THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

Example Problems for
Chapter 10 – Transmission Lines

Observations

These observations provide additional insights concerning the problems and their solutions.

I. Transmission Lines

1. The parameters of a certain transmission line operating at $\omega = 6\times10^8$ rad/s are L = 0.35 uH/m, C = 40 pf/m, G = 75 μS/m, and R = 17 Ω/m. Find γ, α, β, λ, and Z_0.

   Answers: $\gamma = \alpha + j\beta$, α = 0.0943 Np/m, β = 2.247 rad/m, λ = 2.8 m, and $Z_0 = 93.6 - j3.64\Omega$

   Observations:

   1. This problem focuses on recognition of the relationship between the parameters of the circuit model of the transmission line (given) and the parameters of the transmission line equations (sought). In particular, the series impedance $Z = R + j\omega L$ and the shunt admittance $Y = G + j\omega C$ are the basis for the expressions $\gamma = \alpha + j\beta = \sqrt{ZY}$ and $Z_0 = = \sqrt{Z/Y}$.
   2. Ideally, the learner can also sketch the equivalent circuit model without reference and explain the physical significance of each of the four elements, *e.g.*, the inductance per unit length L in H/m accounts for the magnetic energy associated with the flow of current along the line.
   3. Here, the operating frequency corresponds to 95.5 MHz. The free space wavelength would be 3.14 m which is a bit longer than the line wavelength of 2.8 m. This implies that there is some light dielectric structure surrounding the transmission line and that the velocity of propagation is a bit slower than it would be in free space.
   4. The small imaginary component of the characteristic impedance Z_0 indicates that the line is slightly lossy and that α is non-zero. While Im(Z_0) doesn't contain enough information to estimate α, multiplying Z_0 by Y will yield $\gamma = \alpha + j\beta$.

2. A 50-Ω load is attached to a 50-m section of the transmission line of problem 1 and a 100-W signal is applied to the input end of the line. Evaluate:

   i. the distributed line loss in dB/m.

   ii. the reflection coefficient at the load.

   iii. the power that is dissipated by the load resistor.

   iv. the power that is returned to the input.

Answers: A = 0.819 dB/m, Γ_0 = 0.302 ∠ 3.08 rad = 176.6 deg, P_L = 0.0073 W,
$P_{refl} = 5.89 \times 10^{-8}$ W

Observations:

1. This problem focuses on the manner in which power can be lost as a signal propagates along a transmission line and how to perform the associated calculations in either dB or linear units. Key points include:

   a. Power can be lost through ohmic or conductive losses in the line, *i.e.*, *line loss*. Line loss α is given in Np/m. Convert this to line loss A in dB/m by multiplying by 8.686 dB/Np. Total loss in dB is given by multiplying A by the line length.

   b. In linear units, loss and gain are reciprocals. In dB, loss and gain are the negative of each other. *E.g.*, a loss of 2 is a gain of 1/2. A loss of 3 dB is a gain of –3dB.

   c. If power gain is less than 1 in linear units or less than 0 dB in logarithmic units, the gain is usually interpreted as a loss.

   d. Power can be lost through reflection at an impedance mismatch, *i.e.*, *return loss*. The reflected power is given by $P_r = |\Gamma|^2 P_i$. The process does not dissipate power, so conservation of power requires that the transmitted power is given by $P_t = 1 - |\Gamma|^2 P_i$.

   e. $\Gamma_0 = (Z_L - Z_0)/(Z_L + Z_0)$ and $|\Gamma|^2 = |\Gamma_0|^2$.

2. In this case, the power dissipated by the load resistor is the product of:

   a. the power at the input,

   b. the power gain associated with forward transmission along the line, and,

   c. the power gain associated with the impedance mismatch.

   We can either multiply these quantities in linear units or add them in dB or logarithmic units.

3. In this case, the power returned to the input is the product of:

   a. the power at the input,

   b. the power gain associated with forward transmission along the line,

   c. the reflection power gain associated with the impedance mismatch (often referred to as the *return loss*), and,

   d. the power gain associated with backward transmission along the line.

   Once again, we can either multiply these quantities in linear units or add them in dB or logarithmic units.

4. Reminder: dB is an expression of power gain in logarithmic units. dBm and dBW are expressions of absolute power in logarithmic units. dBm compares a given power level to 1 mW. dBW compares a given power level to 1 W.

5. In this case, the reduction in signal strength or *attenuation* associated with *line loss* is modest when expressed in dB/m but rather large across the entire line length of 50 m: 41 dB in loss or -41 dB in gain. That means that only 0.008% of the power that

enters the transmission line appears at the end. 99.991% is dissipated by ohmic or conductive losses.

