THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

Chapter 11 – Uniform Plane Waves – Example Problems

Strategies

The purpose of these example problems is to help you master some of fundamental techniques used to analyze uniform plane waves. Many of the concepts will be critical to our study of guided waves.

Try these problems before we review the solutions in class. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

1. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in the positive z direction through a perfect dielectric with relative permittivity = 2.5 and relative permeability = 1. Give the corresponding Helmholtz equations and find expressions for the field components of the wave and intrinsic impedance of the medium.

   Strategy:

   Given: The electric field strength, frequency, and direction of a uniform plane wave. The relative permittivity and permeability of the medium.

   Sought: The corresponding Helmholtz equations, expressions for the field components of the wave, and the intrinsic impedance of the medium.

   Steps:

   1. Sketch and label the problem geometry (the electric and magnetic field vectors and the Poynting vector) as an aid to understanding. Assume unbounded space. Polarization is not specified so is arbitrary.
   2. The Helmholtz equations for the electric and magnetic field and how they are derived from Maxwell's equation using vector identities are presented in the lecture slides.
   3. The Helmholtz equations are second order differential equations. The simplest solution is a complex exponential in time and distance which is uniform in directions perpendicular to the direction of propagation. See the lecture slides.
   4. To find the intrinsic impedance, take the curl of **E** to find **H**, and then take the ratio of **E** to **H**. Assume a time variation of the form $e^{j\omega t}$ so that $\frac{\partial}{\partial t} = j\omega$.

   Consilium est demonstratum.

2. A medium has relative permittivity = 2.5, relative permeability = 1 and conductivity = 50 S/m. Find the intrinsic impedance of the medium and the velocity of propagation, wavelength, loss tangent, and complex propagation constant of a 50 MHz plane wave that is travelling through it.

   Strategy:

   Given: The frequency of a uniform plane wave. The relative permittivity, permeability and conductivity of the medium.

   Sought: The intrinsic impedance of the medium. The velocity of propagation, wavelength, loss tangent, and complex propagation constant of a 50 MHz plane wave that is travelling through the medium.

   Steps:

   1. Sketch and label the problem geometry (the electric and magnetic field vectors and the Poynting vector) as an aid to understanding. Assume unbounded space. The polarization is arbitrary.
   2. Find the complex propagation constant $\gamma = \alpha + j\beta$, which is a function of the frequency of the wave and the permittivity, permeability and conductivity of the medium. The derivation is presented in the lecture slides.
   3. Find the velocity of propagation and wavelength, which are functions of ω and/or β.
   4. Find the loss tangent, which is the ratio of conduction to displacement current and a function of the conductivity and permittivity of the medium and the frequency of the wave.
   5. Find the intrinsic impedance, which is the ratio of electric to magnetic field strength and is a function of the conductivity, permittivity and permeability of the medium and the frequency of the wave.

   Consilium est demonstratum.

3. Consider an AWG 30 copper wire of length 15 cm. What is the skin depth and resistance at 2 GHz? How deeply does the current penetrate? What are the attenuation and phase constants?

   Strategy:

   Given: An AWG 30 copper wire of length 15 cm. The frequency of the field.

   Sought: The skin depth and resistance at 2 GHz. How deeply the current penetrates. The attenuation and phase constants?

   Steps:

   1. Sketch and label the problem geometry (cross section of the wire) as an aid to understanding.
   2. Find the diameter of AWG 30 wire and the material properties of copper in the Appendices to the textbook.

3. Find the complex propagation constant $\gamma = \alpha + j\beta$, which is a function of the frequency of the wave and the permittivity, permeability and conductivity of the medium.
4. Find the skin depth $= \ 1/\alpha$. The effective penetration is 5 times the skin depth.
5. Find the effective cross-sectional area of the wire at 2 GHz which is the product of the circumference of the wire and the skin depth.
6. Find the resistance of the wire which is the ratio of the length of the wire to the product of the conductivity of the copper and the effective cross-sectional area of the wire.

Consilium est demonstratum.

4. A plane wave with electric field strength of 10 V/m and frequency of 5 GHz is travelling in free space in the positive *z* direction. Calculate the peak and time averaged power density that passes through *z* = 0 and the total power that passes through an aperture of dimensions 50 cm x 50 cm.

Strategy:

Given: The electric field strength, frequency, and direction of a uniform plane wave. The relative permittivity and permeability of the medium.

Sought:

Steps:

1. Sketch and label the problem geometry (the electric and magnetic field vectors and the Poynting vector) and the aperture as an aid to understanding.
2. Recognize that this is a follow-on to problem 1, and its solution can be used as a starting point.
3. Find the peak power density which is expressed in terms of the product of the electric and magnetic field strength.
4. Find the time-averaged power density which involves integrating the peak power density over one period.
5. Find the total power by integrating the time-averaged power density over the aperture.

Consilium est demonstratum.

5. A hollow tubular conductor is constructed from a type of brass having a conductivity of 1.2×10^7 S/m. The inner and outer radii are 9 and 10 mm, respectively. Calculate the resistance per metre length at a frequency of (*a*) DC; (*b*) 20 MHz; (*c*) 2 GHz.

   Strategy:

   Given: The geometry of the conductor. The material properties of the conductor.

   Sought: the resistance per metre length at a frequency of (*a*) DC; (*b*) 20 MHz; (*c*) 2 GHz.

   Steps:

   1. Sketch and label the problem geometry (cross section of the wire) as an aid to understanding.
   2. Recognize that the good conductor approximation applies.
   3. Calculate the skin depth at each frequency.
   4. Calculate the effective cross-sectional area at each frequency.
   5. Calculate the resistance at each frequency.

   Consilium est demonstratum.

6. (*a*) Most microwave ovens operate at 2.45 GHz. Assume that $\sigma = 1.2 \times 10^6$ S/m and $\mu_r = 500$ for the stainless-steel interior, and find the depth of penetration. (*b*) Let $E_s = 50\angle 0°$ V/m at the surface of the conductor, and plot a curve of the amplitude of E_s versus the angle of E_s as the field propagates into the stainless steel.

   Strategy:

   Given: The properties of the stainless-steel interior, the frequency of operation, the strength at the electric field at the surface of the conductor.

   Sought: The depth of penetration. A plot of the amplitude of E_s versus the angle of E_s as the field propagates into the stainless steel

   Steps:

   1. Sketch and label the problem geometry as an aid to understanding.
   2. Recognize that the good conductor approximation can be applied.
   3. Calculate α from knowledge of the material properties of stainless steel.
   4. Recognize that $E_s = E_0 e^{-\gamma z}$, $\alpha = \beta$, and the phase angle $= \beta z$.

   Consilium est demonstratum.

7. Consider a left circularly polarized wave in free space that propagates in the forward *z* direction. The electric field is given by the appropriate form of

$$\mathbf{E}_s = E_0\left(\mathbf{a}_x \pm j\mathbf{a}_y\right)e^{-j\beta z} \tag{100}$$

Determine (*a*) the magnetic field phasor, $\mathbf{H}_s$; (*b*) an expression for the average power density in the wave in W/m^2 by direct application of Eq. (77).

Strategy:

Given: The phasor representation of the electric field.

Sought: The phasor representation of the magnetic field. An expression for the average power density in the wave in W/m^2

Steps:

1. Sketch and label the problem geometry as an aid to understanding.
2. Recognize that the perfect dielectric/free space approximation can be apploied.
3. Take the curl of **E** to find **H**.
4. Find the peak power density which is expressed in terms of the product of the electric and magnetic field strength.
5. Find the time-averaged power density which involves integrating the peak power density over one period.

Consilium est demonstratum.