THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

A Supplement to

Chapter 11 – The Uniform Plane Wave

in W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, McGraw-Hill, 2019, pp. 369-408.

The purposes of this supplement are:

· to assist the reader in identifying key points to be recognized as they apply the SQ3R (Survey, Question, Read, Recite, Review) process to reading and reviewing the chapter and

· to provide comments and supplemental information that fill in apparent gaps in the textbook.

Introduction

The uniform plane wave represents the simplest, but still exceedingly practical, propagating electromagnetic wave. This chapter considers: 1) wave propagation in free space, 2) wave propagation in dielectrics, 3) power flow associated with an electromagnetic wave, 4) wave propagation in good conductors, and 5) polarization of electromagnetic waves.

In this chapter, the wave is assumed to be propagating in unbounded media. In Chapter 12, reflection and transmission of plane waves incident upon boundaries between different materials is considered.

11.1 Wave Propagation in Free Space

The section links the concept of waves propagating along transmission lines (as introduced in Chapter 10) to the concept of uniform plane waves propagating in unbounded media.

While it is useful to be aware that the linkage to Chapter 10 exists, detailed knowledge of this linkage is not a prerequisite for full understanding of Chapter 11. Indeed, most intermediate textbooks follow the sequence that we are pursuing in ELEC 311 – propagation in unbounded media followed by propagation along transmission lines and waveguides. Accordingly, any references to Chapter 10 can be noted for interest but otherwise ignored for now.

11.1.1 Wave Equation for the Uniform Plane Wave

This section:

- introduces Maxwell's equations in point form for a *sourceless medium*
- suggests how one might infer the possibility of propagating electromagnetic waves *and their properties* by inspecting these equations
- postulates the existence of a specific type and particularly simple form of propagating wave, *i.e., uniform plane waves*, that satisfy these equations

- demonstrates that if a uniform plane wave is given by $\mathbf{E} = E_x\mathbf{a}_x$ and $\mathbf{H} = H_y\mathbf{a}_y$, one can begin with Maxwell's equations and determine the second-order partial differential equation (in t and z) that E_x must satisfy
- demonstrates that the resulting second-order partial differential equation is, in fact, the wave equation

Comments

This section and the next are largely anticipated by Problem 9.22 from the course problem set:

> In a sourceless medium in which $\mathbf{J} = 0$ and $\rho_v = 0$, assume a rectangular coordinate system in which $\mathbf{E}$ and $\mathbf{H}$ are functions only of z and t. The medium has permittivity ϵ and permeability μ.
>
> (*a*) If $\mathbf{E} = E_x\mathbf{a}_x$ and $\mathbf{H} = H_y\mathbf{a}_y$, begin with Maxwell's equations and determine the second-order partial differential equation (in t and z) that E_x must satisfy.
>
> (*b*) Show that $E_x = E_0 \cos(\omega t - \beta z)$ is a solution of that equation for a particular value of β.
>
> (*c*) Find β as a function of given parameters.

11.1.2 Solutions of the Wave Equation

This section:

- gives the general form for forward- and backward-propagating waves that are solutions to the wave equation given in §11.1.1
- demonstrates the significance of the wave number k_0 in the factor $\cos(\omega t - k_0 z)$
  - k_0 is essentially identical to the phase constant β
- suggests how one can interpret the simultaneous variation in time and space implied by the factor $\cos(\omega t - k_0 z)$:
  - at a fixed point in space (*i.e.,* z is constant), one will observe field strength to vary sinusoidally in time with frequency $f = \omega/2\pi$
  - at a fixed point in time, one will observe zero crossings to move in the positive z direction with a velocity given by $v = \omega/k_0 = \omega/\beta$
  - overall, the wave will be periodic with wavelength $\lambda = 2\pi/k_0 = 2\pi/\beta$

Comments

Being able to:

- visualize the wave,
- interpret the simultaneous variation in time and space implied by the factor $\cos(\omega t - k_0 z)$
- interpret the parameters f, v, λ in terms of $k_0 = \beta$

will be key in the sections to follow.

In Problem 9.22, we showed how β depends on the material properties (constitutive parameters) of the medium.

11.1.3 Vector Helmholtz Equation in Free Space

This section:

- assumes a time-harmonic solution that varies as $e^{j\omega t}$ which allows us to replace $\frac{\partial}{\partial t}$ by $j\omega$
- this allows us to express the vector wave equation in a more compact form which is referred to as the vector Helmholtz equation

Comments

This a bit similar to the Laplace transform: we have simplified the expression by replacing a derivative by a parameter.

11.1.4 Relation Between E and H; Intrinsic Impedance

This section:

- shows that there is a fixed relationship between electric and magnetic field strength that, in the case of a uniform plane wave propagating in unbounded media, is given by the *intrinsic impedance* of the medium
- shows that the intrinsic impedance of free space is a function of its permittivity and permeability

Comments

In follow-on sections, we'll see that the intrinsic impedance is also a function of the conductivity of the medium.

