THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

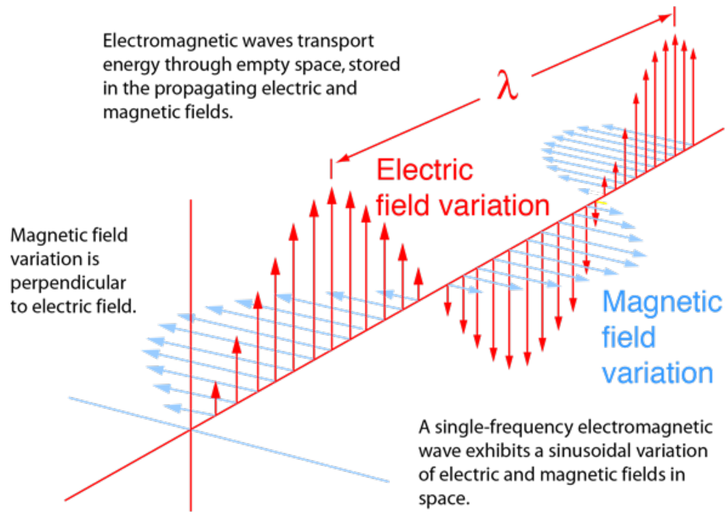
2025 W1

Chapter 11 – Uniform Plane Waves

What you need to know!

A compilation of course performance objectives with detailed enabling objectives.

Where formulas are cited, be certain that you can identify each quantity and its units, and sketch figures that describe the scenario.



1. Given the field strength or power density of a uniform plane wave travelling in a given direction through a medium with given permittivity, permeability and conductivity, give the corresponding Helmholtz equations and find expressions for all of the field components of the wave.
   - Recognize that we are assuming time-harmonic fields with time dependence $e^{j\omega t}$ so we can replace $\partial/\partial t$ with $j\omega$ in Maxwell's equations
   - Recognize that the vector Helmholtz equation is $\nabla^2\mathbf{E} = \gamma^2\mathbf{E} = -k^2\mathbf{E}$ where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ and $\gamma = jk = \alpha + j\beta$ is the complex propagation constant
   - Recognize that, in the general case of a uniform plane wave travelling in the *z* direction, $\mathbf{E} = (E_x\mathbf{a_x} + E_y\mathbf{a_y})\, e^{j(\omega t \pm \beta z)}$ (this is the *complex instantaneous form*) and we can compute **H** from $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$, *i.e.,* $\mathbf{H} = \frac{j}{\omega\mu}\nabla \times \mathbf{E}$
   - Recognize that from this, we can conclude that:
     1. the electric and magnetic field vectors and the direction of propagation are mutually orthogonal and
     2. the ratio of the electric to magnetic field strength is given by

$$\eta = \sqrt{j\omega\mu/(\sigma + j\omega\epsilon)}$$

- Recognize that the *phasor form* results from dropping the $e^{j\omega t}$ factor from the *complex instantaneous form*
- Recognize that the *complex instantaneous form* can be recovered by re-inserting the $e^{j\omega t}$ factor into *phasor form*
- Recognize that the *observable form* involves taking the real part of the *complex instantaneous form*
- Recognize that from this and the principles described in Performance Objective 4 below, one can find expressions for all of the field components of the wave, the direction of propagation, and the Poynting vector

2. Given the permittivity, permeability and/or conductivity of free space, a perfect dielectric, partially conducting medium, or a good conductor, find the impedance of the medium and the velocity of propagation, wavelength, loss tangent, and complex propagation constant of a uniform plane wave that is travelling through it.
   - Recognize that we are assuming time-harmonic fields with time dependence $e^{j\omega t}$ so we can replace $\partial/\partial t$ with $j\omega$ in Maxwell's equations
   - Recognize that the vector Helmholtz equation is $\nabla^2\mathbf{E} = \gamma^2\mathbf{E} = -k^2\mathbf{E}$ where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ and $\gamma = jk = \alpha + j\beta$ is the complex propagation constant
   - Recognize that:
     - the velocity of propagation is given by ω/β and
     - the wavelength is given by $2\pi/\beta$
   - Recognize that, in the general case,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

     but these can be simplified, sometimes considerably, for the special cases of a perfect dielectric, good dielectric, and good conductor
     - In a pure dielectric, $\alpha = 0$ and $\beta = \omega\sqrt{\mu\epsilon}$
     - In a lossy material, the magnetic field lags by $\theta = \delta/2$ where $\tan\delta = \sigma/\omega\epsilon$.
     - In a good conductor, $\alpha = \beta$ and the maximum lag of 45 degrees (corresponding to an infinite loss tangent or $\delta = 90$ degrees) is observed.
   - From Performance Objective 1 above, the impedance of the medium (intrinsic impedance) in the general case is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

but this can be simplified, sometimes considerably, for the special cases of:
free space, a perfect dielectric, a good dielectric, and a good conductor
where the loss tangent, $\sigma/\omega\epsilon$, is 0, 0, < 0.1, or >> 1, respectively

- In a pure (or perfect or ideal) dielectric, the intrinsic impedance is real and is given by

$$\eta = \sqrt{\mu/\epsilon}$$

  and the electric and magnetic field are in phase.

