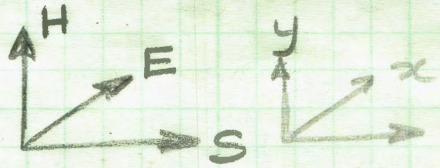
I. PLANE WAVES

1. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in the positive z direction through a perfect dielectric with relative permittivity = 2.5 and relative permeability = 1. Give the corresponding Helmholtz equations and find expressions for the field components of the wave and intrinsic impedance of the medium.



E AND H BOTH LIE IN THE x-y PLANE.

THE PROBLEM DOESN'T SPECIFY THE POLARIZATION OF THE WAVE, SO WE ASSUME WITHOUT LOSS OF GENERALITY THAT THE WAVE IS HORIZONTALLY POLARIZED.

$$\boxed{\begin{aligned} \nabla^2 E - \gamma^2 E &= 0 \\ \nabla^2 H - \gamma^2 H &= 0 \end{aligned}}$$

HELMHOLTZ EQUATIONS

$$\gamma = \alpha + j\beta = 0 + j\,\omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{2.5}} = \boxed{238\,\Omega}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi \times 5\times10^9 \times \sqrt{2.5}}{3\times10^8}$$

THESE WERE NOT REQUESTED!

$$= 165.6 \text{ rad/m} = 2\pi/\lambda$$

$$\lambda = 2\pi/\beta = 3.80 \text{ cm}$$

$$E_x = \boxed{10 \text{ V/m}}$$

$$H_y = 10/238 = \boxed{42.0 \text{ mA/m}}$$

TOPS

2. A medium has relative permittivity = 2.5, relative permeability = 1 and conductivity = 50 S/m. Find the intrinsic impedance of the medium and the velocity of propagation, wavelength, loss tangent, and complex propagation constant of a 50 MHz plane wave that is travelling through it.

$\epsilon_r = 2.5 \quad \mu_r = 1 \qquad \sigma = 50 \text{ s/m}$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\epsilon + \sigma}}$$

$$\gamma = \sqrt{j\omega\mu(j\omega\epsilon + \sigma)} = \alpha + j\beta$$

$$\beta = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{\beta} \qquad v = \frac{\omega}{\beta} = f\lambda$$

$$\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{50}{2\pi \times 50 \times 10^6 \times 8.85 \times 10^{-12} \times 2.5}$$

$$= 7193$$

$\therefore$ THE MEDIUM IS A VERY GOOD CONDUCTOR AND WE CAN USE THE FOLLOWING APPROXIMATIONS

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma} \angle 45^\circ$$

$$= \alpha + j\beta \qquad \alpha = \beta = \frac{|\gamma|}{\sqrt{2}} = \sqrt{\pi f \mu \sigma}$$

$$\beta = \frac{2\pi}{\lambda} \qquad \lambda = \frac{2\pi}{\beta}$$

CALCULATE THE NUMBERS

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ = \sqrt{\frac{2\pi \times \cancel{50} \times 10^6 \times 4\pi \times 10^{-7}}{\cancel{50}}}$$

$$\boxed{= 2.81\angle 45^\circ\ \Omega}$$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 50 \times 10^6 \times 4\pi \times 10^{-7} \times 50}$$

$$\boxed{= 99.3\ /m}$$

$$\gamma = \boxed{99.3\ Np/m + j\,99.3\ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{99.3} = \boxed{0.0633 m}$$

$$v = f\lambda = \frac{\omega}{\beta} = 0.063 \times 50 \times 10^6$$

$$= \boxed{3.164 \times 10^6\ m/s}$$

3. Consider an AWG 30 copper wire of length 15 cm. What is the skin depth and resistance at 2 GHz? How deeply does the current penetrate? What are the attenuation and phase constants?

