THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves

Solutions to the Drill Problems from Chapter 11– Uniform Plane Waves

D11.1 The electric field amplitude of a uniform plane wave propagating in the $\mathbf{a}_z$ direction is 250 V/m. If $\mathbf{E} = E_x\mathbf{a}_x$ and $\omega = 1.00$ Mrad/s, find: (*a*) the frequency; (*b*) the wavelength; (*c*) the period; (*d*) the amplitude of **H**.

Strategy:

Given: The electric field amplitude and direction, direction of propagation and angular frequency of a uniform plane wave.

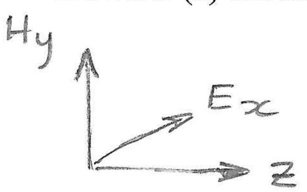
Sought: (*a*) the frequency; (*b*) the wavelength; (*c*) the period; (*d*) the amplitude of **H**.

Steps:

1. Sketch and label the problem geometry as an aid to understanding. Assume free space.
2. To find *f*, we need to convert angular frequency to frequency.
3. To find λ when the medium is lossless, we need to start with the velocity of propagation and the frequency.
4. To find *T*, we need to start with *f*.
5. To find the amplitude of **H**, we need to start with **E** and η.

Answers: (*a*) 159 kHz; (*b*) 1.88 km; (*c*) 6.28 μs; (*d*) 0.663 A/m

1.



$$\frac{|E_x|}{|H_y|} = 377\,\Omega = \eta_0 \qquad \omega = 10^6 \text{ rad/s} \qquad E_x = 250 \text{ V/m}$$

2. $f = \frac{\omega}{2\pi} = 159.155 \text{ kHz}$

3. $c = f\lambda \quad \lambda = c/f$

$$\lambda = \frac{3\times 10^8}{159{,}155} = 1885 \text{ m} = 1.885 \text{ km}$$

4. $$T = \frac{1}{f} = \frac{1}{159{,}155} = 6.283\times 10^{-6} \text{ sec} = 6.28\ \mu\text{s}$$

5. $$|H_y| = \frac{|E_x|}{377} = 0.663 \text{ A/m}$$

$e^{-j\beta z}$

D11.2 Let $H_s = (2\angle -40° \mathbf{a}_x - 3\angle 20° \mathbf{a}_y)e^{-j0.07z}$ A/m for a uniform plane wave traveling in free space. Find: (*a*) ω; (*b*) H_x at *P* (1, 2, 3) at $t = 31$ ns; (*c*) |**H**| at $t = 0$ at the origin.

Strategy:

Given: An expression for the phasor that describes the magnetic field component of a uniform plane wave.

Sought: (*a*) ω; (*b*) H_x at *P* (1, 2, 3) at $t = 31$ ns; (*c*) |**H**| at $t = 0$ at the origin.

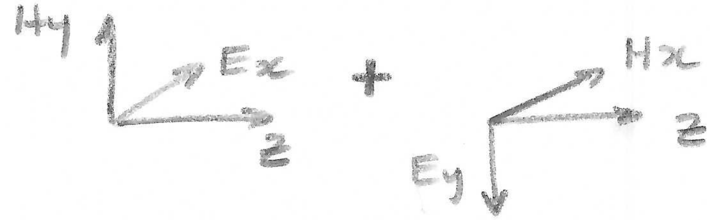
Steps:

1. Sketch and label the problem geometry as an aid to understanding. Assume free space.
2. To find ω, recall that the velocity of propagation depends on ω and β, and the velocity of propagation in free space is known.
3. To find H_x at *P* (1, 2, 3) at $t = 31$ ns, convert the expression for H from phasor form to complex instantaneous form, evaluate at the appropriate time and space coordinates, and take the *x* component.
4. To find |**H**| at $t = 0$ at the origin, evaluate the complex instantaneous form at the appropriate time and space coordinates and then take the absolute value.

   For more information of phasor form, complex instantaneous form, etc., see §10.5 of the textbook.