- The criterion that $\sigma/\omega\epsilon$ in a good dielectric is < 0.1 is arbitrary; some authors use 0.01 instead.

- In a lossy material, the intrinsic impedance is complex; the magnitude can be simplified to yield

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1+(\sigma/\omega\epsilon)^2}}$$

  while phase angel (which corresponds to the magnetic field lag) is given by

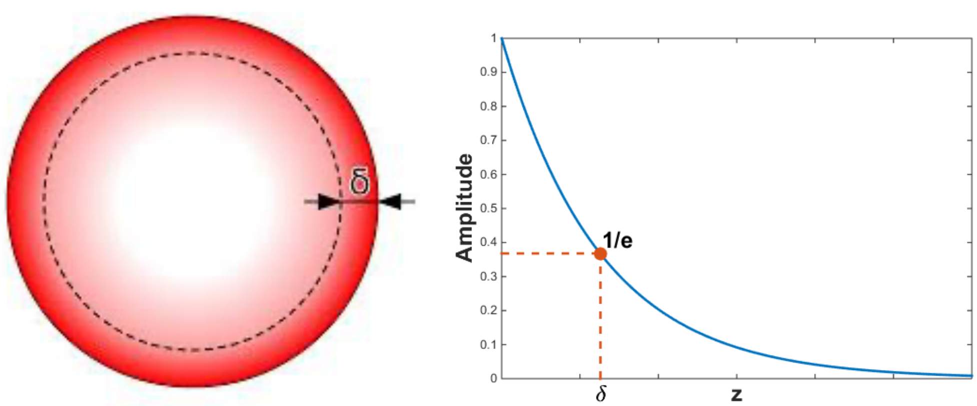
$$\theta = \frac{1}{2}\operatorname{atan}\frac{\sigma}{\omega\epsilon}.$$

- In a good conductor, the intrinsic impedance is complex and is given by

$$\eta = \sqrt{j\omega\mu/\sigma}$$

  and the maximum lag of θ = 45 degrees is observed.

3. Given a cylindrical wire conductor with specified diameter and length, and permittivity, permeability and conductivity, calculate the field distribution, skin depth and resistance of the wire at a specified frequency. (See §11.4)



Skin depth is defined as the depth at which the amplitude of the wave has been reduced by $1/e$.

- Recognize that the electric field and the current density that it drives both decay with depth z as $e^{-\alpha z}$

- Recognize that if α is sufficiently large, as in the case of any good conductor, the current is almost completely contained within a *very* narrow annulus around the outside of the cylinder
- Recognize that the electric field strength and current density decay to less than 1% of their surface values at 5δ and that this is sometimes referred to as the penetration depth
- Recognize that the total current within that annulus is given by the current density at the surface, J_0, multiplied by the area $A_{\text{eff}} = 2\pi r\delta$, where *r* is the radius of the cylinder and $\delta = 1/\alpha = 1/\sqrt{\pi f\mu\sigma}$ is the skin depth, which is a function of $\epsilon, \mu, \sigma,$ and ω
- Recognize that the AC resistance of such a wire is given by $R_{AC} = L/\sigma A_{\text{eff}}$
- Recognize that because $A_{\text{eff}} < A_{\text{phy}}$, $R_{AC} > R_{DC}$

4. Given the parameters of a uniform plane wave, calculate the power density that passes through a given point and the total power that passes through a given area.
   - Recognize that the instantaneous power density is given by the Poynting vector,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

   - Recognize that the time-averaged power density in a perfect dielectric is given by

$$\langle S \rangle = \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2\eta} E_0^2$$

   and in a lossy dielectric is given by

$$\langle S_z \rangle = \frac{1}{2} Re(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \cos\theta_\eta \ .$$

   - Recognize that the total power (in W) that passes through a given area is given by the product of the power density (in W/m^2) and the area (in m^2) where it is assumed that the power density is uniform, as it always is for a uniform plane wave.