THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W2

Strategies for Solving the Drill Problems from Chapter 10 – Transmission Lines

The purpose of the eight drill problems from Chapter 10 is to help you master fundamental techniques used to analyze transmission lines.

Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

The numerical answers below are from the text. Do you agree with them?

D10.1 At an operating radian frequency of 500 Mrad/s, typical circuit values for a certain transmission line are: R = 0.2 Ω/m, L = 0.25 μH/m, G = 10 μS/m, and C = 100 pf/m. Find (a) α, (b) β, (c) λ, (d) v_p, (e) Z_0.

Strategy:

Given: The operating frequency and circuit parameters *R, L, G* and *C* for a transmission line.

Sought: (a) α, (b) β, (c) λ, (d) v_p, (e) Z_0.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find α and β, we need to find γ which is as function of *R, L, G* and *C*.
3. To find λ and v_p, we need to evaluate 2π/β and ω/β, respectively.
4. To find Z_0, we need to find the square root of the series impedance *Z* over the shunt admittance *Y*.
5. In all cases, indicate the name and units of each parameter.

   Consilium est demonstratum.

Answers: (a) 2.25 mNp/m; (b) 2.50 rad/m; (c) 2.51 m; (d) 2×10^8 m/s; (e) 50.0 - j0.0350 Ω

Observations

1. This problem focuses on recognition of the relationship between the parameters of the circuit model of the transmission line (given) and the parameters of the transmission line equations (sought). In particular, the series impedance $Z = R + j\omega L$ and the shunt admittance $Y = G + j\omega C$ are the basis for the expressions $\gamma = \alpha + j\beta = \sqrt{ZY}$ and $Z_0 = = \sqrt{Z/Y}$.
2. Ideally, the learner can also sketch the equivalent circuit model without reference and explain the physical significance of each of the four elements, *e.g.,* the inductance per unit length *L* in H/m accounts for the magnetic energy associated with the flow of current along the line.

3. Here, the operating frequency corresponds to 95.5 MHz. The free space wavelength would be 3.14 m which is a bit longer than the line wavelength of 2.8 m. This implies that there is some light dielectric structure surrounding the transmission line and that the velocity of propagation is a bit slower than it would be in free space.
4. The small imaginary component of the characteristic impedance Z_0 indicates that the line is slightly lossy and that α is non-zero. While Im(Z_0) doesn't contain enough information to estimate α, multiplying Z_0 by Y will yield $\gamma = \alpha + j\beta$.

D10.2 Two transmission lines are to be joined end to end. Line 1 is 30 m long and is rated at 0.1 dB/m. Line 2 is 45 m long and is rated at 0.15 dB/m. The joint is not well done and imparts a 3-dB loss. What percentage of the input power that reaches the output of the combination?

Strategy:

Given: The length and attenuation constants of two transmission lines that are to be joined end to end and the loss associated with joint.

Sought: The percentage of the input power that reaches the output of the combination.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find the loss of the transmission line assembly, find the sum of the attenuation of each transmission line in dB. This is equivalent to multiplying the power gain of each element in linear units.
3. To find the percentage of the input power that reaches the output of the combination, convert α, the attenuation in dB, back into a power ratio using the expression $10^{-\alpha/10}$ and multiply by 100 to obtain a percentage.

   Consilium est demonstratum.

Answer: 5.3%

Observations:

1. This problem focuses on the manner in which power can be lost as a signal propagates along a transmission line and how to perform the associated calculations in either dB or linear units. Key points include:
   a. Power can be lost through ohmic or conductive losses in the line, *i.e.*, *line loss*. Line loss α is given in Np/m. Convert this to line loss A in dB/m by multiplying by 8.686 dB/Np. Total loss in dB is given by multiplying A by the line length.
   b. In linear units, loss and gain are reciprocals. In dB, loss and gain are the negative of each other. *E.g.,* a loss of 2 is a gain of 1/2. A loss of 3 dB is a gain of –3dB.
   c. If power gain is less than 1 in linear units or less than 0 dB in logarithmic units, the gain is usually interpreted as a loss.
   d. Power can be lost through reflection at an impedance mismatch, *i.e., return loss*. The reflected power is given by $P_r = |\Gamma|^2 P_i$. The process does not dissipate power, so conservation of power requires that the transmitted power is given by $P_t = 1 - |\Gamma|^2 P_i$.
   e. $\Gamma_0 = (Z_L - Z_0)/(Z_L + Z_0)$ and $|\Gamma|^2 = |\Gamma_0|^2$.

