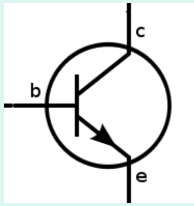
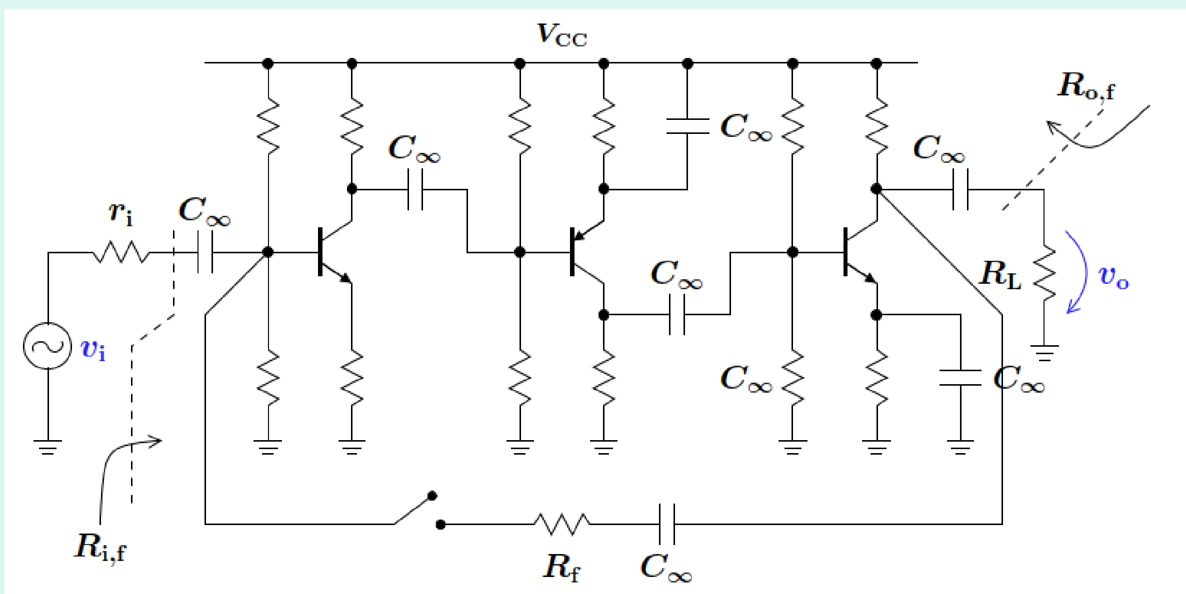
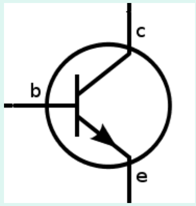


ELEC 301 - CT and DT filters

L34 - Dec 2, 2025

Instructor: Edmond Cretu



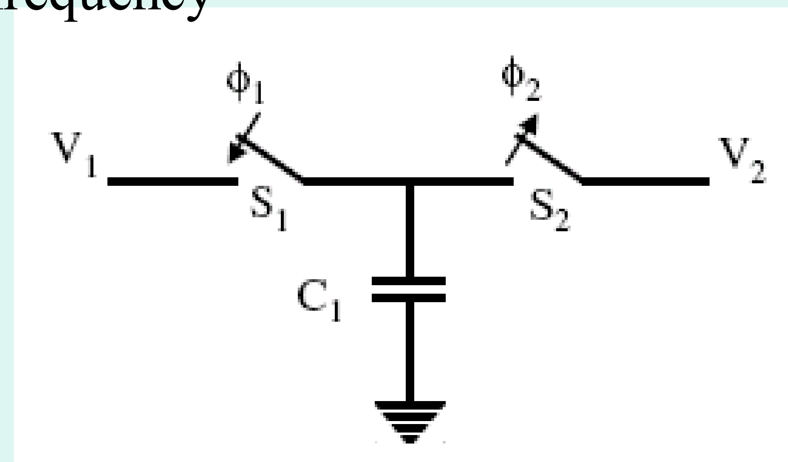


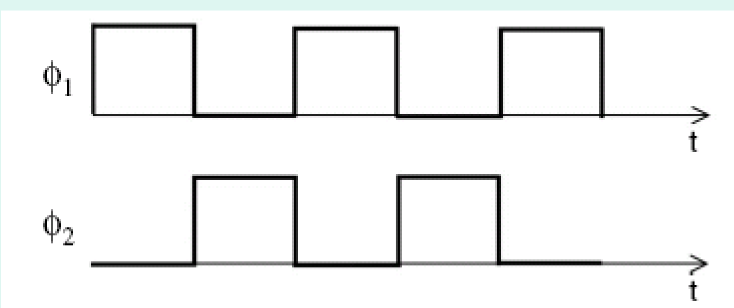
Recall

- CT active oscillators: phase-shift (RC), RLC (resonator), Wien bridge (RC), quadrature (inverting + noninverting integrators), LC (Hartley, Colpitts)
- Discrete-time approach - switched-capacitors (SC) circuits - emulate large R = C+fast switching

Switched-capacitor resistor

- Important: non-overlapping clock phases
- small C => large $R_{eq,LF}$ + Equivalent resistance value changed by clock frequency





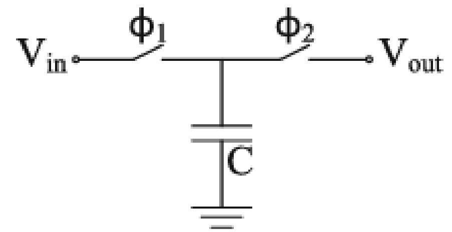
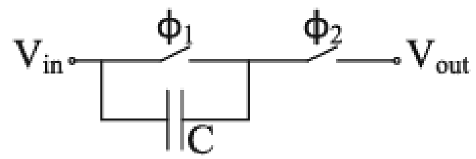
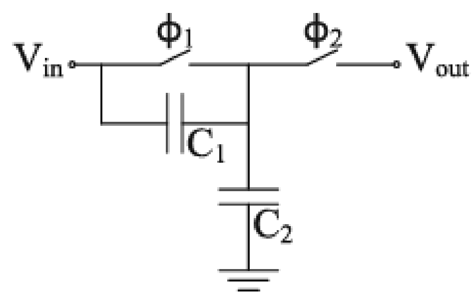
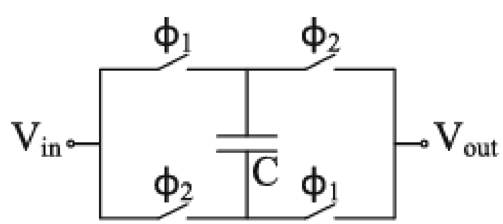
- Charge transfer from V1 to V2 as dictated by clock