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}}$$

$$= \boxed{1.478\ \mu m}$$

THE CURRENT ACTUALLY PENETRATES TO 5δ OR $7.39\ \mu m$ BEFORE ITS EXPONENTIAL DECAY RENDERS IT INSIGNIFICANT.

$$\alpha = \frac{1}{\delta} = \boxed{6.77 \times 10^5\ Np/m}, \quad \beta = \boxed{6.77 \times 10^5\ rad/m}$$

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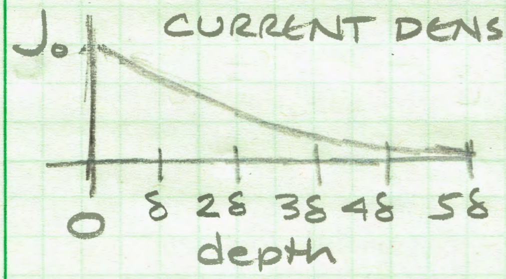
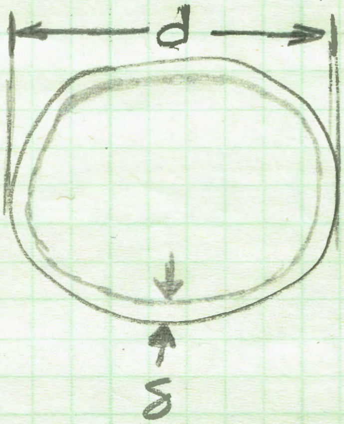


$L = 0.15\,m$

$\sigma = 5.8\times10^{7}\,S/m$

$$R = \frac{L}{A\sigma}$$



$A \approx \pi d \delta \qquad \delta \ll d$

FOR AWG 30 WIRE,

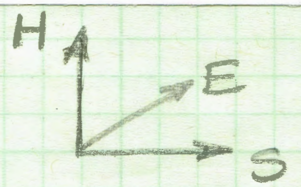
$d = 0.255\,mm \gg \delta$

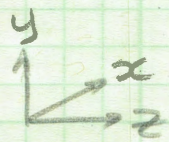
$$A \approx \pi\, 0.255\times10^{-3} \times 1.478\times10^{-6}$$

$$= 1.1840\times10^{-9}\,m^2$$

$$R = \frac{0.15}{1.184\times10^{-9}\times5.8\times10^{7}} = \boxed{2.18\,\Omega}$$

4. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in free space in the positive *z* direction. Calculate the peak and time averaged power density that passes through *z* = 0 and the total power that passes through an aperture of dimensions 50 cm x 50 cm.





$\eta = 120\pi\,\Omega$

$= 377\,\Omega$

$$S = E\times H = \frac{|E|^2}{\eta_0} e^{j2\omega t} \quad W/m^2$$

THE PEAK VALUE IS $10^2/377 = 0.265\,W/m^2$

$$S_{avg} = \frac{1}{2} E\times H^* = \frac{|E|^2}{2\eta_0} \quad W/m^2$$

$$= 0.1325\,W/m^2$$

TOPS

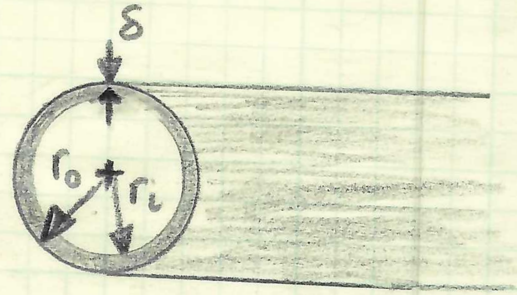
THE TOTAL POWER THAT PASSES THROUGH AN APERTURE OF SIZE 50 x 50 cm $= 0.25 \, m^2$ IS

$$P = S_{avg} A = \frac{0.1325}{4}$$

$$\boxed{= 0.0331 \, W}$$

TOPS

11.23 A hollow tubular conductor is constructed from a type of brass having a conductivity of 1.2 × 10^7 S/m. The inner and outer radii are 9 and 10 mm, respectively. Calculate the resistance per metre length at a frequency of (*a*) DC; (*b*) 20 MHz; (*c*) 2 GHz.