2. In this case, the power dissipated by the load resistor is the product of:
    a. the power at the input,
    b. the power gain associated with forward transmission along the line, and,
    c. the power gain associated with the impedance mismatch.

   We can either multiply these quantities in linear units or add them in dB or logarithmic units.

3. In this case, the power returned to the input is the product of:
    a. the power at the input,
    b. the power gain associated with forward transmission along the line,
    c. the reflection power gain associated with the impedance mismatch (often referred to as the *return loss*), and,
    d. the power gain associated with backward transmission along the line.

   Once again, we can either multiply these quantities in linear units or add them in dB or logarithmic units.

4. Reminder: dB is an expression of power gain in logarithmic units. dBm and dBW are expressions of absolute power in logarithmic units. dBm compares a given power level to 1 mW. dBW compares a given power level to 1 W.
5. In this case, the reduction in signal strength or *attenuation* associated with *line loss* is modest when expressed in dB/m but rather large across the entire line length of 50 m: 41 dB in loss or -41 dB in gain. That means that only 0.008% of the power that enters the transmission line appears at the end. 99.991% is dissipated by ohmic or conductive losses.
6. By comparison, the reduction in signal strength or *attenuation* associated with *return loss* is modest in absolute terms. 91% of the power that is incident upon the load is delivered to the load. Only 9% is reflected.

D10.3 What voltage standing wave ratio results when $\Gamma = \pm\frac{1}{2}$?

Strategy:

Given: Γ

Sought: s

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find s, evaluate $(1+|\Gamma|)/(1-|\Gamma|)$

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Answer: 3

D10.4 A 50-ohm lossless line has a length of 0.4λ. The operating frequency is 300 MHz. A load $Z_L = 40 + j30\ \Omega$ is connected at $z = 0$, and the Thevenin equivalent source at $z = -l$ is $12\angle 0°$ V in series with $Z_{Th} = 50 + j0\ \Omega$. Find: (*a*) Γ; (*b*) s; (*c*) Z_{in}.

Strategy:

Given: A transmission line configuration comprising a Thevenin equivalent source, a load, and a transmission line.

Sought: (*a*) Γ; (*b*) *s*; (*c*) Z_{in}.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. Because this problem occurs before the sections dealing with the Smith chart, we should use analytical rather than graphical solutions.
3. To find $\Gamma = \frac{V_r}{V_i}$, evaluate $\frac{Z_L - Z_0}{Z_L + Z_0}$.
4. To find $s = \frac{V_{max}}{V_{min}}$, evaluate $\frac{1+|\Gamma|}{1-|\Gamma|}$.
5. To find $Z_{in}(\ell = 0.4\lambda)$, evaluate $Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}$.

   Consilium est demonstratum.

Answers: (*a*) 0.333∠90°; (*b*) 2.00; (*c*) 25.5 + *j* 5.90 Ω

D10.5 For the transmission line of Problem D10.4, also find: (*a*) the phasor voltage at $z = -l$; (*b*) the phasor voltage at $z = 0$; (*c*) the average power delivered to Z_L.

Strategy:

Given: The transmission line configuration of Problem D10.4.

Sought: (*a*) the phasor voltage at $z = -l$; (*b*) the phasor voltage at $z = 0$; (*c*) the average power delivered to Z_L.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find $V_{s,\text{in}}$, the phasor voltage at $z = -l$, aka the input voltage, find the source current at the input and multiply by Z_{in}.
3. To find $V_{s,L}$ the phasor voltage at $z = 0$, aka the load voltage, find V_0^+ from equation (105) and then apply (104).
4. To find the average power delivered to Z_L, find the source current at the input. Square it, multiply by the real part of the input resistance, and multiply by one-half.

