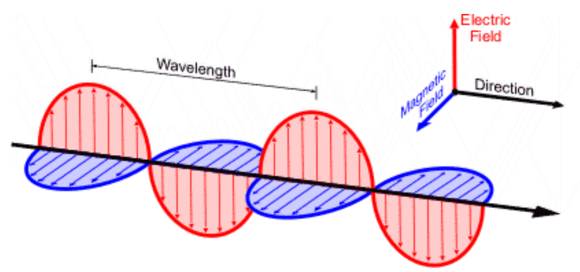
THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

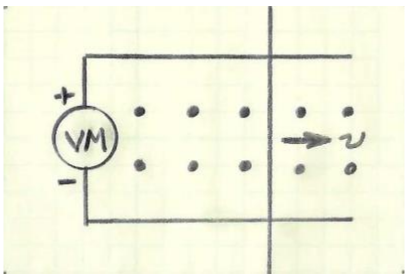
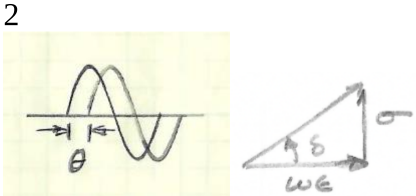
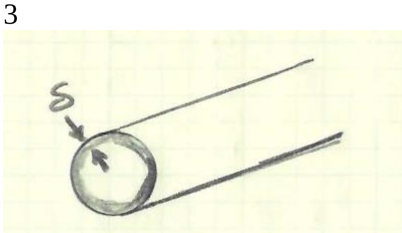
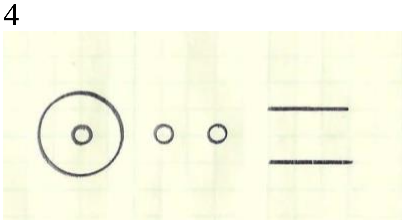
ELEC 311 – Electromagnetic Fields and Waves

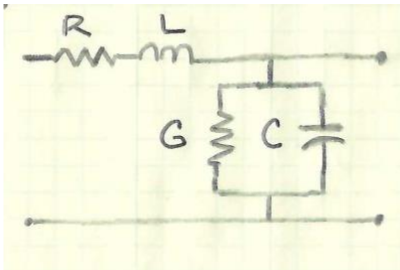
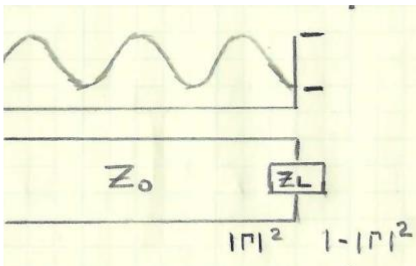
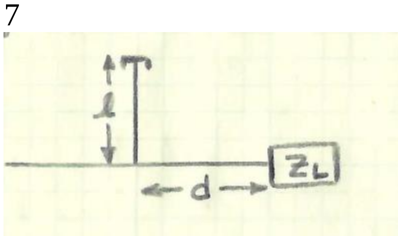
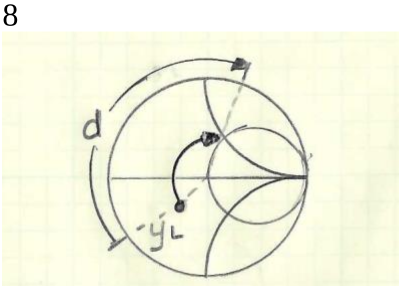
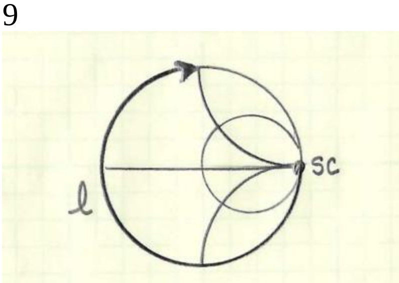
2025 W1