$$i = \frac{\Delta Q}{\Delta t} = \frac{N \cdot C_1 (V_1 - V_2)}{\Delta t}$$

$$\Rightarrow R_{eq,LF} = \frac{V_1 - V_2}{i_{LF}} = \frac{1}{C_1 f_{clk}}$$

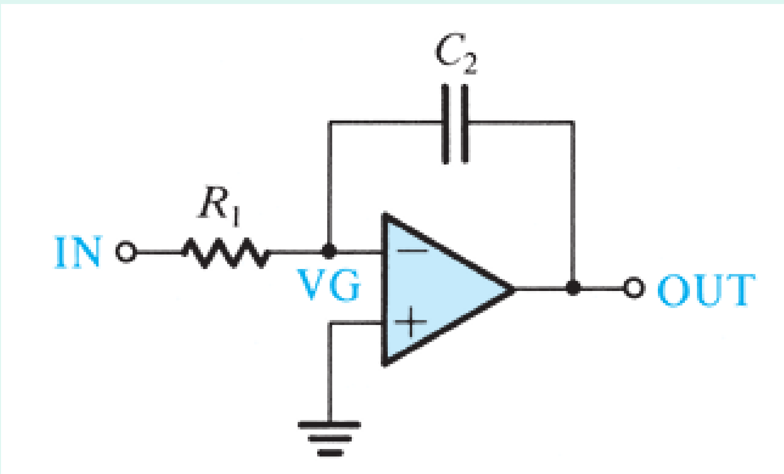
SC resistor emulation circuits

- Alternative ways to mimic a LF resistor through charge transfer

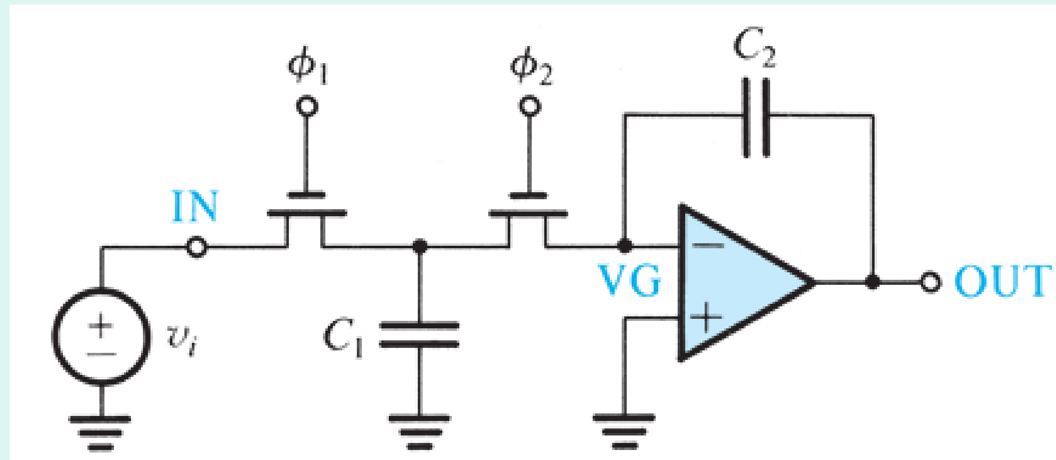
Circuit	Schematic	R_{eq}	$Q(\varphi_1)$	$Q(\varphi_2)$
Parallel		$\frac{T}{C}$	$V_{in}C$	$V_{out}C$
Series		$\frac{T}{C}$	0	$(V_{in} - V_{out})C$
Series-Parallel		$\frac{T}{C_1 + C_2}$	0 $V_{in}C_2$	$(V_{in} - V_{out})C_1$ $V_{out}C_2$
Bilinear		$\frac{1}{4}\frac{T}{C}$	$(V_{in} - V_{out})C$	$(V_{out} - V_{in})C$

Exm: integrator circuit

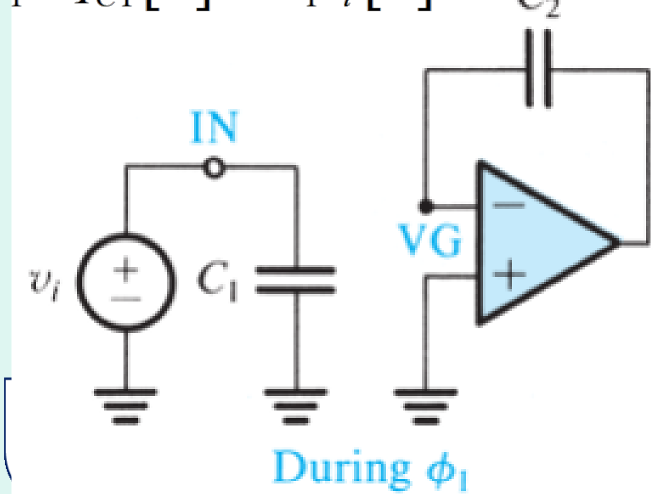
Continuous-time active integrator:

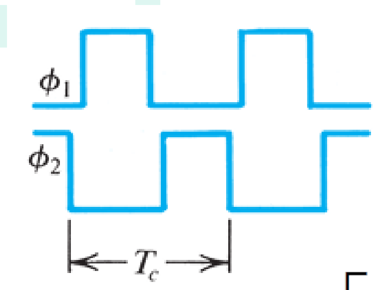


Switched-capacitor integrator (f_C>>f_H):