Answers: (*a*) 21.0 Mrad/s; (*b*) 1.934 A/m; (*c*) 3.22 A/m

1. This problem can be viewed as two orthogonally polarized plane waves superimposed on each other.



2. $\beta = 0.07$ rad/m

$v = \omega/\beta = 3\times10^8$ m/s

$\omega = \beta v = \boxed{2.1\times10^7 \text{ rad/s}}$

$-40° = -2\pi/9$ rad

$-160° = -8\pi/9$ rad $(= -180° + 20°)$

3. $H_x = 2e^{j(\omega t + \theta_x)}e^{-j\beta z}$

$H_y = -3e^{j(\omega t + \theta_y)}e^{-j\beta z}$

$H_x(t,z) = 2\cos(\omega t + \theta_x - \beta z)$

$= 2\cos(2.1\times10^7 \times 31\times10^{-9} - 2\pi/9 - 0.07\times 3) = \boxed{1.934 \text{ A/m}}$

4. $H_x(0,0) = 2\cos(-2\pi/9)$

$H_y(0,0) = 3\cos(-8\pi/9)$

$|H| = \sqrt{H_x^2 + H_y^2}$

$= \boxed{3.21 \text{ A/m}}$

D11.3 A 9.375-GHz uniform plane wave is propagating in polyethylene (see Appendix C of the textbook for the details). If the amplitude of the electric field intensity is 500 V/m and the material is assumed to be lossless, find: (*a*) the phase constant; (*b*) the wavelength in the polyethylene; (*c*) the velocity of propagation; (*d*) the intrinsic impedance; (*e*) the amplitude of the magnetic field intensity.

Strategy:

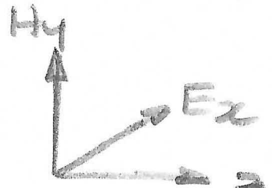
Given: The frequency and amplitude of a uniform plane wave propagating in polyethylene.

Sought: (*a*) the phase constant; (*b*) the wavelength in the polyethylene; (*c*) the velocity of propagation; (*d*) the intrinsic impedance; (*e*) the amplitude of the magnetic field intensity.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find β, recall that $\gamma = \alpha + j\beta$ is a function of the constitutive parameters of the material.
3. To find λ, recall that $\lambda = \frac{2\pi}{\beta}$.
4. To find v_p, recall that $v_p = \omega/\beta$.
5. To find η, recall that for a lossless material, $\eta = \sqrt{\frac{\mu}{\epsilon}}$.
6. To find |**H**|, recall that $\eta = E/H$.

Answers: (*a*) 295 rad/m; (*b*) 2.13 cm; (*c*) 1.99×10^8 m/s; (*d*) 251 Ω; (*e*) 1.99 A/m



$f = 9.375 \times 10^9$ Hz $\quad \epsilon_r = 2.26$

$|E_x| = 500$ V/m $\quad \mu_r = 1$ (NON MAGNETIC)

POLYETHYLENE - ASSUME LOSSLESS

$\eta = 377/\sqrt{2.26} = 250.8\ \Omega$

CAN USE SIMPLIFIED EXPRESSIONS FOR THE LOSSLESS CASE

$\alpha = 0 \quad \beta = \omega\sqrt{\mu\epsilon}$

2. $\beta = \omega\sqrt{\mu\epsilon}$

$= \frac{2\pi \times 9.375 \times 10^9 \sqrt{2.26}}{3 \times 10^8}$

$= 295$ rad/m

3. $\lambda = 2\pi/\beta = 2\pi/295$

$= 0.0213\text{ m} = \boxed{2.13\text{ cm}}$

4. $v = \frac{\omega}{\beta} = \frac{2\pi \times 9.375 \times 10^9}{295}$

$= \boxed{1.997 \times 10^8\text{ m/s}}$

5. $\eta = 377/\sqrt{2.26} = \boxed{250.8\ \Omega}$

6. $|H| = |E|/\eta$

$= 500/250.8$

$= \boxed{1.994\text{ A/m}}$

REAL PART OF PERMITIVITY

D11.4 Given a nonmagnetic material having $\epsilon'_r = 3.2$ and $\sigma = 1.5 \times 10^{-4}$ S/m, find numerical values at 3 MHz for the (*a*) loss tangent; (*b*) attenuation constant; (*c*) phase constant; (*d*) intrinsic impedance.

Strategy:

Given: The properties of a material.

Sought: the (*a*) loss tangent; (*b*) attenuation constant; (*c*) phase constant; (*d*) intrinsic impedance at 3 MHz.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find the loss tangent, recall that $\tan\delta = \frac{\sigma}{\omega\epsilon}$.
3. To find the attenuation and phase constants, recall that that $\gamma = \alpha + j\beta$ is a function of the constitutive parameters of the material.
4. To find the intrinsic impedance at 3 MHz, recall that η is a function of the constitutive parameters of the material and will be complex because the material is lossy.

Answers: (*a*) 0.28; (*b*) 0.016 Np/m; (*c*) 0.11 rad/m; (*d*) 207 $\angle 7.8°$ Ω

1.