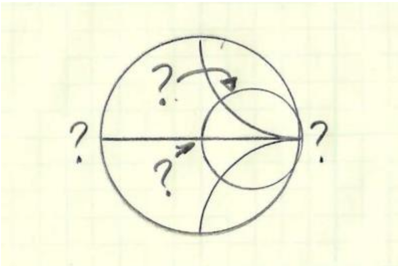
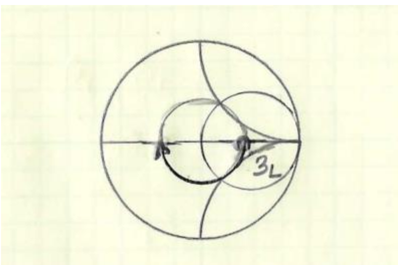
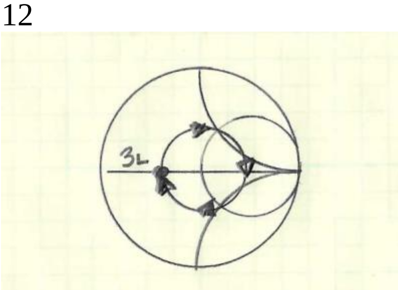
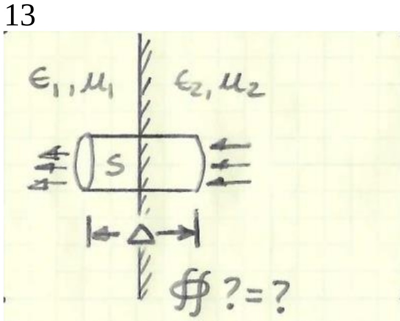
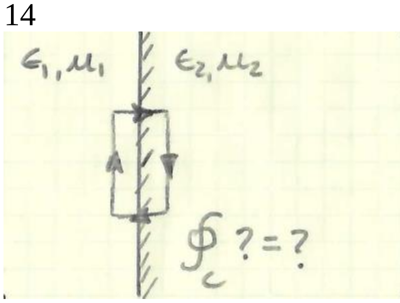
Twenty-Four Problem Scenarios - Solutions

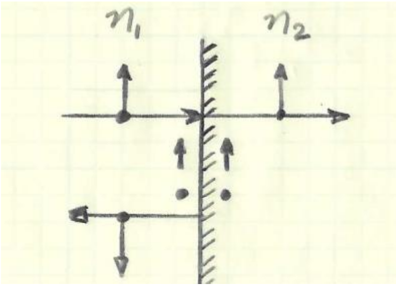
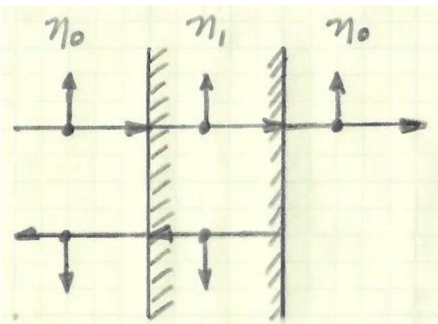
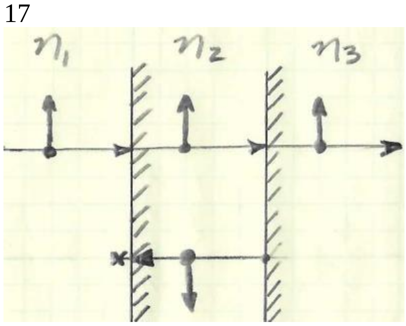
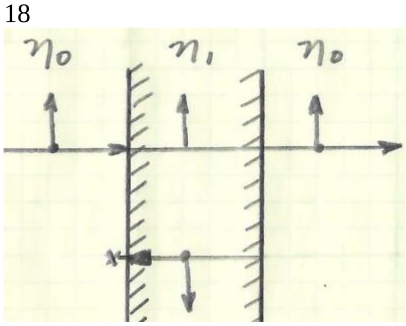
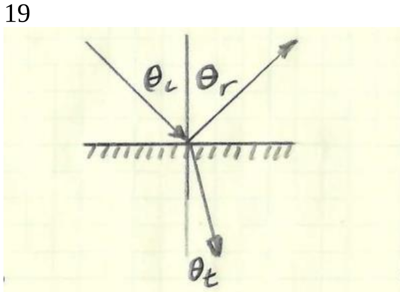