   Consilium est demonstratum.

Answers: (*a*) 4.14∠8.58° V; (*b*) 6.32∠−125.6° V; (*c*) 0.320 W

D10.6 A load $Z_L = 80 - j100\ \Omega$ is located at $z = 0$ on a lossless 50-Ω line. The operating frequency is 200 MHz, and the wavelength on the line is 2 m. (*a*) If the line is 0.8 m in length, use the Smith chart to find the input impedance. (*b*) What is *s*? (*c*) What is the distance from the load to the nearest voltage maximum? (*d*) What is the distance from the input to the nearest point at which the remainder of the line could be replaced by a pure resistance?

Strategy:

Given: A transmission line configuration comprising a load and transmission line.

Sought: (a) the input impedance, (b) s, (c) the distance from the load back to the first voltage maximum, (d) the distance from the source forward to the first location at which the wave impedance is real.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. Normalize the wave impedance and plot it on the Smith Chart. Draw the Γ circle that passes through z_L.
3. To find Z_{in}, walk back towards the generator by a distance corresponding to the length of the line in wavelengths, find z_{in} and multiply the result by Z_0.
4. To find *s*, find Γ and evaluate $(1+|\Gamma|)/(1-|\Gamma|)$ or use the scales at the bottom.
5. To find (c), walk back along the Γ circle (clockwise) until one reaches the first point where $x = 0$ and r is maximum.
6. To find (d), walk forward along the Γ circle (counter clockwise) until one reaches the first point where $x = 0$.

   Consilium est demonstratum.

Answers: (*a*) 79 + *j*99 Ω; (*b*) 4.50; (*c*) 0.0397 m; (*d*) 0.760 m

D10.7 Standing wave measurements on an air-filled lossless 75-Ω line show maxima of 18 V and minima of 5 V. The first voltage minimum is located at a scale reading of 17 cm; the second minimum occurs at 37 cm. Find: (*a*) *s*; (*b*) *λ*; (*c*) *f*; (*d*) Γ_L; (*e*) Z_L.

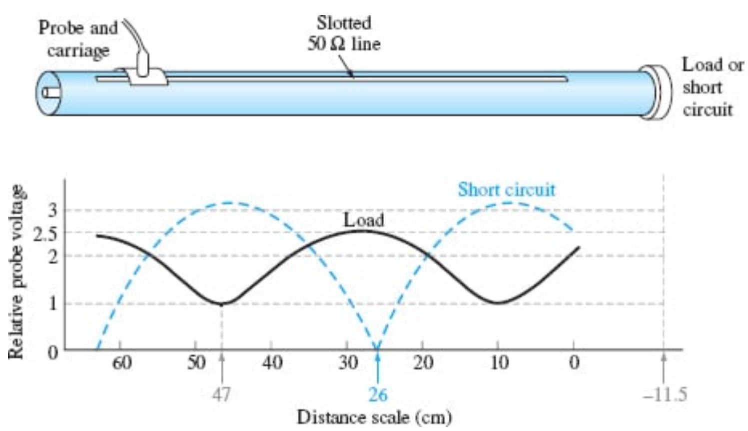
Strategy:

Given: Standing wave measurements on an air-filled lossless 75-Ω line.

Sought: (*a*) *s*; (*b*) *λ*; (*c*) *f*; (*d*) Γ_L; (*e*) Z_L

Steps:

1. Sketch and label the problem geometry as an aid to understanding. The sketch will be similar to Figure 10.15 but with the load at 0 cm.



2. To find *s*, find V_{max}/V_{min}
3. To find *λ*, find the separation between the voltage minima; this corresponds to *λ*/2 so multiply by 2.

4. To find f, find v/λ. Because the line is air-filled, $v = c$.
5. Assume the load is located at a scale reading of 0 cm, as suggested by Figure 10.15.
6. Plot Γ at the first voltage minimum where $|\Gamma| = (s-1)/(s+1)$ and Z_w is both real and reaches its minimum value.
7. To find Γ_L and Z_L, find the point that corresponds to moving along the Γ circle from the first voltage minimum towards the load (counterclockwise) by a distance in wavelengths corresponding to 17 cm/λ(cm).

