THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W2*

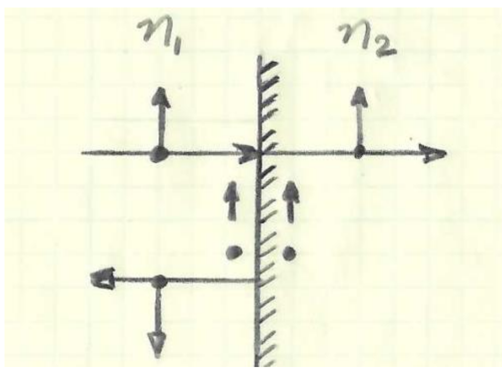
Chapter 12 – Plane Wave Reflection and Dispersion

What you need to know!

A compilation of course performance objectives with detailed enabling objectives.

Where formulas are cited, be certain that you can identify each quantity and its units and sketch figures that describe the scenario.

1. Given a plane wave normally incident on the boundary between two media with specified properties, find or derive expressions for, and determine the strength of, the transmitted and reflected waves.



- Recognize that the voltage reflection and transmission coefficients are the result of applying the following conditions:

$$E_i + E_r = E_t$$
$$H_i + H_r = H_t$$

These equalities reflect the boundary conditions at the interface.

$$\frac{E_i}{H_i} = \eta_1$$
$$\frac{E_r}{H_r} = \eta_1$$
$$\frac{E_t}{H_t} = \eta_2$$

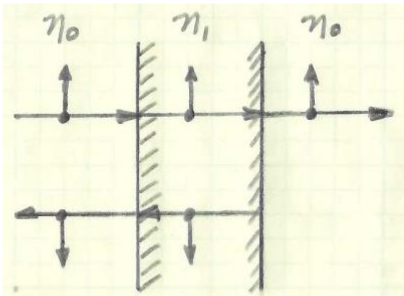
These equalities reflect the material properties of the media.

yielding the following results

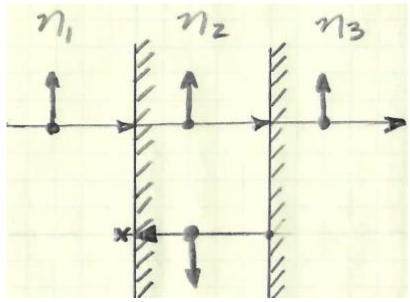
$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

These are the voltage reflection and transmission coefficients, respectively.

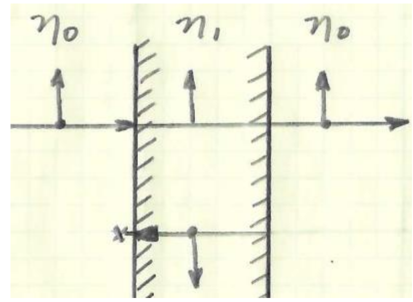
2. Given a plane wave normally incident from a first region onto a dielectric slab (a second region of finite thickness) with specified material properties, and backed by a third region with specified material properties, derive expressions for and determine the strength of the transmitted and reflected waves. Recognize the special cases of a quarter-wave and half-wave transformer.



Propagation through a dielectric slab



Propagation through a quarter-wave slab



Propagation through a half-wave slab

- Recognize that wave impedance at a given point is the vector sum of the electric field strength in the incident and reflected travelling wave divided by the vector sum of the corresponding magnetic field strengths
- Recognize that the voltage at a given point on the line for the incident and reflected waves are given by

$$E_i(z) = E_{i0}e^{-j\beta z} \text{ and } E_r(z) = E_{r0}e^{j\beta z}$$

- Recognize that the voltage reflection coefficient at a given point on the line is given by

$$\Gamma(z) = \frac{E_r(z)}{E_i(z)} = \Gamma_0 e^{j2\beta z}$$

or

$$\Gamma(\ell) = \Gamma_0 e^{-j2\beta\ell}$$

depending on whether the location is identified by the value of *z* at that point or the distance ℓ from the point to *z =0*.

