THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

Chapter 11 – Uniform Plane Waves – Example Problems

The purpose of these example problems is to help you master some of fundamental techniques used to analyze uniform plane waves. Many of the concepts will be critical to our study of guided waves.

Try these problems before we review the solutions in class. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

1. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in the positive z direction through a perfect dielectric with relative permittivity = 2.5 and relative permeability = 1. Give the corresponding Helmholtz equations and find expressions for the field components of the wave and intrinsic impedance of the medium.

2. A medium has relative permittivity = 2.5, relative permeability = 1 and conductivity = 50 S/m. Find the intrinsic impedance of the medium and the velocity of propagation, wavelength, loss tangent, and complex propagation constant of a 50 MHz plane wave that is travelling through it.

3. Consider an AWG 30 copper wire of length 15 cm. What is the skin depth and resistance at 2 GHz? How deeply does the current penetrate? What are the attenuation and phase constants?

4. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in free space in the positive z direction. Calculate the peak and time averaged power density that passes through $z = 0$ and the total power that passes through an aperture of dimensions 50 cm x 50 cm.

5. A hollow tubular conductor is constructed from a type of brass having a conductivity of 1.2×10^7 S/m. The inner and outer radii are 9 and 10 mm, respectively. Calculate the resistance per metre length at a frequency of (*a*) DC; (*b*) 20 MHz; (*c*) 2 GHz.

6. (*a*) Most microwave ovens operate at 2.45 GHz. Assume that $\sigma = 1.2 \times 10^6$ S/m and $\mu_r = 500$ for the stainless-steel interior, and find the depth of penetration. (*b*) Let $E_s = 50\angle 0°$ V/m at the surface of the conductor, and plot a curve of the amplitude of E_s versus the angle of E_s as the field propagates into the stainless steel.

7. Consider a left circularly polarized wave in free space that propagates in the forward *z* direction. The electric field is given by the appropriate form of

$$\mathbf{E}_s = E_0\left(\mathbf{a}_x \pm j\mathbf{a}_y\right)e^{-j\beta z} \tag{100}$$

Determine (*a*) the magnetic field phasor, $\mathbf{H}_s$; (*b*) an expression for the average power density in the wave in W/m^2 by direct application of Eq. (77).