$\rho = 1.2 \times 10^7$ S/m
$r_o = 10^{-2}$ m = 10 mm
$r_i = 9 \times 10^{-3}$ m = 9 mm

a. DC resistance / m

$$R_{DC} = \frac{l}{\rho A} = \frac{1}{1.2 \times 10^7 \pi (10^{-2})^2 (1 - 0.9^2)}$$

$$= 1.4 \times 10^{-3}\ \Omega/m$$

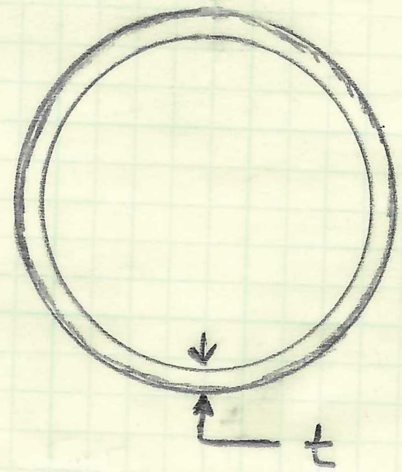
b./c. $R_{ac} = \dfrac{l}{\rho A} = \dfrac{l}{\rho\, 2\pi r_o \delta}$

$$\delta = \frac{1}{\sqrt{\pi f \mu_o \sigma}}$$

$f = 20$ MHz

$$R_{20M} = \frac{1}{1.2 \times 10^7 \times 2 \times \pi \times 10^{-2} \times 3.25 \times 10^{-5}}$$

$$= 4.08 \times 10^{-2}\ \Omega/m$$



DC	1mm
20 MHZ	$\delta = 3.25 \times 10^{-5}$ m
2 GHZ	$\delta = 3.25 \times 10^{-6}$ m

$f = 2$ GHz

$$R_{2G} = \frac{1}{1.2 \times 10^7 \times 2 \times \pi \times 10^{-2} \times 3.25 \times 10^{-6}}$$

$$= 0.408\ \Omega/m$$

TOPS

11.24 (*a*) Most microwave ovens operate at 2.45 GHz. Assume that $\sigma = 1.2 \times 10^6$ S/m and $\mu_r = 500$ for the stainless-steel interior, and find the depth of penetration. (*b*) Let $E_s = 50\angle 0°$ V/m at the surface of the conductor, and plot a curve of the amplitude of E_s versus the angle of E_s as the field propagates into the stainless steel.

a.

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\alpha} = \text{SKIN DEPTH}$$

$$= \frac{1}{\sqrt{\pi \times 2.45 \times 10^9 \times 1.2 \times 10^6 \times 500 \times 4\pi \times 10^{-7}}}$$

