mapping back into CT

- The average current flowing between IN and the virtual ground (VG):

$$i_{av} = \frac{C_1 v_i}{T_c} = C_1 f_c v_i$$

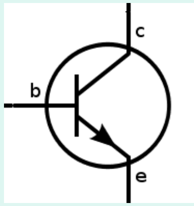
If $f_c >> f_H$ (max frequency of the input analog voltage v_i), then the equivalent LF resistance:

$$R_{eq} = \frac{1}{C_1 f_c}$$

Equivalent integrator time constant:

$$\tau = R_{eq} C_2 = \frac{C_2}{C_1} T_c$$

- Implementation aspects: the accuracy of capacitor ratios in IC technology $C_2/C_1 \sim 0.1\%$ (C_1, C_2 ~ 0.05…100pF)
- with $f_c \sim 100$kHz, $C_2/C_1 \sim 10$ => $\tau \sim 10^{-4}$s


c
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