11.2 Wave Propagation in Dielectrics

This section:

- extends our analytical treatment of the uniform plane wave to propagation in a dielectric of permittivity ϵ and permeability μ.
- assumes the medium to be homogeneous (having constant μ and ϵ with position) and isotropic (in which μ and ϵ are invariant with field orientation).

Comments

None.

11.2.1 Propagation in Lossy Media

This section:

- defines k in terms of the material properties of the medium
- acknowledges that k can be complex, yielding $jk = \alpha + j\beta$
- notes that:
  - α can be interpreted as an attenuation constant (Np/m)
  - β can be interpreted as a phase constant (rad/m)
- gives expressions for $\boldsymbol{E}(z, t)$ in terms of α and β

- gives expressions for α and β in terms of ω, μ, and ϵ
- gives expressions for phase velocity, intrinsic impedance, wavelength, etc. in terms of ω, μ, and ϵ

Comments

In the follow-on sections, we look at how these relatively complex expressions simplify for good dielectrics and conducting media. Note that some books call the propagation constant $\gamma = \alpha + j\beta$

11.2.2 Propagation in Conducting Media

This section:

- uses complex permittivity $\epsilon = \epsilon' + \mathrm{j}\epsilon''$ to account for the conductivity of the media where $\epsilon'' = \frac{\sigma}{\omega}$ while many other books use σ directly.
- recalls that the loss tangent is the ratio of conduction current to displacement current

Comments

None.

11.2.3 Good Dielectric Approximation

This section presents a detailed analysis to show that a material may be considered to be a good dielectric if $\frac{\sigma}{\omega\epsilon} < 0.1$.

Comments

None.

11.3 Poynting's Theorem and Wave Power

This section:

- shows how to find the power flow associated with an electromagnetic wave
- defines the Poynting vector **S** and instantaneous power density (W/m^2)
- derives Poynting's theorem
- derives an expression for time-averaged power density (W/m^2)

Comments

None.

11.4 Propagation in Good Conductors

This section considers the simplifications that occur when wave propagation occurs in a good conductor, *i.e.,* the loss tangent is very high or, equivalently, the conduction current >> the displacement current.

Comments

None.

11.4.1 Good Conductor Approximations

This section:

- notes that when the loss tangent is very high or, equivalently, the conduction current >> the displacement current, then $\alpha = \beta = \sqrt{\pi f \mu \sigma}$
- gives an expression for **J** in terms of **E**

Comments

None.

11.4.2 Skin Effect

This section:

- defines the skin depth
- gives the relationship between skin depth and the attenuation constant
- gives numerical examples at various frequencies for various materials

Comments

The authors are not particularly clear about why $\delta = \frac{1}{\alpha}$.

We can think of skin depth as the depth to which current density would penetrate into a conductor if its density was uniform and the total current remained unchanged. This suggests that

$$J_0 \, \delta = \int_0^\infty J_0 e^{-\alpha z} \, dz$$

The authors note that "all fields in a good conductor such as copper are essentially zero at distances greater than a few skin depths from the surface." In practice, we generally assume that the field is negligible at depths more than 5δ.

11.4.3 Intrinsic Impedance and Power Density in Good Conductors

This section gives expressions for the intrinsic impedance and power density that are encountered in good conductors.

Comments

None.

11.4.4 Skin Effect Resistance in Conductors

This section derives expressions for the AC or skin effect resistance of conductors with:

- a rectangular cross section
- a circular cross section

Comments

None.

11.5 Wave Polarization

This section:

- defines polarization as a description of "the manner in which the amplitude and direction of the electric field vector changes over time as observed at a fixed point in space"
- notes that the polarization of an electromagnetic wave is as fundamental as its wavelength, phase velocity and power

Comments

In the general case, the electric field vector traces an elliptical locus. Linear and circular polarization are special cases of elliptical polarization.

As we shall see in Chapter 12, the polarization of an electromagnetic wave must be accounted for when predicting reflection and transmission of the wave obliquely incident upon a boundary between two media.

In *ELEC 411 – Antennas and Propagation*, you will look at electromagnetic wave polarization in more depth and detail, and be introduced to concepts such as the Poincaré sphere, partial polarization, Stokes parameters, etc.

11.5.1 Linear Polarization

This section:

- defines linear polarization
- gives a general expression for a linearly polarized electromagnetic wave
- notes that any polarization state can be described in terms of mutually perpendicular components of the electric field and their relative phase
- notes that the mutually perpendicular components of the electric field will be in phase if the wave is linearly polarized

Comments

None.

11.5.2 Phase-Displaced Field Components: Elliptical Polarization

This section:

- defines elliptical polarization
- gives a general expression for an elliptically polarized electromagnetic wave
- notes that the mutually perpendicular components of the electric field will not be in phase if the wave is elliptically polarized

Comments

None.

11.5.3 Circular Polarization

This section:

- defines circular polarization
- gives a general expression for a circularly polarized electromagnetic wave
- introduces the notion of right and left-handed circular (and elliptical) polarization
- notes that the mutually perpendicular components of the electric field will equal in amplitude and in quadrature if the wave is circularly polarized

Comments

None.