THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

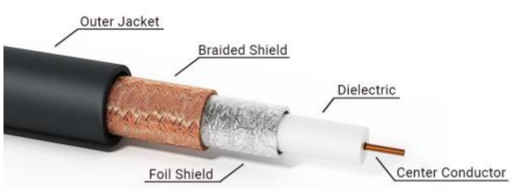
2025 W1*

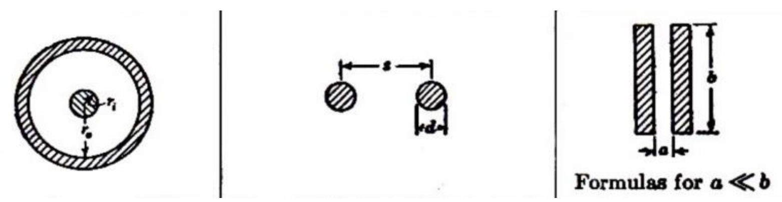
Chapter 10 – Transmission Lines

What you need to know!

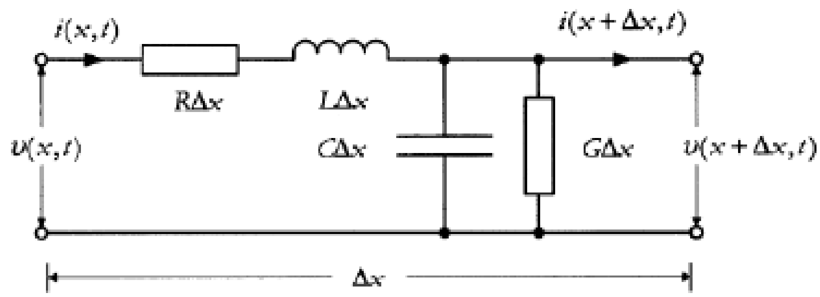
A compilation of course performance objectives with detailed enabling objectives.

Where formulas are cited, be certain that you can identify each quantity and its units and sketch figures that describe the scenario.





1. Given the geometry of a uniform coaxial line, ladder line or parallel-plate waveguide, calculate the parameters *R, L, G* and *C* of the lumped- and distributed-element transmission line models.



- Recognize that:
  - *R* relates the current in the line to the ohmic losses in the conductor
  - *L* relates the current in the line to the energy stored by the surrounding magnetic field
  - *G* relates the voltage across the line to the conductive losses in the dielectric
  - *C* relates the voltage across the line to the energy stored by the surrounding electric field
- Be familiar with the table from Ramo *et al.* that gives the cross-sectional geometry of common uniform transmission lines and gives formulas for the parameters *R, L, G* and *C* of the incremental transmission line model in terms of the dimensions of, and separation between, the conductors.

- To check your answers, consider using the online calculator at

  https://learnemc.com/EXT/calculators/TL_Calculator/index.html .

- Recognize that for the case of a uniform transmission line, these parameters depend only on the cross-sectional geometry of the line and are independent of the length (as suggested by the units being Ω/m, H/m, S/m, and F/m).
- Recognize that the formulas for *R, L, G* and *C* for the case of a parallel-plate waveguide can be derived from first principles in a straightforward manner.
- Recognize that the formulas for *R, L, G* and *C* for the coaxial line or ladder line cases can be obtained by applying a conformal mapping to the geometry and formulas that apply to parallel-plate waveguide.
- Recognize that $R + j\omega L$ is *Z*, the *series impedance,* and $G + j\omega C$ is *Y*, the *shunt admittance* of the line.

2. Given the parameters *R, L, G* and *C* of the incremental transmission line model, calculate the propagation constant $\gamma = \alpha + j\beta$, characteristic impedance Z_0, velocity of propagation *v,* and the rates of voltage and power decay on the line.