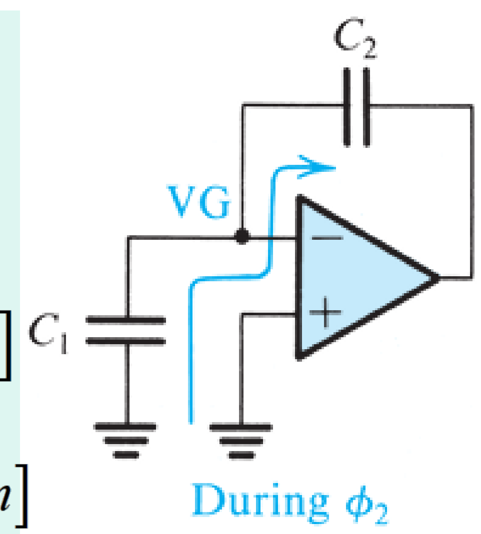
$$\varphi_1 : q_{C1}[n] = C_1 v_i[n]$$





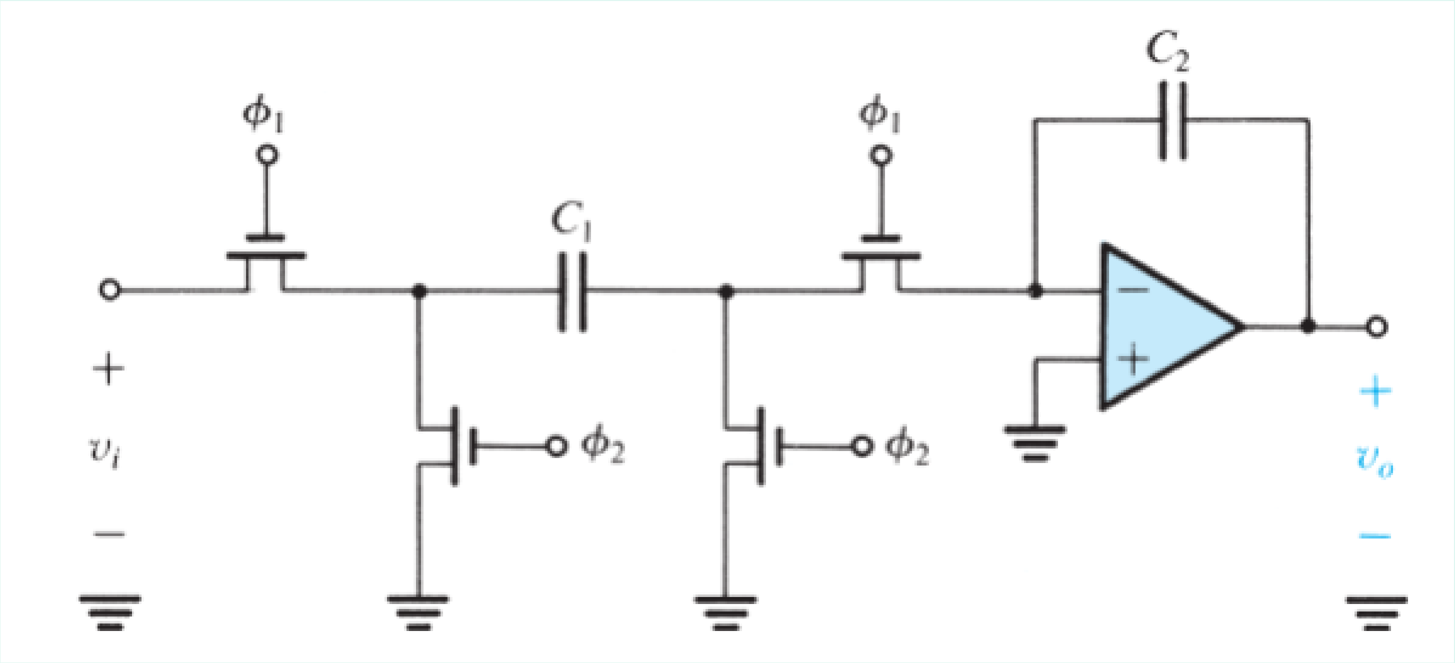
$$\varphi_2 : q_{C2}\left[n+\frac{1}{2}\right] = q_{C1}[n]$$

$$v_o\left[n+\frac{1}{2}\right] = v_o[n] - \frac{C_1}{C_2} v_1[n]$$



Improved SC inverting integrator

- SC integrator insensitive to stray capacitances

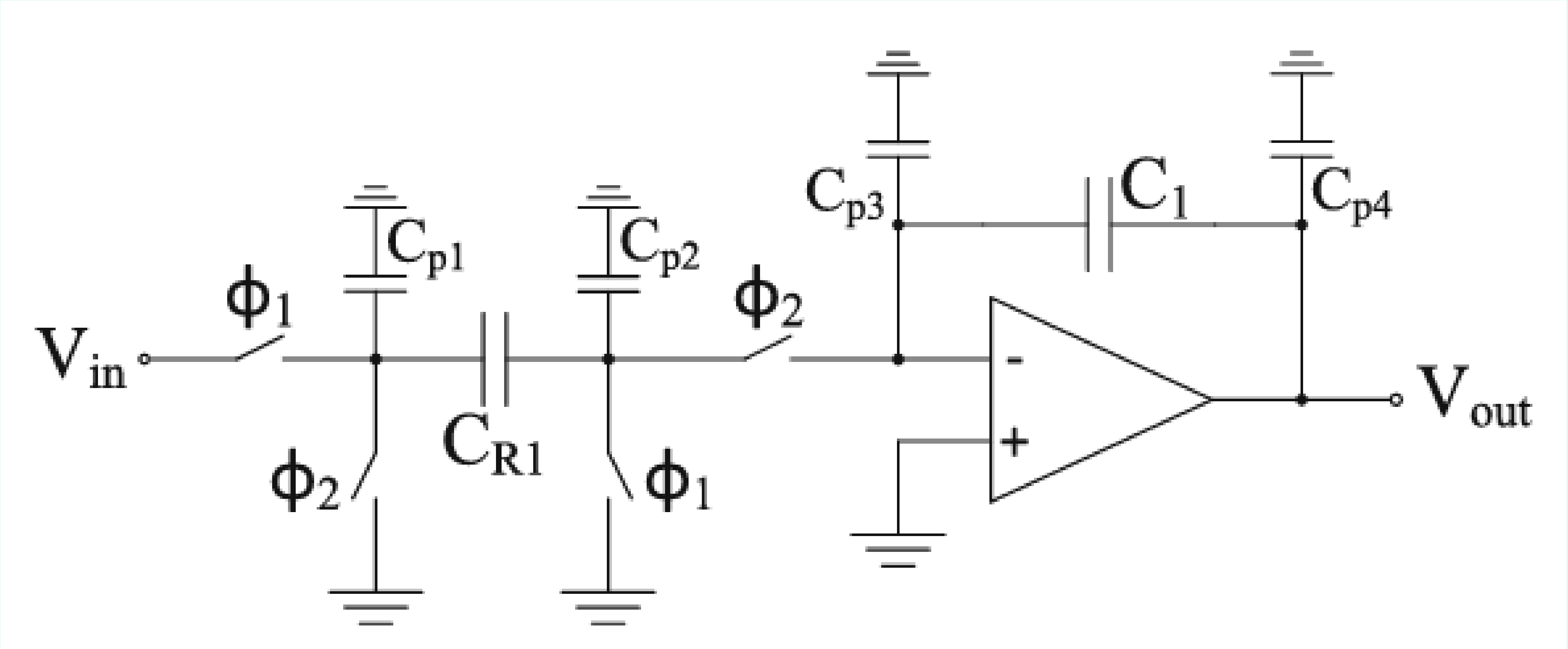


$$\varphi_1 : q_1[n] = C_1 v_i[n],\ q_2[n] = q_2\left[n - \frac{1}{2}\right] + q_1[n]$$

$$\varphi_2 : q_1\left[n + \frac{1}{2}\right] = 0,\ q_2\left[n + \frac{1}{2}\right] = -C_2 v_0\left[n + \frac{1}{2}\right]$$

SC parasitic insensitive integrator

- C_{pi} - stray (parasitic capacitances)