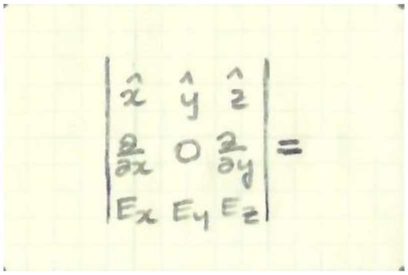
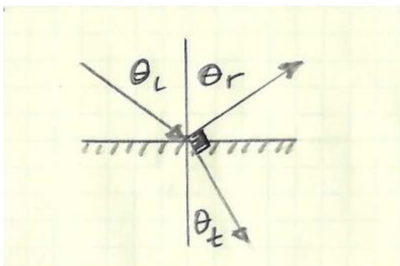
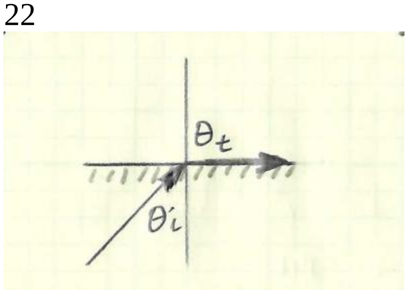
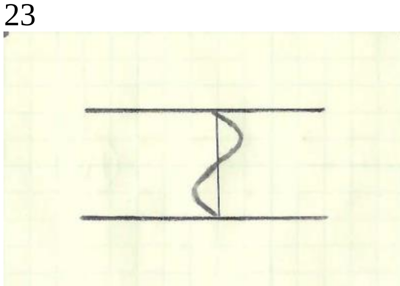
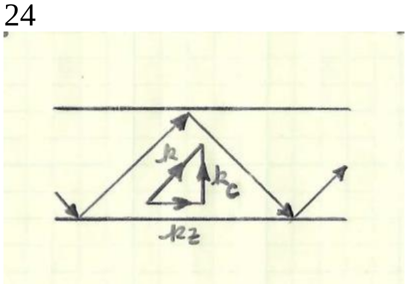
In ELEC 311, it is *not sufficient* to simply recognize and interpret equations. Here is a compilation of 24 *problem scenarios* that you need to be able to recognize, interpret, and solve.

Scenario	Interpretation
1  emf = $-N\, d\Phi/dt$	This problem scenario likely describes… … the emf induced due to motion of the conducting rod as it moves along the conducting rails in a constant magnetic field. The magnetic flux density B may be constant, but the total magnetic flux Φ increases with time as the enclosed area increases. The energy required to induce the emf comes from the force required to move the rod along the rails! If a scheme doesn't require that force be applied to change the flux, *e.g.*, switch-based schemes, then no emf will be generated.
2  $\theta = \delta/2$ where $\tan\delta = \sigma/\omega\epsilon$	This problem scenario likely describes… … the phase shift between the electric and magnetic fields in either a lossy material or good conductor. In a pure dielectric, the electric and magnetic field are in phase. In a lossy material, the magnetic field lags by θ. In a good conductor, the maximum lag of 45 degrees is observed. Note the relationship between the phase shift and the loss tangent.
3  $\delta = \dfrac{1}{\alpha} = \dfrac{1}{\sqrt{\pi f \mu \sigma}}$	This problem scenario likely describes… … the effect of skin depth on the AC (or RF) resistance of a wire. Because the time-varying electric field decays exponentially and rapidly within the conductor, the current is confined to a narrow ring about the outer circumference. The actual current distribution can be replaced by an annulus with constant current density equal to that observed at the surface. The thickness of the annulus or <u>skin depth</u> δ is chosen such that the total current through the annulus is the same as that through the original exponentially decaying ring.
4 	This problem scenario likely describes… … the three basic two-conductor TEM transmission lines. They are, from left to right, the coaxial line, the parallel wire or ladder line, and the parallel plate line. In ELEC 311, we expect you to be able to use supplied tables of formulas to determine the parameters R, L, G and C of the equivalent circuit model in terms of the dimensions and materials of a given transmission line.