   Consilium est demonstratum.

Answers: (*a*) 3.60; (*b*) 0.400 m; (*c*) 750 MHz; (*d*) $0.57\angle 130$; (*e*) $24.2 + j32.6\ \Omega$

Observations:

1. This problem focuses on use of the Smith Chart to reveal the minima and maxima of the voltage standing wave, which is an interference pattern between the incident and reflected waves.
2. The phase of Γ rotates twice as fast as the phase of either the incident or reflected voltage because it is the result of dividing $V_{ref,0}e^{j\beta z}$ by $V_{inc,0}e^{-j\beta z}$ yielding $V_{ref,0}/V_{inc,0}\ e^{j2\beta z}$.

D10.8 A normalized load, $z_L = 2 - j1$, is located at $z = 0$ on a lossless 50-Ω line. Let the wavelength be 100 cm. (*a*) A short-circuited stub is to be located at $z = -d$. What is the shortest suitable value for d? (*b*) What is the shortest possible length of the stub? Find s: (*c*) on the main line for $z < -d$; (*d*) on the main line for $-d < z < 0$; (*e*) on the stub.

Strategy:

Given: A transmission line configuration comprising a load, transmission line and short-circuited stub.

Sought: The (a) location and (b) length of the stub that will result in a matched line and the VSWR on the main line (c) before and (d) after the stub, and (e) on the stub.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. Plot z_L on a Smith Chart and draw the Γ circle that passes through that point.
3. Transform z_L to y_L by finding the point on the opposite side of the circle.
4. Walk back towards the generator (clockwise) until the Γ circle intersects the $g = 1$ circle; the distance traversed is the distance d.
5. Find the value of b at that point.
6. Starting at the short-circuit point, walk back along the $|\Gamma| = 1$ circle until the susceptance is $-b$. That is the length of the stub that will cancel out the imaginary component of the wave admittance at $z = -d$.
7. Find $|\Gamma|$ on the main line before and after the stub, and on the stub. Use the scales at the bottom of the Smith Chart to convert $|\Gamma|$ to s.

   Consilium est demonstratum.

Answers: (*a*) 12.5 cm; (*b*) 12.5 cm; (*c*) 1.00; (*d*) 2.62; (*e*) ∞

Observations:

1. This problem focuses on use of the Smith Chart to design a single-stub matching network.
2. Learners are expected to be familiar with the goal of single-stub matching – move along the line until the normalized wave admittance is $y = 1 + jb$ and then place a stub with input admittance equal to $-jb$ in parallel to yield a net $y_{in} = 1$. The result is a matched line.
3. There are two solutions because the constant-Γ circle crosses the $g = 1$ circle in two places. Often problems will ask learners to determine the shortest one, as here.
4. One can extend either d or l by $\lambda/2$ and still obtain a correct result. (Extending the length of either is not desirable and should be avoided.)
5. The VSWR on the section
6. Learners are expected to be familiar with the following design steps:
   a. Find the normalized value of load impedance and plot it on the Smith Chart. Draw the constant-Γ circle.
   b. Find the normalized load admittance by finding the point on the opposite size of the constant-Γ circle.
   c. Move back towards the generator until one reaches the $g = 1$ circle. This is where the stub must be placed. Determine the distance in wavelengths from the load to this point by reading the scales on the outside of the Smith Chart. Convert to distance in cm by multiplying by the wavelength on the line (here, equal to the free space wavelength because the line is air-filled.)
   d. Find the value of B at this point. Determine the length of short-circuited stub required to realize a susceptance of $-B$ by moving towards the generator from the short-circuit point on the Smith chart.
   e. Recall that the short-circuit point on the admittance chart is located at the right side of the Smith chart, *not* the left side.
   f. When the stub is placed in parallel with the transmission line where normalized $Y_{in} = 1 + jB$, the susceptance of the stub and the susceptance of the transmission line at that point will cancel out and normalized input admittance will become $Y_{in} = 1$. Multiply Y_{in} by $Y_0 = 1/Z_0$ to obtain the actual value.