6. By comparison, the reduction in signal strength or *attenuation* associated with *return loss* is modest in absolute terms. 91% of the power that is incident upon the load is delivered to the load. Only 9% is reflected.

3. A parallel-wire transmission line is constructed of #0 AWG copper wire (diameter = 0.3249 inches, conductivity = 58 MS/m), with a 12-inch separation in air.

   a. Neglecting internal inductance, find the per-meter values of L, C, G, R at DC and R at 2 MHz.

   b. Find the characteristic impedance, propagation constant (attenuation and phase), velocity of propagation, and wavelength for operation at 1 kHz.

Answers:

a. L = 1.72 μH/m, C = 6.47 pF/m, G = 0 S/m, $R_{DC} = 6.44\times10^{-4}$ Ω/m, $R_{AC} = 2.846\times10^{-2}$ Ω/m

b. $Z_0 = 515.7 - j15.37$ Ω

c. $\gamma = \alpha + j\beta = (6.25\times10^{-7} + j2.10\times10^{-5})$ /m, $\lambda = 2.997\times10^{5}$ m

Observations:

1. This problem focuses on recognition of the relationship between the parameters of the circuit model of the of transmission line (given) and the parameters of the transmission line equations (sought). In particular, the series impedance $Z = R + j\omega L$ and the shunt admittance $Y = G + j\omega C$ are the basis for the expressions $\gamma = \alpha + j\beta = \sqrt{ZY}$ and $Z_0 = = \sqrt{Z/Y}$. Once β is known, it is a relatively simple matter to calculate $\lambda = 2\pi/\beta$ and $u = v_p = \omega/\beta = f\,\lambda$.

2. As always, it is expected that the learner will be completely familiar with the names, conventional symbols, and customary units used to describe each parameter, and the physical significance of each parameter.

3. The only new piece here is the AC or RF resistance, which is a preview of material to be covered in Chapter 13. At AC or RF frequencies, the electric field attenuates significantly as it penetrates into the conductor. As a result, the current is effectively confined to a narrow annulus just within the circumference of the wire. This reduction in the effective cross-sectional area of the wire significantly raises the resistance of the wire. More details may be found in §13.1 of the textbook.

4. Find the outer diameter of an air-filled 50-Ω coaxial line with inner diameter 0.3249”.

Answer: $d_0 = 0.7476”$

Observation:

1. This problem focuses on recognition of the relationship between the cross-sectional geometry of a coaxial line and the characteristic impedance of the line.
2. It is expected that the learner will be familiar with the effects of changing the design parameters of the line, *e.g.*, increasing the relative permittivity of the dielectric and/or increasing the diameter of the inner conductor will reduce Z_0.

II. Smith Charts

1. A 50-Ω high-frequency air-filled lossless line is used at a frequency where $\lambda = 15$ cm with a load at $z = 0$ of $Z_L = (70 + j45)\ \Omega$. Use the Smith Chart to find:

   a. Γ_R,

   b. VSWR,

   c. Distance to the first voltage maximum from the load,

   d. Distance to the first voltage minimum from the load,

   e. The wave impedance at V_{max},

   f. The wave impedance at V_{min},

   g. The input impedance (and admittance) for a section of line that is 7.5 cm long.

Answers: (*a*) $\Gamma_R = 0.38 \angle 45.8$ deg; (*b*) $s = 2.22$; (*c*) $d_{max} = 0.063\lambda = 0.945$ cm; (*d*) $d_{min} = 0.313\lambda = 4.695$ cm; (*e*) $R_{max} = 110\ \Omega$; (*f*) $R_{min} = 22.7\ \Omega$; (*g*) $Y_{in} = 0.010 + j\,0.0065$ S and $Z_{in} = 70 + j\,45\ \Omega$

Observations:

1. This problem focuses on recognition of the properties of the Smith Chart and the constant-Γ circle.
2. Because the line is filled with air, the wavelength on the line is the same as in free space.
3. For parts a and b, measure the radius of the constant-Γ circle using the scales at the bottom of the Smith chart.
4. The voltage maximum and minimum occur when the impedance is real, *i.e.*, where the constant-Γ circle crosses the horizontal or $x = 0$ axis. Voltage maximum occurs for the higher resistance while voltage minimum occurs for the lower resistance. These points correspond to the normalized value of the wave impedance that was asked for.

5. To find the input impedance, plot the normalized impedance of the load on the Smith Chart and then trace the constant-Γ circle in the clockwise direction, *i.e.*, back toward the generator for the appropriate number of wavelengths.
6. Multiply the value of the normalized impedance on the Smith Chart by Z_0 (here, $Z_0 = 50\ \Omega$) in order to obtain the actual impedance.
7. To find the normalized admittance that corresponds to the normalized impedance, find the point on the constant-Γ circle that is on the opposite side, *i.e.*, the same radius but 180 degrees away.

