THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

Example Problems for
Chapter 10 – Transmission Lines

These hints are meant to be read in conjunction with both the problems (if one gets stuck) and the solutions (in order to better appreciate the underlying concepts).

I. Transmission Lines

1. The parameters of a certain transmission line operating at $\omega = 6 \times 10^8$ rad/s are $L = 0.35$ uH/m, $C = 40$ pf/m, G = 75 µS/m, and R = 17 Ω/m. Find γ, α, β, λ, and Z_0.

   Hints

   1. The expressions for $\gamma = \alpha + j\beta$ and Z_0 in terms of $R + j\omega L$ and $G + j\omega C$ are given in equations (41) and (47) of Chapter 10. See also the chapter brief.

   2. Once β is known, it is a relatively simple matter to calculate $\lambda = 2\pi/\beta$ and $u = \omega/\beta = f\,\lambda$.

2. A 50-Ω load is attached to a 50-m section of the transmission line of problem 1 and a 100-W signal is applied to the input end of the line. Evaluate:

   i. the distributed line loss in dB/m.

   ii. the reflection coefficient at the load.

   iii. the power that is dissipated by the load resistor.

   iv. the power that is returned to the input.

   Hints

   1. Line loss in Np/m is given by α. Convert this to dB/m by multiplying by 8.686 dB/Np.

   2. The expressions for $\gamma = \alpha + j\beta$ and Z_0 in terms of $R + j\omega L$ and $G + j\omega C$ are given in equations (41) and (47) of Chapter 10. See also the chapter brief.

   3. The expression for the *voltage reflection coefficient* in terms of Z_L and Z_0 is given in equation (73) of Chapter 10.

   4. The *power reflection coefficient* is $P_r/P_i = |\Gamma|^2$.

   5. Conservation of power requires that the *power transmission coefficient* be $P_t/P_i = 1 - |\Gamma|^2$.

3. A parallel-wire transmission line is constructed of #0 AWG copper wire (diameter = 0.3249 inches, conductivity = 58 MS/m), with a 12-inch separation in air.

    a. Neglecting internal inductance, find the per-meter values of *L, C, G, R* at DC and *R* at 2 MHz.

Hint – Correctly interpret the transmission line formula sheet.

    b. Find the characteristic impedance, propagation constant (attenuation and phase), velocity of propagation, and wavelength for operation at 1 kHz.

Hints

    1. The expressions for $\gamma = \alpha + j\beta$ and Z_0 in terms of $R + j\omega L$ and $G + j\omega C$ are given in equations (41) and (47) of Chapter 10. See also the chapter brief.

    2. Once β is known, it is a relatively simple matter to calculate $\lambda = 2\pi/\beta$ and $u = \omega/\beta = f\lambda$.

4. Find the outer diameter of an air-filled 50-Ω coaxial line with inner diameter 0.3249 inches.

    Hint - Correctly interpret the transmission line formula sheet.

II. Smith Charts

1. A 50-Ω high-frequency air-filled lossless line is used at a frequency where $\lambda = 15$ cm with a load at $z = 0$ of $Z_L = (70 + j45)$ Ω. Use the Smith Chart to find:

    a. Γ_R,

    b. VSWR,

    c. Distance to the first voltage maximum from the load,

    d. Distance to the first voltage minimum from the load,

    e. The wave impedance at V_{max},

    f. The wave impedance at V_{min},

    g. The input impedance (and admittance) for a section of line that is 7.5 cm long.

Hints

    1. Because the line is filled with air, the wavelength on the line is the same as in free space.

    2. For parts a and b, measure the radius of the constant-Γ circle using the scales at the bottom of the Smith chart.

3. The voltage maximum and minimum occur when the impedance is real, i.e., where the constant-Γ circle crosses the horizontal or $x = 0$ axis. Maximum occurs for the higher resistance while minimum occurs for the lower resistance. These correspond to the wave impedance that was asked for.

4. To find the input impedance, plot the normalized impedance of the load on the Smith Chart and then trace the constant-Γ circle in the clockwise direction, *i.e.,* back toward the generator for the appropriate number of wavelengths.

5. Multiply the value of the normalized impedance on the Smith Chart by Z_0 (here, $Z_0 = 50\ \Omega$) in order to obtain the actual impedance.

6. To find the normalized admittance that corresponds to the normalized impedance, find the point on the constant-Γ circle that is on the opposite side, *i.e.,* same radius but 180 degrees around.

2. Use a single short-circuit shunt stub to match the configuration described in Problem 1.

   Hints

   1. Find the normalized value of load impedance and plot it on the Smith Chart. Draw the constant-Γ circle.

   3. Find the normalized load admittance by finding the point on the opposite size of the constant-Γ circle.

   4. Move back towards the generator until one reaches the $g = 1$ circle. This is where the stub must be placed. Determine the distance in wavelengths from the load to this point by reading the scales on the outside of the Smith Chart. Convert to distance in cm by multiplying by the wavelength on the line (here, equal to the free space wavelength because the line is air-filled.)

   5. Find the value of B at this point. Determine the length of short-circuited stub required to realize a susceptance of $-B$ by moving towards the generator from the short-circuit point on the Smith chart.

   6. Recall that the short-circuit point on the admittance chart is located at the right side of the Smith chart, *not* the left side.

   7. When the stub is placed in parallel with the transmission line where normalized $Y_{in} = 1 + jB$, the susceptance of the stub and the susceptance of the transmission line at that point will cancel out and normalized input admittance will become $Y_{in} = 1$. Multiply Y_{in} by $Y_0 = 1/Z_0$ to obtain the actual value.

8. Use a Smith Chart to demonstrate the operation of half-wave and quarter-wave transformers.

   Hint – See Section 10.12 of the textbook.

9. A high-frequency 50-Ω lossless line is 100.0 cm long with a relative dielectric constant, $\varepsilon_R = 2.49$. At 800 MHz, the input impedance of the terminated line is measured as $Z_{in} = (10 + j25)\Omega$. Use a Smith Chart to find the value of the terminating load.

Hints

1. Because the line is filled with dielectric, the wavelength is $1/\sqrt{\varepsilon_r}$ of its value in air.

2. Plot the normalized impedance on the Smith Chart and then trace the constant-Γ circle in the counter-clockwise direction, *i.e.,* towards the load, by an appropriate number of wavelengths.

3. One full circle around the Smith Chart = one-half wavelength.