- Recognize that the propagation constant $\gamma = \alpha + j\beta$ is a parameter of the Helmholtz equation and α and β are expressed in units of Np/m and rad/m, respectively.
- Recognize that the Helmholtz equation is a consequence of the *telegraphist's equations* relate the change in voltage with distance to the change in current with time, and vice versa, *i.e.,*

$$\frac{\partial V}{\partial z} = -(RI + L\frac{\partial I}{\partial t}). \qquad \frac{\partial I}{\partial z} = -(GV + C\frac{\partial V}{\partial t})$$

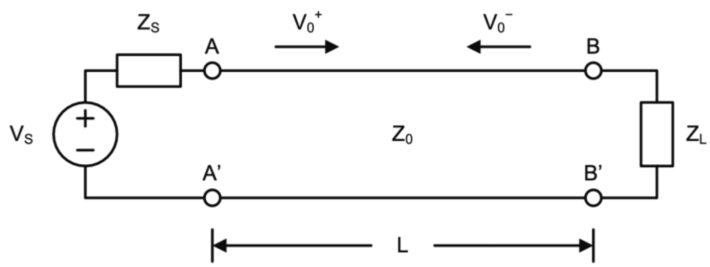
  that are derived by applying Kirchoff's Voltage and Current Laws to the equivalent circuit for the uniform transmission line.

- Recognize that $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$ and takes on a particularly simple form if the line is lossless, *i.e.,* $\gamma = j\beta = j\omega\sqrt{LC}$ or $\beta = \omega\sqrt{LC}$
- Recognize that $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}}$ and takes on a particularly simple form if the line is lossless, *i.e.,* $Z_0 = \sqrt{\frac{L}{C}}$
- Recognize that $v = \frac{\omega}{\beta}$ and that phase and group velocities are identical for TEM modes.
- Recognize that if the line is lossless, $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$
- Recognize that the velocity factor (VF) is the ratio of the actual propagation velocity *v* to the speed of light *c*. It depends on the material properties of the dielectric that lies between the inner and outer conductor. For a solid dielectric with relative permittivity ϵ_r, it is generally given by $1/\sqrt{\epsilon_r}$.
- Recognize that $\lambda = \frac{2\pi}{\beta}$ which is equal to $\frac{v}{f}$ only for the lossless case

- Recognize that 1 Np = 8.686 dB or 1 dB = 0.1151 Np
- Recognize that power decays twice as fast as voltage or current, *i.e.,*

$$V(z) = V_0 e^{-\alpha z} \text{ and } I(z) = I_0 e^{-\alpha z} \text{ but } P(z) = P_0 e^{-2\alpha z}$$

3. Given a uniform transmission line terminated by a specified load impedance, determine the voltage reflection coefficient and the voltage standing wave ratio that describes the standing wave that appears on the line, and the fraction of power returned from, and delivered to, the load.



This figure shows the source voltage V_s and source impedance Z_s, a transmission line of length L with characteristic impedance Z_0, and both forward and backward travelling waves of amplitude V_0^+ and V_0^-, respectively, and the load impedance, Z_L.

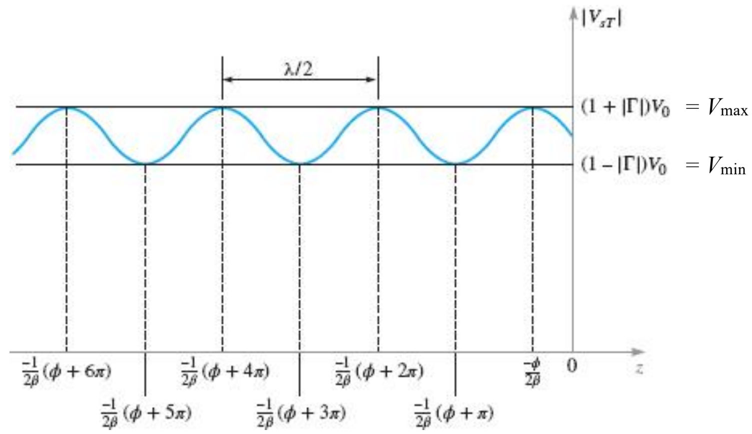
- Recognize that the voltage reflection coefficient Γ_0, *i.e.,* Γ at $z = 0$, is obtained by *simultaneously* matching the sum of voltage of the incident wave and the voltage of the reflected wave to the voltage across the load *and* the sum of current in the incident wave and the current in the reflected wave to the current through the load, yielding