$\omega = 2\pi f$
$= 6\pi \times 10^6$ rad/s
$\mu = \mu_0$
$= 4\pi \times 10^{-7}$ H/m
$\epsilon = \epsilon_r \epsilon_0$
$= 2.832 \times 10^{-11}$ F/m

2. $\tan\delta = \dfrac{1.5 \times 10^{-4}}{2\pi \times 3 \times 10^6 \times 3.2 \times 8.85 \times 10^{-12}} = 0.281$
$= \sigma/\omega\epsilon$

3. $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)} = 0.0156$ Np/m

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)} = 0.1135$ rad/m

4. $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}} = 204.8 + j\,28.22$

$|\eta| = 206.7\ \Omega$ $\quad \angle\eta = \theta_\eta = 7.85°$

REAL PART OF PERMITTIVITY

D11.5 Consider a material for which $\mu_r = 1$, $\epsilon'_r = 2.5$, and the loss tangent is 0.12. If these three values are constant with frequency in the range 0.5 MHz $\leq f \leq$ 100 MHz, calculate: (*a*) σ at 1 and 75 MHz; (*b*) λ at 1 and 75 MHz; (*c*) v_p at 1 and 75 MHz.

Strategy:

Given: The constitutive properties of a material in the range 0.5 MHz $\leq f \leq$ 100 MHz.

Sought: σ, λ and v_p at 1 and 75 MHz.

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. To find σ, recall that $\tan\delta = \frac{\sigma}{\omega\epsilon}$.
3. To find λ, recall that $\lambda = \frac{2\pi}{\beta}$.
4. To find v_p, recall that $v_p = \omega/\beta$.
5. To find β, recall that $\gamma = \alpha + j\beta$ is a function of the constitutive parameters of the material.

Answers: (*a*) 1.67×10^{-5} and 1.25×10^{-3} S/m; (*b*) 190 and 2.53 m; (*c*) 1.90×10^{8} m/s twice

$\mu_r = 1$
$\epsilon_r = 2.5$
$\tan\delta = 0.12$

$\sigma = \omega\epsilon\tan\delta$

$\lambda = 2\pi/\beta$
$v_p = \omega/\beta$

f (MHz)	σ (S/m)	β (rad/m)	λ (m)	v_p (m/s)
1	1.668×10^{-5}	0.0332	189.3	1.89×10^{8}
75	1.251×10^{-3}	2.488	2.525	1.89×10^{8}

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}$$

$\tan\delta$

$= 5.28 \times 10^{-9}$ s/m

constant for $0.5 \leq f \leq 100$ MHz
only need to calculate once.

D11.6 At frequencies of 1, 100, and 3000 MHz, the dielectric constant of ice made from pure water has values of 4.15, 3.45, and 3.20, respectively, while the loss tangent is 0.12, 0.035, and 0.0009, also respectively. If a uniform plane wave with an amplitude of 100 V/m at $z = 0$ is propagating through such ice, find the time-average power density at $z = 0$ and $z = 10$ m for each frequency.

Strategy:

Given: The dielectric constant and loss tangent of ice at three frequencies, and the amplitude of a plane wave that propagates through it.

Sought: The time-average power density at $z = 0$ and $z = 10$ m for each frequency.

Steps:

1. Sketch and label the problem geometry as an aid to understanding. Assume free space.
2. To find the time-average power density, recall that in a lossy medium,

I believe that my answers are more accurate.

$$\langle S_z \rangle = \frac{1}{2} Re(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \cos\theta_\eta$$

Answers: 27.1 and 25.7 W/m²; 24.7 and 6.31 W/m²; 23.7 and 8.63 W/m²

1.

$\tan\delta = \frac{\sigma}{\omega\epsilon}$

DIELECTRIC CONSTANT $\equiv \epsilon_r$

$|E_x| = 100$ V/m @ $z = 0$

GIVEN			CALCULATED			RESULTS	
f(MHz)	ϵ_r	$\tan\delta$	α	$\lvert\eta\rvert$	$\theta_\eta = \frac{\delta}{2}$	$\langle S_0 \rangle$	$\langle S_{10} \rangle$
1	4.15	0.12	0.0026	185.7	3.42	26.87	25.53
100	3.45	0.035	0.0681	203.0	1.00	24.62	6.31
3000	3.20	0.0009	0.0506	210.7	0.0258	23.72	8.63
MHz	—	—	Np/m	Ω	deg.	W/m²	W/m²

2. TO CALCULATE $\langle S_z \rangle$, WE NEED TO FIND $\alpha, |\eta|, \theta_\eta$

ALL OTHER QUANTITIES ARE GIVEN.