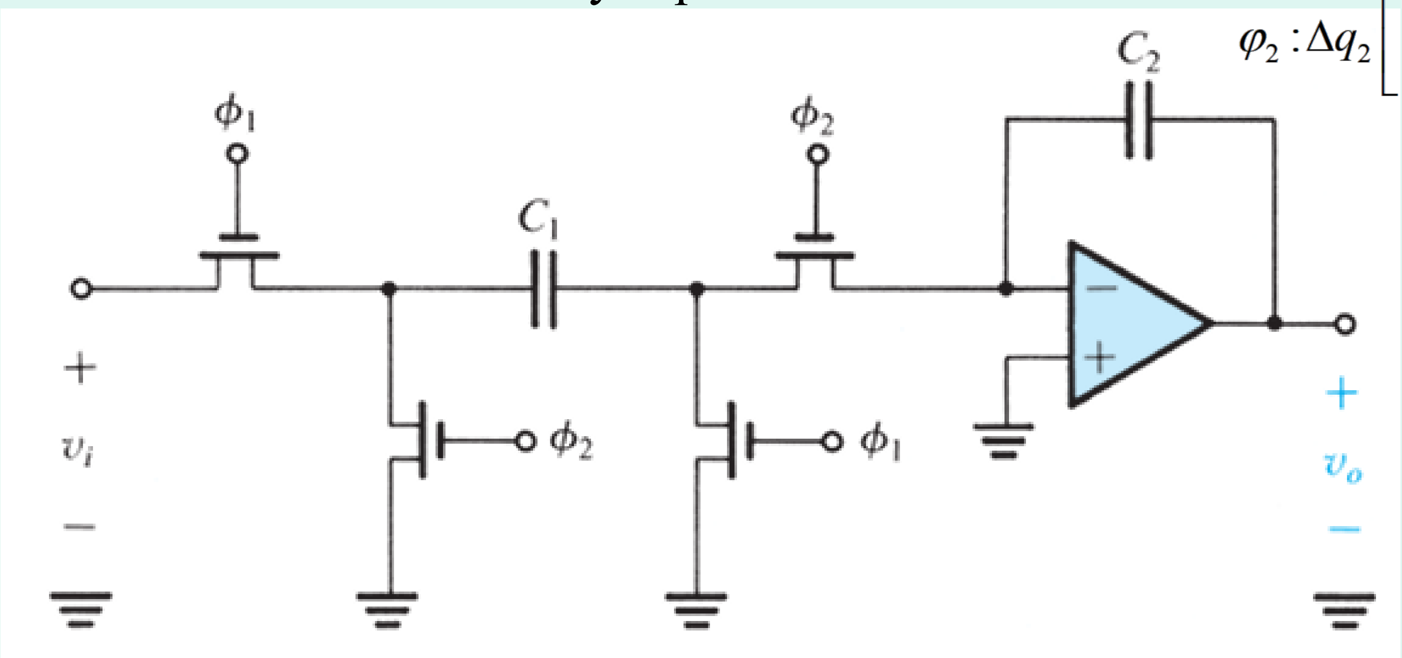


Improved non-inverting SC integrator

- Two-integrator-loop active filter - pair of complementary SC integrators (inverting + non-inverting)
- circuit insensitive to stray capacitances

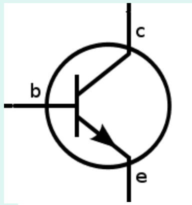
$$\varphi_1 : q_1[n] = v_i[n] C_1$$

$$\varphi_2 : \Delta q_2\left[n+\frac{1}{2}\right] = -q_1[n] = -C_1 v_i[n]$$



Non-inverting SC integrator:

$$v_o\left[n+\frac{1}{2}\right] = v_o[n] - \frac{\Delta q_2\left[n+\frac{1}{2}\right]}{C_2} = v_o[n] + v_i[n]\frac{C_1}{C_2}$$



Comparison of active RC and SC filters

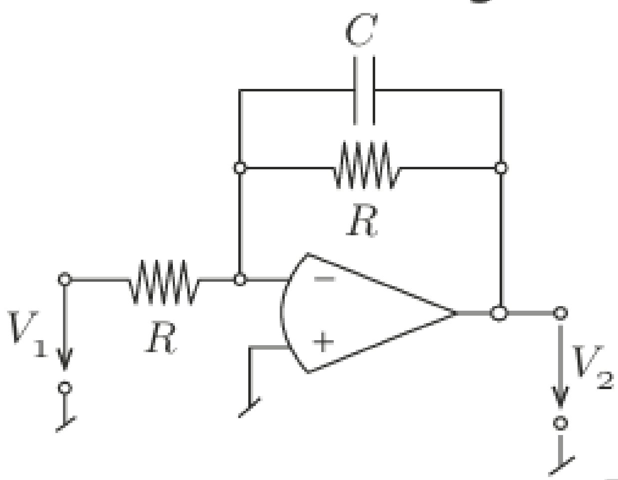
	Active RC (conventional)	SC
-Integrate on an MOS Chip?	Not possible for precision filter.	Readily possible.
-Is tuning necessary?	Yes.	No.
-Noise?	Depends on impedance level and amplifiers.	Depends on capacitance level and amplifiers. Generally larger than with Active RC.

Source: Moschytz[2019]Analog circuit theory and filter design

Active RC integrator block

The integrator is a basic building block for MOS VLSI networks and filters.

1. Active RC Integrator



$$\frac{V_2}{V_1} = \frac{1}{1+sT}$$

$$T = RC$$

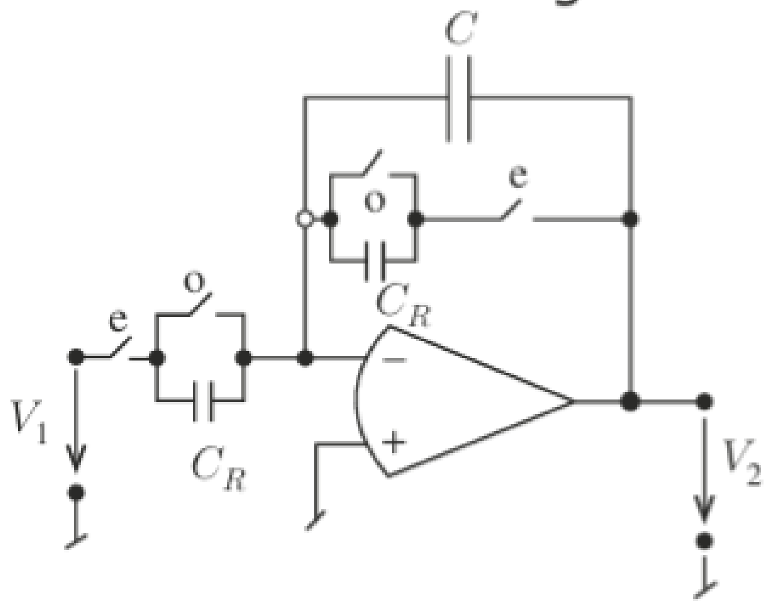
Required Chip area:		
Opamp:		$0.04\ mm^2$
Resistors (100 MΩ)	$2 \times$	$3\ mm^2$
Capacitor (10 pF):		$0.02\ mm^2$
Total Chip Area:		$6.06\ mm^2$

$$T = 1ms,\ C = 10\ pF\ \Rightarrow R = \frac{T}{C} = 100\ M\Omega$$

Source: Moschytz[2019]Analog circuit theory and filter design

Active SC integrator block

2. Active SC Integrator



$$\frac{V_2^e}{V_1^e} = \frac{1}{1 + pC\frac{\tau}{C_R}} = \frac{1}{1 + (1 - z^{-2})\frac{C}{C_R}}$$

$$f_s = \frac{1}{T} = 10kHz$$

$$p = \frac{1 - z^{-2}}{\tau}$$

$$T = 2\tau$$

Required Chip area:	Opamp:	$0.04\ mm^2$
	Resistors (100 MΩ)	$0.02\ mm^2$
	Capacitor (10 pF):	$2 \times 0.002\ mm^2$
	Switch:	$4 \times 0.005\ mm^2$
	Total Chip Area:	$0.084\ mm^2$