$$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Recognize that a standing wave forms because the phasors that describe the incident and reflected travelling waves rotate in *opposite directions*; the *maximum* voltage occurs at points on the line where the phasors are in phase while the *minimum* voltage occurs where the phasors are out of phase.
- Recognize that V_{max} occurs when V_i and V_r add in phase ($V_{max} = |V_i| + |V_r|$) and that V_{min} occurs when V_i and V_r add out of phase ($V_{min} = |V_i| - |V_r|$).
- Recognize that the ratio of the maximum to minimum voltage is given by

$$s = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- Recognise that $|\Gamma|^2$ gives the fraction of power returned from, and $1 - |\Gamma|^2$ gives the fraction of power delivered to, the load.



Plot of a voltage standing wave, *i.e.,* the superposition of forward and backward traveling waves with difference amplitudes and phases, on a transmission line.

4. Given a uniform transmission line terminated by a specified load impedance, determine the wave impedance at any arbitrary position along the line.
   - Recognize that wave impedance at a given point is the vector sum of the voltages in the incident and reflected travelling wave divided by the vector sum of the corresponding currents.
   - Recognize that the voltage at a given point on the line for the incident and reflected waves are given by

$$V_i(z) = V_{i0}e^{-j\beta z} \text{ and } V_r(z) = V_{r0}e^{j\beta z}$$

   - Recognize that the voltage reflection coefficient at a given point on the line is given by

$$\Gamma(z) = \frac{V_r(z)}{V_i(z)} = \Gamma_0 e^{j2\beta z} \quad or$$

$$\Gamma(\ell) = \Gamma_0 e^{-j2\beta\ell}$$

depending on whether the location is identified by the value of *z* at that point or the distance ℓ from the point to *z =0*.

   - Recognize that the wave impedance is given by

$$Z_{in}(z) = \frac{V_i(z) + V_r(z)}{I_i(z) + I_r(z)}$$

with care given to ensure that the correct sign is used for current, which becomes

$$Z_{in}(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)} \quad or$$

$$Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}$$

which are different but equivalent to the expressions given in the textbook and the chapter supplement

   - Recognize that the Smith Chart allows us to perform these calculations graphically!