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)}$$

$$\left.\begin{array}{l} \omega = 2\pi f \\ \mu\epsilon = \mu_0\epsilon_0\epsilon_r \\ \frac{\sigma}{\omega\epsilon} = \tan\delta \end{array}\right\} \checkmark$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\omega\epsilon(\tan\delta + j)}}$$

$\sigma = \omega\epsilon\tan\delta$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}}, \quad \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

$$\theta_\eta = \delta/2$$

$|\eta|$ should be close to $377/\sqrt{\epsilon_r}$

← IT TAKES SOME EFFORT TO PROVE THIS, BUT IT'S NOT IMPOSSIBLE.

D11.7 A steel pipe is constructed of a material for which $\mu_r = 180$ and $\sigma = 4 \times 10^6$ S/m. The two radii are 5 and 7 mm, and the length is 75 m. If the total current $I(t)$ carried by the pipe is $8 \cos \omega t$ A, where $\omega = 1200\pi$ rad/s, find: (*a*) the skin depth; (*b*) the effective resistance; (*c*) the dc resistance; (*d*) the time-average power loss.

Strategy:

Given: A detailed description of the geometry and composition of a steel pipe and the total current $I(t)$ that it carries.

Sought: (*a*) the skin depth; (*b*) the effective or ac resistance; (*c*) the dc resistance; (*d*) the time-average power loss.

Steps:

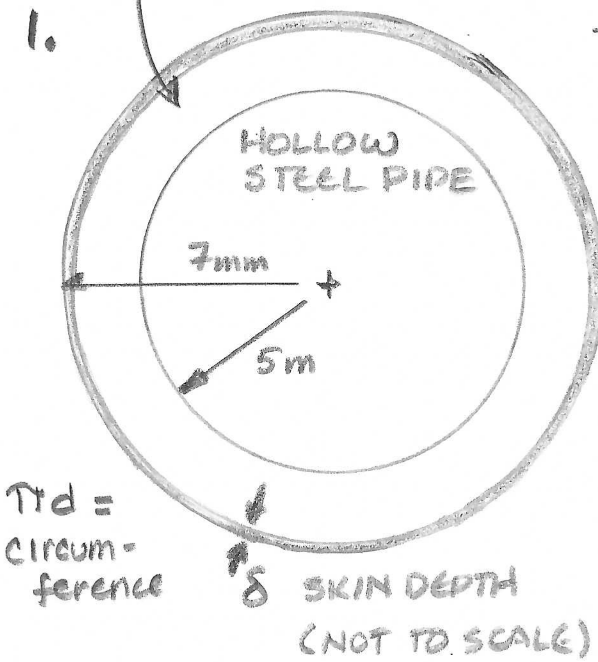
1. Sketch and label the problem geometry as an aid to understanding.
2. To find skin depth δ, take the inverse of the attenuation constant α. (Why?)
3. To find the effective or ac resistance, assume the current flows through an annulus of thickness δ around the outer circumference of the wire and apply the relation $R = \frac{L}{\sigma A}$.
4. To find the dc resistance, assume the current flows through the entire cross-section of the wire and apply the relation $R = \frac{L}{\sigma A}$.
5. To find the time-average power loss, apply the relation $P_L = \frac{1}{2}|I|^2 R$.

Answers: (*a*) 0.766 mm; (*b*) 0.557 Ω; (*c*) 0.249 Ω; (*d*) 17.82 W

$\sigma = 4\times10^6$ S/m

$\mu_r = 180$

1.



πd = circumference

AREA OF ANNULUS = $\pi d \delta$

TOTAL CURRENT = $8\cos\omega t$ A

$\omega = 1200\pi$ rad/s

$f = 600$ Hz

2. $$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi\, 600\, 180\, 4\pi\times10^{-7}\, 4\times10^{6}}}$$

$$= 7.66\times10^{-4}\,\text{m} = \boxed{0.766\ \text{mm}}$$

3. $$R_{AC} = \frac{L}{\sigma A_{AC}} = \frac{L}{\sigma\, \underbrace{\pi d \delta}_{\text{AREA OF ANNULUS}}}$$

$$= \frac{75}{4\times10^{6}\, \pi\, 14\times10^{-3}\, 0.766\times10^{-3}}$$

$$= \boxed{0.5565\ \Omega}$$

4. $$R_{DC} = \frac{L}{\sigma A} = \frac{L}{\sigma \pi (r_o^2 - r_i^2)}$$

$$= \frac{75}{4\times10^{6}\, \pi\, ((7\times10^{-3})^2 - (5\times10^{-3})^2)} = \boxed{0.2487\ \Omega}$$

5. $$P_L = \frac{1}{2}|I|^2 R$$

$$= \frac{1}{2}\, 8^2\, 0.5565$$

$$= \boxed{17.81\ \text{W}}$$