- Recognize that the wave impedance is given by

$$\eta_{in}(z) = \frac{E_i(z) + E_r(z)}{H_i(z) + H_r(z)}$$

with appropriate care given to ensure that the correct sign is used for magnetic field strength, which becomes

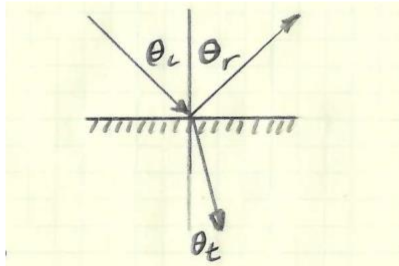
$$\eta_{in}(z) = \eta_1 \frac{\eta_0 - j\eta_1 \tan(\beta z)}{\eta_1 - j\eta_0 \tan(\beta z)}$$

or

$$\eta_{in}(\ell) = \eta_1 \frac{\eta_0 + j\eta_1 \tan(\beta\ell)}{\eta_1 + j\eta_0 \tan(\beta\ell)}$$

- Recall that $\eta_{in}\,\eta_2 = \eta_1^2$ for a quarter-wave transformer.
- Recall that $\eta_{in} = \eta_2$ for a half-wave transformer.

3. Given a TE- or TM-polarized plane wave obliquely incident on the boundary between two media with specified properties, derive expressions for and determine the strength of the transmitted and reflected waves.



- Recall that the *plane of propagation* includes the normal to the surface and the direction of propagation.
- Recall that the angles of incidence, θ_i, reflection, θ_r, and transmission θ_t are measured with respect to the normal to the surface.
- Recall the expressions for waves propagating obliquely with respect to the coordinate axes, *e.g.*,

$$\mathbf{E}_s = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$$

  where **r** corresponds to the direction of propagation and the vector **k** has magnitude equal to the phase constant β.
- Recognize that Transverse Electric or TE-polarized plane waves have the electric field vector parallel to the boundary or transverse (normal) to the plane of propagation
- Recognize that Transverse Magnetic or TM-polarized plane waves have the magnetic field vector parallel to the boundary or transverse (normal) to the plane of propagation
- Recognize that that the voltage reflection and transmission coefficients are the result of applying the conditions for normal incidence *plus* continuity of the normal components of flux density across the interface, but are otherwise derived in a similar manner

$$\frac{E_0^r}{E_0^i} = \Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\frac{E_0^t}{E_0^i} = \tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\frac{E_0^r}{E_0^i} = \Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\frac{E_0^t}{E_0^i} = \tau_\parallel = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

- Be careful not to confuse the intrinsic impedance η with the refractive index n.

- Recall Snell's laws for reflection

$$\theta_r = \theta_i$$

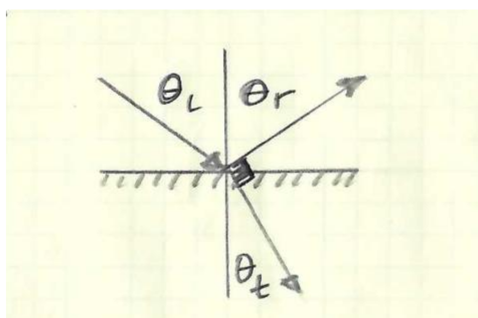
and transmission (refraction)

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{v_1}{v_2}$$

where region 1 is the region of incidence and reflection and region 2 is the region of transmission and v is the velocity of the wave in each region.

4. Demonstrate the separability of TE- and TM-polarized solutions when the problem geometry is invariant in the direction perpendicular to the plane of incidence, recognize the special cases of Brewster's angle and the critical angle, and apply these outcomes to the design of practical structures.

   - Recognize that if a problem geometry is uniform in the *y* direction, $\frac{\partial}{\partial y} = 0$ and the plane of incidence is the *x-z* plane, the equations involving H_x, H_z, E_y define a TE-polarized wave and the equations involving E_x, E_z, H_y define a TM-polarized wave
   - Recognize that for TM-polarized waves only, there exists Brewster's angle, an angle $\theta_B = \tan^{-1}\sqrt{\epsilon_2/\epsilon_1}$, for which $\Gamma = 0$
   - Recognize that for the special case of Brewster's angle, the reflected and refracted (transmitted) waves are separated by 90 degrees.



   - Recognize that for waves propagating from a dense medium into a less dense medium, there exists a critical angle of incidence, $\theta_c = \sin^{-1}\sqrt{\epsilon_2/\epsilon_1}$ where the transmitted wave propagates along the boundary rather than into the less dense medium, *i.e.*, $\theta_t = 90°$.