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Chapter 10 – Smith Charts

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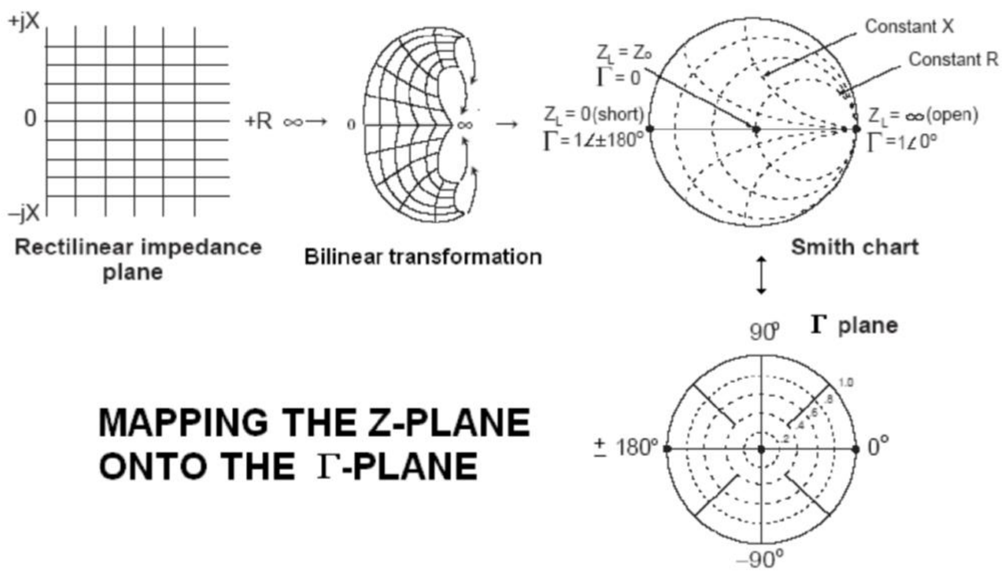
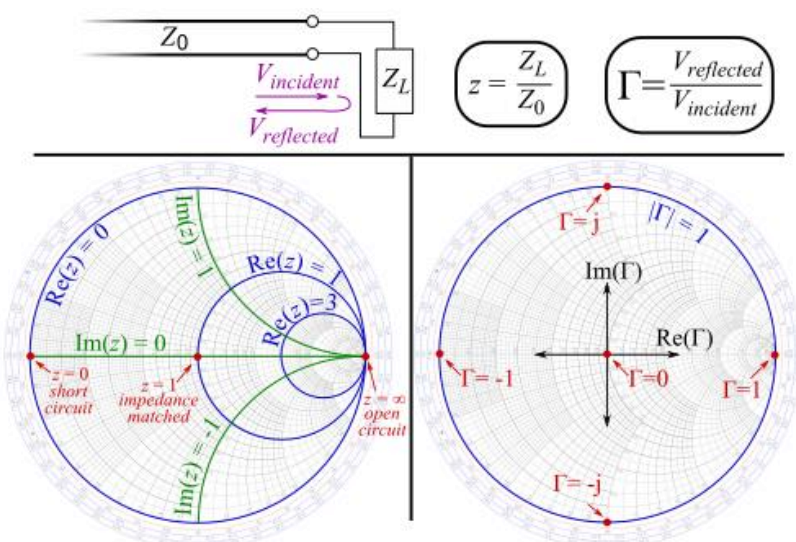
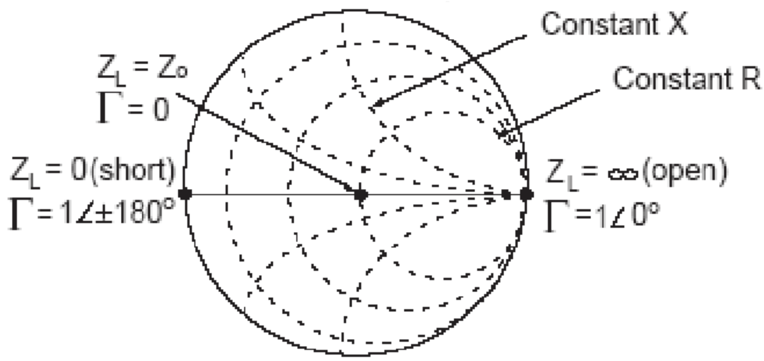
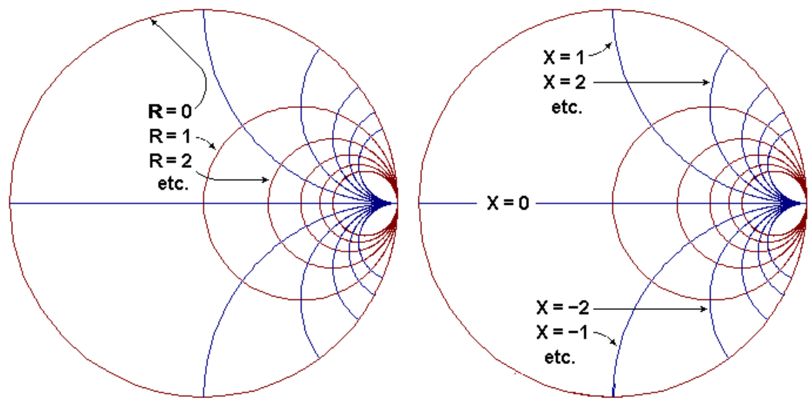




1. Sketch a Smith chart, indicating open circuit, short circuit, matched load, normalized resistances of 0, 1, and 2 and normalized reactances of 0, ± 1 and ± 2.