Source: Moschytz[2019]Analog circuit theory and filter design

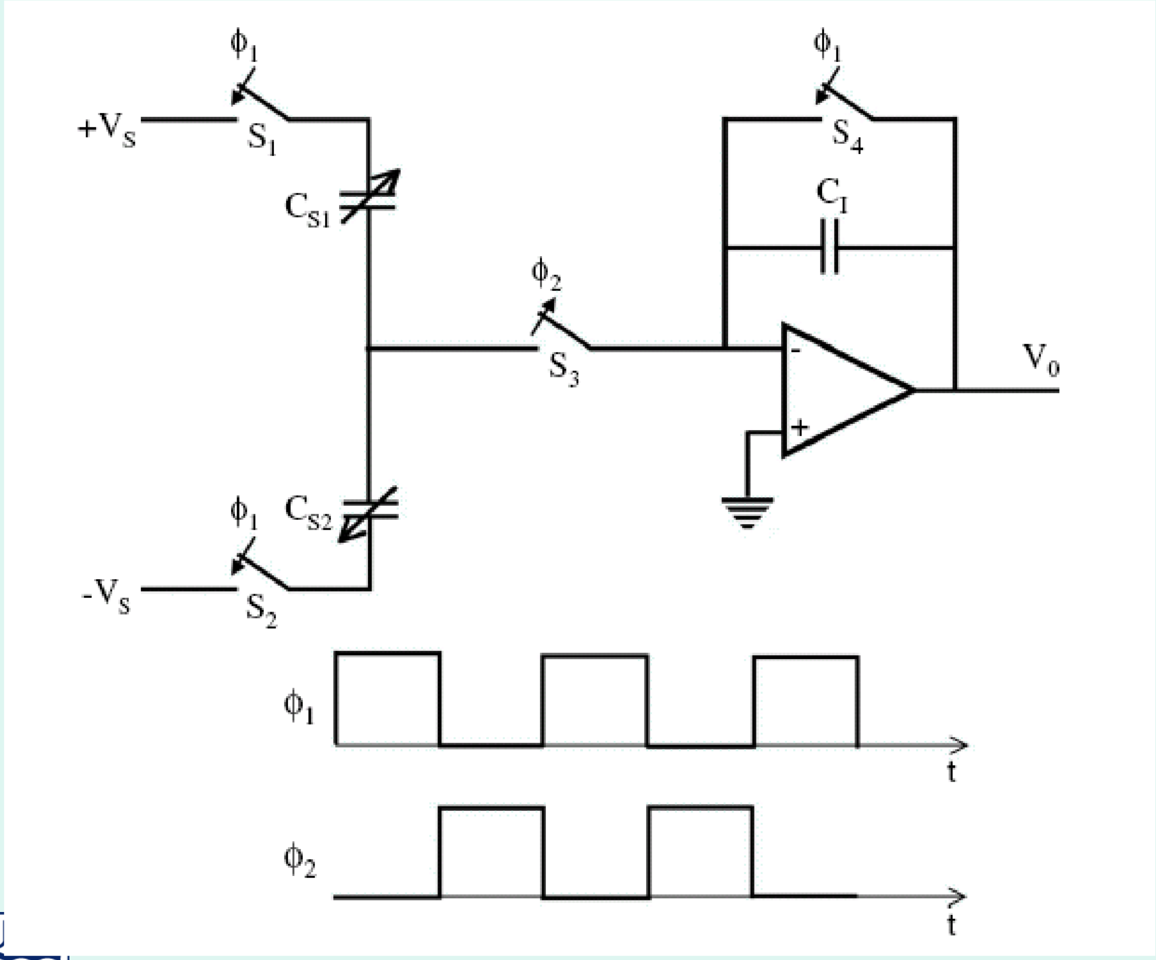
Comparison of active RC and SC integrators

Active RC Integrator	Active SC Integrator
Large Tolerance on R and C:	τ determined by quartz oscillator;
$\frac{\Delta R}{R} \sim 25\%$ $\frac{\Delta C}{C} \sim 10\%$	$\frac{C}{C_R}$ accurate to less than 0.005 %
Large TCR and voltage dependence of R	

Active SC circuit requires much smaller chip area (almost two orders of magnitude) and is far more accurate then Active RC circuits on MOS chip.
Remember: MOS chip circuits cannot be tuned on-chip.

Source: Moschytz[2019]Analog circuit theory and filter design

SC charge transfer - sensor interfaces



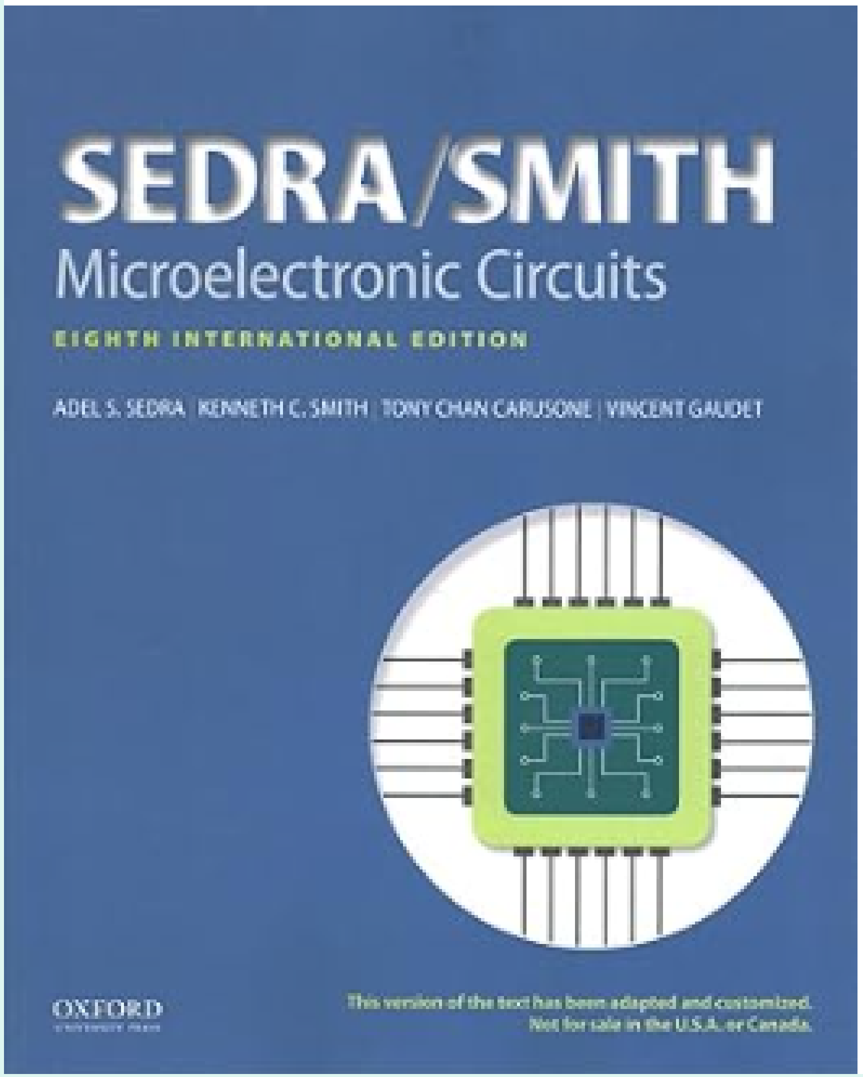
$$V_o = V_s \frac{C_{s1} - C_{s2}}{C_I}$$