$$= 4.15 \times 10^{-7}\,m$$

$$5\delta = \text{PENETRATION DEPTH}:$$

$$= 2.076 \times 10^{-6}\,m.$$

THERE IS NO SIGNIFICANT FIELD BELOW THIS DEPTH

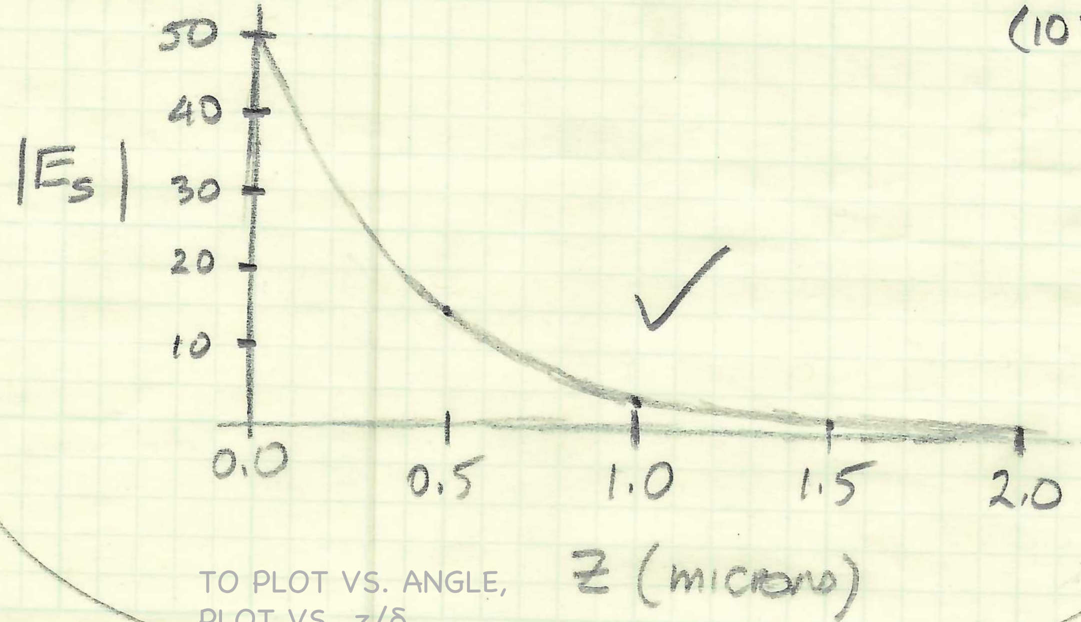
b.

$$E_s(z) = 50\, e^{-\alpha z} e^{-j\beta z}$$

$$= \underbrace{50\, e^{-z/\delta}}_{\text{AMPLITUDE}}\, e^{-j\, z/\delta}$$

ANGLE

$$|E_s(z)| = 50\, e^{-2.41\, z}, \quad z \text{ in microns } (10^{-6} m)$$



TO PLOT VS. ANGLE,
PLOT VS $-z/\delta$

TOPS

11.29 Consider a left circularly polarized wave in free space that propagates in the forward z direction. The electric field is given by the appropriate form of equation (100). Determine (*a*) the magnetic field phasor, $\mathbf{H}_s$; (*b*) an expression for the average power density in the wave in W/m^2 by direct application of Eq. (77).

$$\bar{E}_s = E_o(\bar{a}_x \underbrace{+ j}_{} \bar{a}_y)e^{-j\beta z} \qquad (100)$$

THIS ENSURES THAT $\bar{E}_y$ WILL LEAD $\bar{E}_x$

$$E_o \bar{a}_x \times H_o \bar{a}_y = S\,\bar{a}_z$$

$$jE_o \bar{a}_y \times -jH_o \bar{a}_x = S\,\bar{a}_z$$

a. $$\bar{H}_s = \frac{E_o}{\eta_o}(\bar{a}_y - j\bar{a}_x)e^{-j\beta z}$$

$$+j = e^{j\pi/2}$$

HERE, WE ENFORCE TWO CONDITIONS:

- POWER MUST FLOW IN THE +Z DIRECTION
- E AND H MUST BE IN PHASE

b.

$$\bar{S}_{z,avg} = \frac{1}{2}\mathrm{Re}(\bar{E}_s \times \bar{H}_s^*)$$

$$= \frac{1}{2}\mathrm{Re}\left(E_o(\bar{a}_x + j\bar{a}_y)e^{-j\beta z} \times \frac{E_o}{\eta_o}(\bar{a}_y + j a_x)e^{j\beta z}\right)$$

$$= \frac{1}{2}\frac{E_o^2}{\eta_o}(\bar{a}_x + j\bar{a}_y) \times (\bar{a}_y + j\bar{a}_x)$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & j & 0 \\ j & 1 & 0 \end{vmatrix}$$

$$= \boxed{\frac{E_o^2}{\eta_o}\hat{a}_z \ \mathrm{W/m^2}}$$

$$\hat{a}_z + \hat{a}_z = 2\hat{a}_z$$

TOPS