2. Use a single short-circuit shunt stub to match the configuration described in Problem 1.

Answers: $d_1 = 0.219\lambda = 3.29$ cm, $l_1 = 0.142\lambda = 2.13$ cm;
$d_2 = 0.397\lambda = 5.96$ cm, $l_2 = 0.358\lambda = 5.37$ cm

Observations:

1. This problem focuses on use of the Smith Chart to design a single-stub matching network.
2. Learners are expected to be familiar with the goal of single-stub matching – move along the line until the normalized wave admittance is $y = 1 + jB$ and then place a stub with input admittance equal to $-jB$ in parallel to yield a net $y_{in} = 1$. The result is a matched line.
3. There are two solutions because the constant-Γ circle crosses the $g = 1$ circle in two places. Often problems will ask learners to determine the shortest one.
4. One can extend either d or l by $\lambda/2$ and still obtain a correct result. (Extending the length of either is not desirable and should be avoided.)
5. Learners are expected to be familiar with the following design steps:
   a. Find the normalized value of load impedance and plot it on the Smith Chart. Draw the constant-Γ circle.
   b. Find the normalized load admittance by finding the point on the opposite size of the constant-Γ circle.
   c. Move back towards the generator until one reaches the $g = 1$ circle. This is where the stub must be placed. Determine the distance in wavelengths from the load to this point by reading the scales on the outside of the Smith Chart. Convert to distance in cm by multiplying by the wavelength on the line (here, equal to the free space wavelength because the line is air-filled.)
   d. Find the value of B at this point. Determine the length of short-circuited stub required to realize a susceptance of $-B$ by moving towards the generator from the short-circuit point on the Smith chart.
   e. Recall that the short-circuit point on the admittance chart is located at the right side of the Smith chart, *not* the left side.

f. When the stub is placed in parallel with the transmission line where normalized $Y_{in} = 1 + jB$, the susceptance of the stub and the susceptance of the transmission line at that point will cancel out and normalized input admittance will become $Y_{in} = 1$. Multiply Y_{in} by $Y_0 = 1/Z_0$ to obtain the actual value.

3. Use a Smith Chart to demonstrate the operation of half-wave and quarter-wave transformers.

Observations:

1. This problem focuses on recognition of the properties of the Smith Chart and the goals of impedance matching.
2. Learners are expected to recognize that:

   a. The Smith Chart is a plot of $\Gamma = V_{ref}/V_{inc} = \Gamma_0 e^{j2\beta z}$. The factor of 2 in the exponent is the result of dividing $V_{ref,0}\, e^{j\beta z}$ by $V_{inc,0}\, e^{-j\beta z}$. This means that the phase of Γ rotates twice as fast as the phases of either the incident or reflected wave and a complete rotation of the Smith Chart occurs in just λ/2.

   b. The goal of impedance matching is to achieve an input impedance (wave impedance at the input) equal to the characteristic impedance of the feedline.

   c. There are two points on any constant-Γ circle where the wave impedance is real. Both occur on the horizontal axis that separates the upper and lower halves of the chart. The one on the left side corresponds to the lower value and a voltage minimum. The one on the right side corresponds to the higher value and a voltage maximum.

   d. For a half-wave section, the input and load impedances are identical. The characteristic impedance of the section is immaterial.

   e. For a quarter-wave section, the input and load impedances are related to the characteristic impedance by $Z_0^2 = Z_{in}^2 Z_L^2$. Given any two of these parameters, one can determine the value of the third.

   f. One can use the properties described above to design simple impedance matching networks.

4. A high-frequency 50-Ω lossless line is 100.0 cm long with a relative dielectric constant, $\varepsilon_R = 2.49$. At 800 MHz, the input impedance of the terminated line is measured as $Z_{in} = (10 + j25)\Omega$. Use a Smith Chart to find the value of the terminating load.

Answers: $Z_L = 93.6 - j3.64\Omega$

Observations:

1. Because the line is filled with dielectric, the wavelength is $1/\sqrt{\varepsilon_r}$ of its value in air.
2. Learners are expected to recognize that this is the reverse of what we normally calculate, *i.e.*, the input impedance given the load impedance.
3. Learners are expected to be familiar with the following design steps:

a. Here, we plot the normalized impedance on the Smith Chart and then trace the constant-Γ circle in the counter-clockwise direction, *i.e.*, towards the load, by an appropriate number of wavelengths.

b. As noted above, one full circle around the Smith Chart = one-half wavelength. If the distance is longer than $\lambda/2$, then we need to rotate around the Smith Chart more than once.