$f_{clk} >> f_{analog\ signals}$

Homework: deduce the transfer function

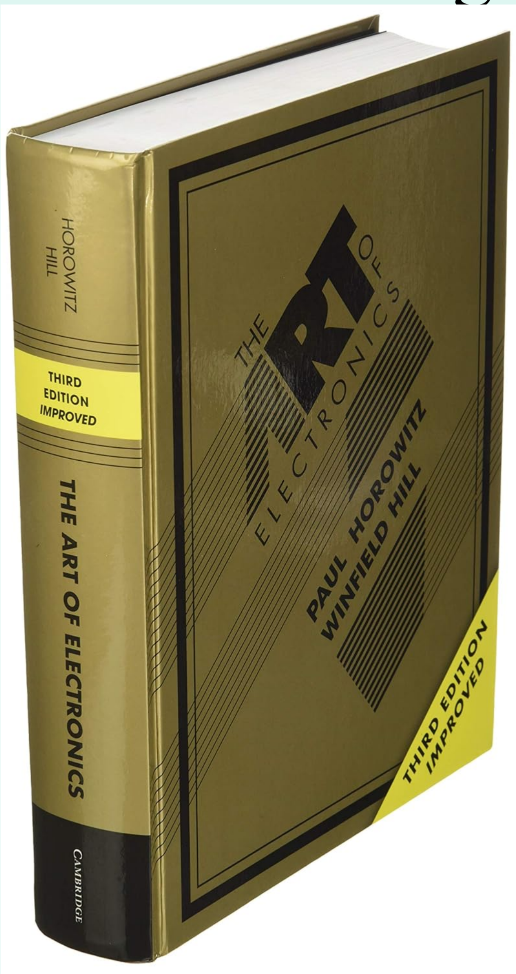
Supplementary resources

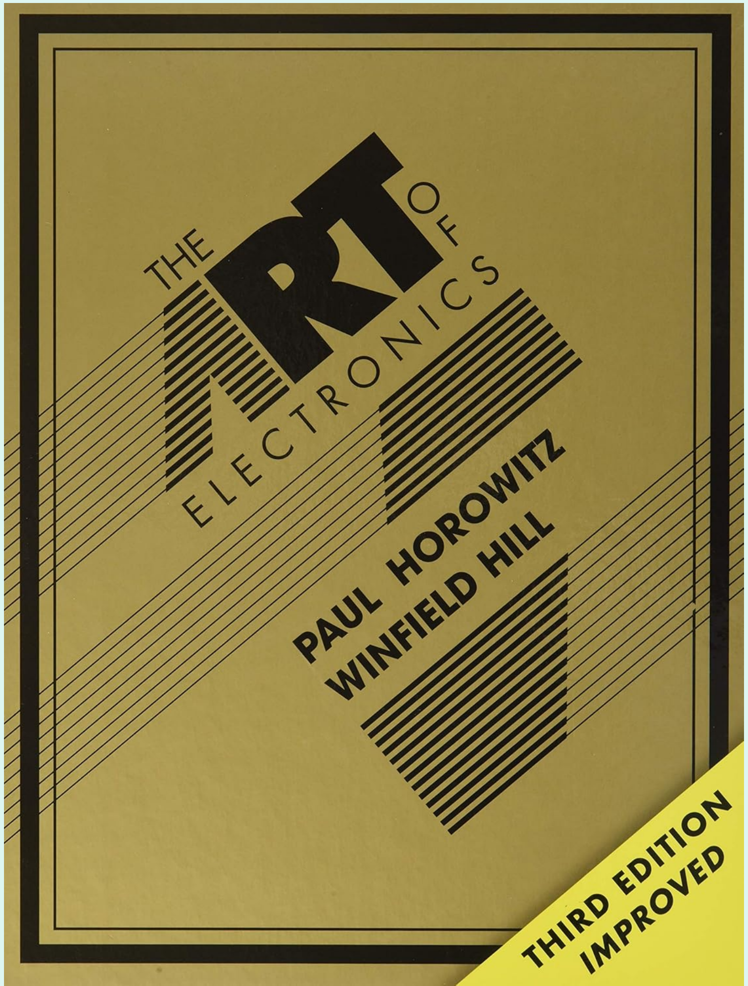
- Sedra&Smith - Microelectronic Circuits, OUP