5  $Z = R + j\omega L, \quad Y = G + j\omega C$ $\gamma = \sqrt{ZY}, \quad Z_0 = \sqrt{Z/Y}$	This problem scenario likely describes… … the equivalent circuit model of a uniform transmission line. Here, the resistance *R* accounts for ohmic losses and *L* accounts for magnetic energy storage associated with current flowing in the conductor. The conductance *G* and capacitance *C* accounts for conductive losses and electric energy storage associated with voltage across the two conductors. Once these parameters are known, one can calculate $\gamma = \alpha + j\beta$ and Z_0, and, importantly, $\lambda = 2\pi/\beta$ and $v = \omega/\beta$.
6  $V_i(z) = V_{i,0}e^{-j\beta z}; V_r(z) = V_{r,0}e^{+j\beta z}$ $V_{max} = 1 + \|\Gamma\|;\ V_{min} = 1 - \|\Gamma\|$	This problem scenario likely describes… … a voltage standing wave on a mis-matched transmission line. Continuity requires that $\frac{V_{i,0}+V_{r,0}}{I_{i,0}-I_{r,0}} = Z_L$ where $\Gamma_0 = \frac{V_{r,0}}{V_{i,0}} = \frac{Z_L - Z_0}{Z_L + Z_0}$. The phasors for V_i and V_r rotate in opposite directions with distance, so will constructively and destructively interfere with a period of $\lambda/2$. The fraction of the incident power reflected from the load is $\|\Gamma\|^2$ while the fraction delivered to the load is $1 - \|\Gamma\|^2$. Here, because a voltage minimum occurs at the load, we can infer that Z_L is real and less than Z_0.
7 	This problem scenario likely describes… … a single stub matching network. Here, a load Z_L is matched to a transmission line with characteristic impedance Z_0 by adding a shunt stub of length ℓ at a distance *d* from the load. The distance is chosen such that at the attach point is $Y_{in} = 1 + jB$. The length of the stub is chosen such that it presents an admittance of $-jB$ which cancels out the susceptive component of Y_{in}.
8 	This problem scenario likely describes… … finding the distance *d* at which to place the shunt stub when designing a stub matching network. Our goal is to start at the (normalized) load admittance y_L and walk along the constant Γ circle towards the generator for a distance *d* until we intercept the $g = 1$ circle. At that point, $y_{in} = 1 + jb$. The length of the stub is chosen such that it presents an admittance of $-jb$ which cancels out the susceptive component of y_{in}.
9 	This problem scenario likely describes… … the design of a shunt-stub with $y_{in} = +j$. Because we're using the Smith chart to describe admittance, short circuit now lies on the right-hand side of the chart. Moving back towards the generator (clockwise) by a distance ℓ yields $y_{in} = +j$.

10 	This problem scenario likely describes… … the key parts of a Smith chart. When the Smith chart is used to described impedance, the outer circle, smaller circle, and centre point (prime centre) describe *z* = 0 + *jx*, *z* = 1+ *jx*, and *r* = 1, respectively. The upper and lower arcs describe *z* = +*j* and *z* = –*j*, respectively. The horizontal line describes *x* = 0. The left-hand point is short circuit. The right-hand point is open circuit. Here, lower case means normalized with respect to Z_0.
11 	This problem scenario likely describes… … a quarter-wave section or transformer. At the point of attachment, the wave impedance equals the real load impedance which in this case is greater than the characteristic impedance Z_0 of the transmission line. After walking back towards the generator by a quarter-wavelength (or 180 degrees on the Smith chart) along the constant Γ circle, the wave impedance is again real but is now lower than Z_0. Here, $Z_{in} = Z_0^2/Z_L$.
12 	This problem scenario likely describes… … a half-wave section or transformer. At the point of attachment, the wave impedance equals the real load impedance which in this case is less than the characteristic impedance Z_0 of the transmission line. After walking back towards the generator (clockwise) by a half-wavelength (or 360 degrees on the Smith chart), $Z_{in} = Z_L$, *i.e.*, no change!
13 	This problem scenario likely describes… … the proof that the normal component of flux density is continuous across a boundary between two dielectrics. Here, we assume that the dielectrics are lossless so there will be no surface charge or current at the interface. According to the divergence theorem, the closed surface integral of the flux entering and leaving the cylinder must be zero. If the caps are parallel to the surface and the cylinder is normal to the surface, only the caps contribute to the integral. The integral is zero only if the normal components of the flux density in the two regions are equal.
14 	This problem scenario likely describes… … the proof that the tangential component of field strength is continuous across a boundary between two dielectrics. Here, we assume that the dielectrics are lossless so there will be no surface charge or current at the interface. According to Stokes' theorem, the closed line integral of the field strength must be zero as the thickness of the loop goes to zero. The integral is zero only if the tangential components of the field strength in the two regions are equal.