2. Analyze λ/2 and λ/4 transformers using a Smith chart.
   - Be able to plot a given (normalized) impedance or admittance on a Smith Chart.
   - Recognize that a circle with its centre at prime corresponds to a plot of $|\Gamma(\ell)|$
   - Recognize that one-half of a revolution corresponds to λ/4 of travel.
   - Recognize that one revolution corresponds to λ/2 of travel
   - Recall that $Z_{in}\,Z_L = Z_0^2$ or, equivalently, $Z_{in}/Z_0 = Z_0/\,Z_L$ for a quarter-wave transformer.
   - Recall that $Z_{in} = Z_L$ for a half-wave transformer.

3. Given a line of length ℓ and characteristic impedance Z_0, and terminated with Z_L, find $Y(\ell)$, $Z(\ell)$, $\Gamma(\ell)$, and s.
   - Begin by normalizing the load impedance so that $z_L = Z_L/Z_0$
   - Plot z_L on the Smith chart
   - Draw a circle centred on $z = 1$ through z_L; the radius of the circle = $|\Gamma|$
   - Use the scales at the bottom of the Smith Chart to convert the radius of the circle to $|\Gamma|$ and s

- Moving around the circle in the clockwise and counter-clockwise directions, is equivalent to moving toward the generator and load, respectively
- The distance in wavelengths can be determined from a pair of scales on the outer circle.
- To find $Y(\ell)$, $Z(\ell)$, $\Gamma(\ell)$, move along the $|\Gamma|$ circle in the appropriate direction for the appropriate distance.
  - Read $z(\ell)$ directly off the Smith Chart grid and then multiply the result by Z_0 to obtain $Z(\ell)$
  - $y(\ell)$ is the point on the $|\Gamma|$ circle that is exactly 180 degrees away from $z(\ell)$; multiply the result by Y_0 to obtain $Y(\ell)$
  - Read $\Gamma(\ell)$ by using: 1) the scale at the bottom of the Smith Chart to convert the radius of the circle to $|\Gamma|$ and 2) the appropriate scale on the outer ring to obtain the phase angle

4. Given Z_0 and Z_L, find d and ℓ required to achieve a single-stub match using a shunt stub.
   - Begin by normalizing the load impedance so that $z_L = Z_L/Z_0$
   - Plot z_L on the Smith chart by interpreting the Smith chart grid as impedance coordinates
   - Draw a circle centred on $z = 1$ through z_L; the radius of the circle = $|\Gamma|$
   - Identify y_L by finding the point on the $|\Gamma|$ circle that is exactly 180 degrees away from z_L; from now on, interpret the Smith chart grid as admittance coordinates
   - Solve for the distance d
     - Move from y_L towards the generator along the $|\Gamma|$ circle
     - Where the $|\Gamma|$ circle intercepts the $g = 1$ circle indicates the distance d at which the stub must be attached
     - Measure this distance in wavelengths by consulting the scales on the outer circle of the Smith Chart
     - To obtain d in standard units, multiply the result by λ
     - The value of b at that point must be cancelled out by the short-circuit stub of length ℓ
   - Solve for the length ℓ
     - Start at the short circuit point on the admittance version of the Smith Chart
     - Move towards the generator (clockwise) along the outer ($G = 0$) circle until one reaches the point at which $y = 0 - jb$
     - Measure the length ℓ in wavelengths from the short circuit point to this point by consulting the scales on the outer circle of the Smith Chart
     - To obtain ℓ in standard units, multiply the result by λ