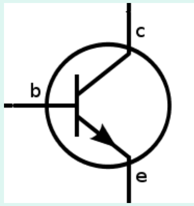


Horrowitz - The Art of Electronics

- Another comprehensive book
- Cambridge University Press

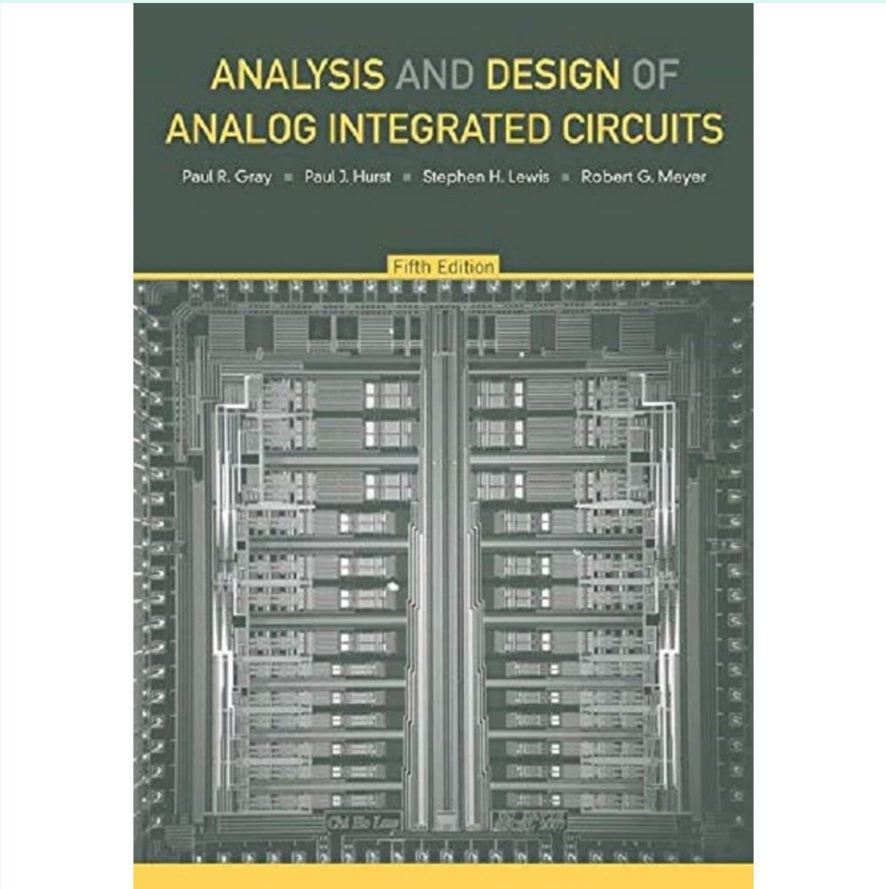


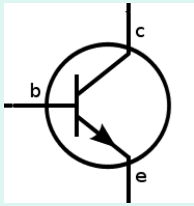




Analog integrated circuits

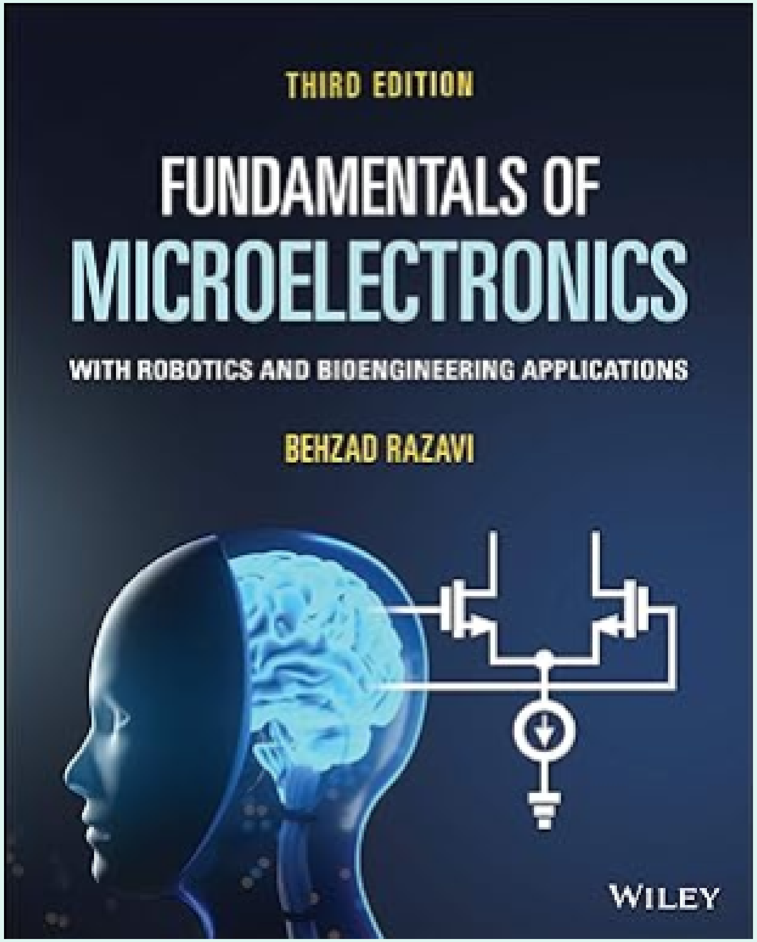
- Gray&Meyer[2000]Analysis and design of analog integrated circuits, 5ed, Wiley
- Covers the design of both bipolar and CMOS analog ICs

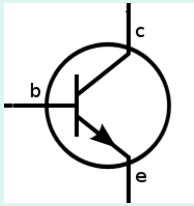




Microelectronics

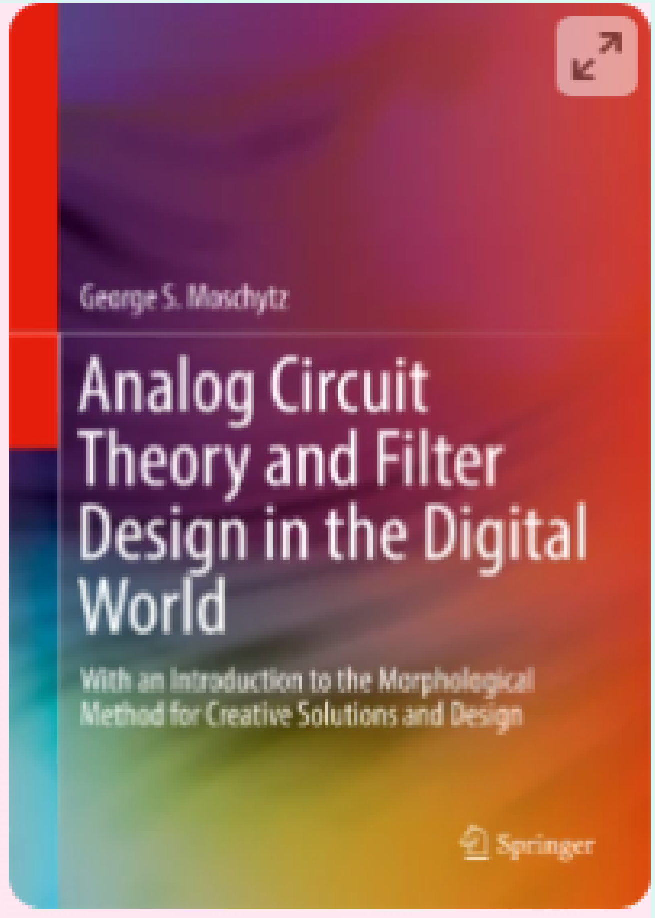
- Razavi[2021] Fundamentals of microelectronics, 3ed, Wiley

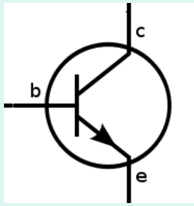




Filter design

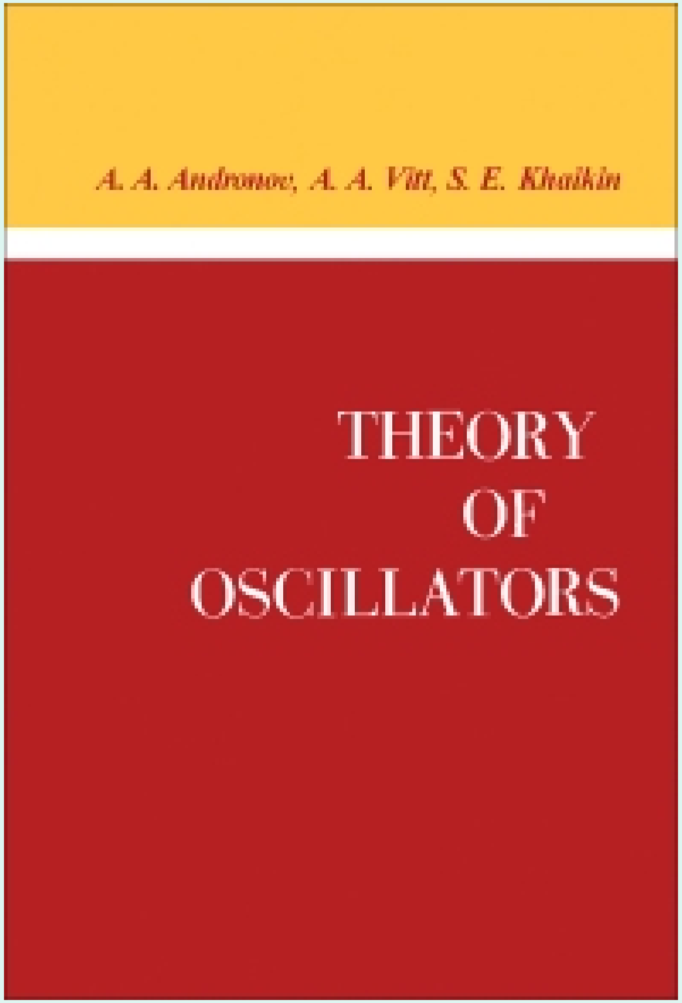
- Moschytz[2019]Analog Circuit Theory and Filter Design in the Digital World, Springer
- https://link.springer.com/book/10.1007/978-3-030-00096-7