15  $\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	This problem scenario likely describes… … wave propagation at the interface between two regions. This scenario involves three waves: incident, reflected and transmitted. Within each region, the ratio of electric to magnetic field in each wave must equal the intrinsic impedance of the region. At the boundary, the sum of the incident and reflected electric (and magnetic) field strengths in region 1 must equal the electric (and magnetic) field strength in region 2. Applying these conditions yields reflection and transmission coefficients Γ and τ in terms of η_2 and η_1.
16  $\|\Gamma_1\|^2$ $1 - \|\Gamma_1\|^2$	This problem scenario likely describes… ... wave propagation through a dielectric slab. This scenario involves five waves: one incident, two reflected and two transmitted. Start by solving for reflection and transmission at the second boundary. Calculate the wave impedance as a function of distance in the second region and use that as the effective impedance at the first boundary when solving for reflection there. The power reflection and transmission coefficients are $\|\Gamma_1\|^2$ while the fraction delivered to the third region is $1 - \|\Gamma_1\|^2$.
17 	This problem scenario likely describes… … wave propagation through a quarter-wave slab. The figure implies that there is no reflection back into the first region. This can only occur if the wave impedance at the first boundary in the slab = η_1. If $\eta_1 > \eta_2 > \eta_3$ and the materials are all loss less (η is real in all cases), this can only occur if the slab is exactly $\lambda_2/4$ thick and $\eta_1\eta_3 = \eta_2^2$.
18 	This problem scenario likely describes… … wave propagation through a half-wave slab. The figure implies that there is no reflection back into the first region. This can only occur if the wave impedance at the first boundary in the slab = η_0. This corresponds to one full revolution around the Smith chart and can only occur if the slab is exactly $\lambda_1/2$ thick.
19 	This problem scenario likely describes… … a general case of a plane wave obliquely incident on a boundary between two dielectrics. Here, the relationship between the angle of reflection and angle of incidence, and the angle of transmission (or refraction) and the angle of incidence are given by Snell's Laws. Because the angle between the angle of reflection and transmission is not 90 degrees, the angle of incidence is *not* Brewster's angle.

20 	This problem scenario likely describes… … the separation of wave solutions into TE and TM polarized components when the problem geometry is uniform in *y*. In the general case of a plane wave obliquely incident on a boundary between two dielectrics, the problem geometry is uniform in the *y*-direction. Accordingly, this result, which is described in detail in a handout under Chapter 12 on Canvas, guarantees that a TM-polarized wave will not transform into TE-polarized reflected or transmitted components!
21  Upper region = 1; Lower region = 2.	This problem scenario likely describes… … incidence at Brewster's angle, a special case of a plane wave obliquely incident on a boundary between two dielectrics. Because the reflected and transmitted rays are 90 degrees apart, the angle of incidence is Brewster's angle, $\theta_B = \tan^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}}$. The reflection coefficient for TM-polarized waves incident at Brewster's angle is zero. This implies that the reflected wave will be purely TE-polarized.
22 	This problem scenario likely describes… … demonstration of the critical angle, *i.e.*, the incidence angle which results in an angle of refraction of 90 degrees When a wave propagates from a denser medium into a less dense medium, the angle of transmission (refraction) is always greater than the angle of incidence. The critical angle marks the onset of total internal reflection which is the basis of fibreoptics and dielectric waveguides.
23 	This problem scenario likely describes… … the standing wave that forms when a higher order mode in a parallel plate waveguide is cut off. Here, the standing wave consists of two half-cycles (or two half wavelengths) so this corresponds to the m = 2 mode. If the separation between the plates is a, the cut-off wavelength λ_c for the m = 2 mode is a.
24  $k = \frac{2\pi}{\lambda}$; $k_c = \frac{2\pi}{\lambda_c}$; $k_z = \frac{2\pi}{\lambda_g} = \beta$	This problem scenario likely describes… … the zig-zag propagation of a higher order mode in a parallel plate waveguide. Here, the wave vector **k** for a given mode can be decomposed into a transverse component $\mathbf{k}_c$ that depends on the cut-off wavelength for that mode (which in turn depends on a, the separation between the plates, and m, the order of the mode) and a longitudinal component $\mathbf{k}_z$ whose magnitude is the phase constant β. The angle of reflection from the plates is always measured with respect to the normal, not the plate.