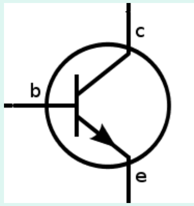




Oscillators

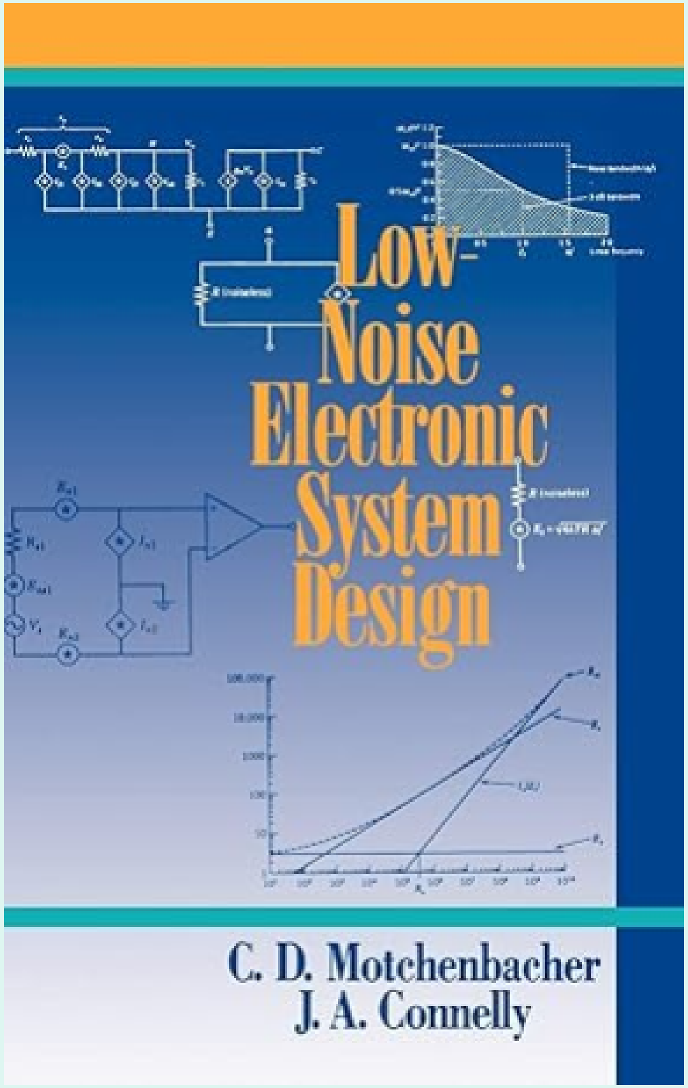
- Andronov[2013]Theory of oscillators, Elsevier
- Analysis from from the physical perspective (conservative systems, dissipative systems, self-oscillating systems), stability, differential equations (time domain)

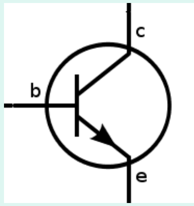




Low-noise design

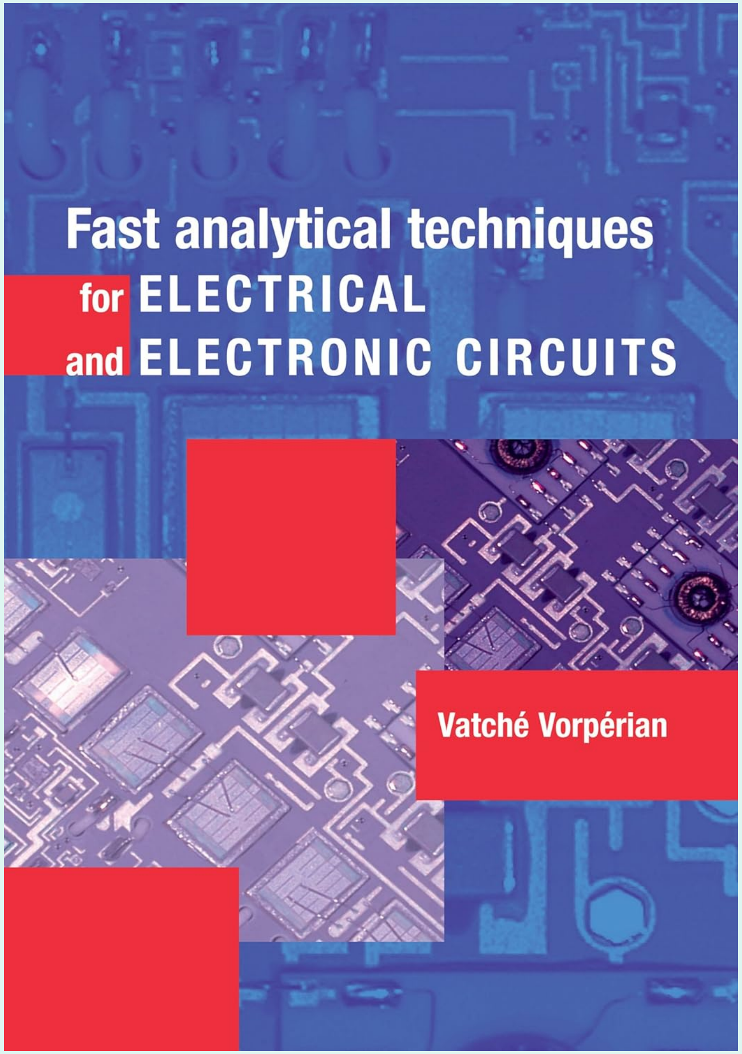
- Motchenbacher[1993] Low-noise electronic system design, Wiley

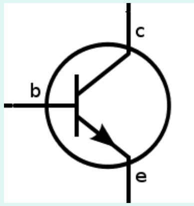




Fast approximations in circuit computation

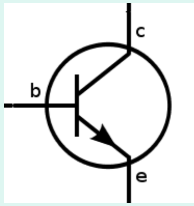
- Vorperian[2002]Fast analytical techniques for electrical and electronic circuits, CUP





Generalized circuits

- Extension to multiphysics systems - design, analysis, synthesis
- Information flow (signals, block diagrams, SFG) vs. energy/power flow (generalized voltage/current ports)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