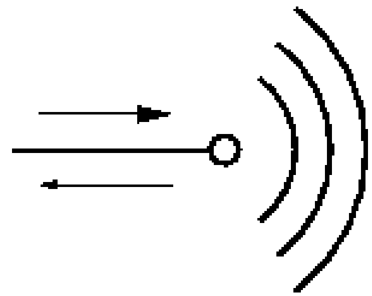

ELEC 311 - Electromagnetic Fields and Waves

Chapter 12

Plane Wave Reflection and Dispersion

Waves and propagation; Maxwell's equations; applications including transmission lines; impedance matching and Smith charts; reflection and refraction; waveguides and antennas. [4-0-0]

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1. Boundary Conditions

- What happens to the field strength and flux density when fields cross the interface between two dielectrics?
- In a source-free region, we can show that:
  - the normal component of flux density is continuous across the interface;
  - the tangential component of field strength is continuous across the interface.
- You have already seen that this can be proven fairly easily by assuming static conditions.
- The results apply equally well to time varying fields.

1 Boundary Conditions for Electric Fields

- In a charge-free region

$$D_{n1} = D_{n2} \ .$$

- If there is surface charge,

$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{n}_{12} = -\rho_s \ .$$

- In all cases,

$$E_{t1} = E_{t2} \ .$$

- In a charge-free region,

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r2}}{\epsilon_{r1}} \ .$$

- Can you prove this using the Divergence Theorem and Stokes Theorem, respectively?

2 Boundary Conditions for Magnetic Fields

- In all cases
$$B_{n1} = B_{n2} \, .$$
- If the surface is current-free,
$$H_{t1} = H_{t2} \, .$$
- If there is a current sheet,
$$(\mathbf{H}_1 - \mathbf{H}_2) \cdot \hat{n}_{12} = \mathbf{K} \, .$$
- If there is no current,
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}} \, .$$
- Can you prove this using the Divergence Theorem and Stokes Theorem, respectively?

Wave Propagation at Boundaries

1. Normal Incidence

2. Oblique Incidence and Snell's Laws

3. Perpendicular or TE Polarization

4. Parallel or TM Polarization

5. Standing Waves

1 Normal Incidence

- When a propagating wave reaches an interface between two different regions, it is partly reflected and partly transmitted.
- Application of the appropriate boundary conditions allows us to determine the transmission and reflection coefficients.
- At the boundary,

$$E_0^i + E_0^r = E_0^t. \quad H_0^i - H_0^r = H_0^t \; .$$

- Furthermore,

$$\frac{E_0^i}{H_0^i} = \eta_1, \quad \frac{E_0^r}{H_0^r} = \eta_1. \quad \frac{E_0^t}{H_0^t} = \eta_2 \; .$$

- These equations can be combined to yield:

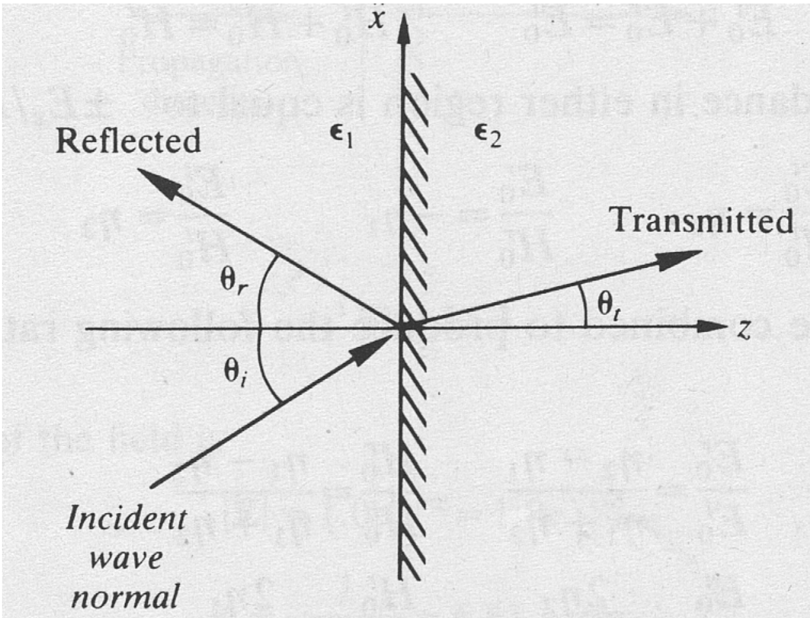
$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}, \quad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}, \quad \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2}, \quad \frac{H_0^t}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2}.$$

- As shown in §4, the intrinsic impedances for various types of media are:

free space	$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\Omega$
perfect dielectric	$\eta = \sqrt{\frac{\mu}{\epsilon}}$
partially conducting medium	$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$
conducting medium	$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

2 Oblique Incidence and Snell's Laws

- A wave incident upon a plane interface between two media will lead to a reflected wave in the first medium and a transmitted wave in the second.



- The *plane of incidence* contains the normal to the interface and the normal to the directions of incident, reflected, and transmitted wave propagation.

- Snell's law of reflection is

$$\theta_i = \theta_r \, .$$

- Snell's law of refraction is

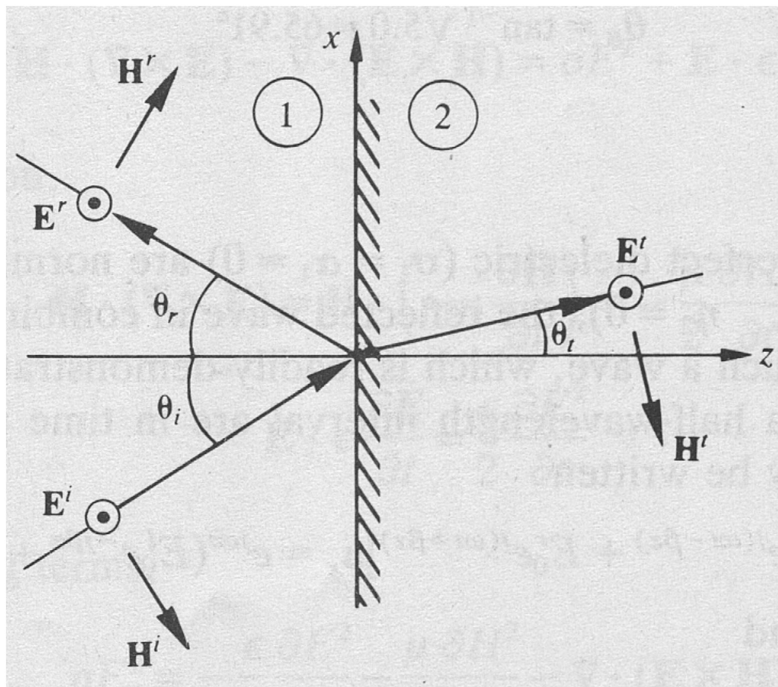
$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}} = \frac{v_1}{v_2} \, .$$

- Note this is the ratio of the speeds of light in the two media!

- What is the *critical angle of incidence* observed when a wave propagates from the denser medium into the less dense medium?

3 Perpendicular or TE Polarization

- For perpendicular or TE or s polarization, $\mathbf{E}$ is perpendicular to the plane of incidence.



- At the interface between two regions,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}\,.$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}.\,.$$

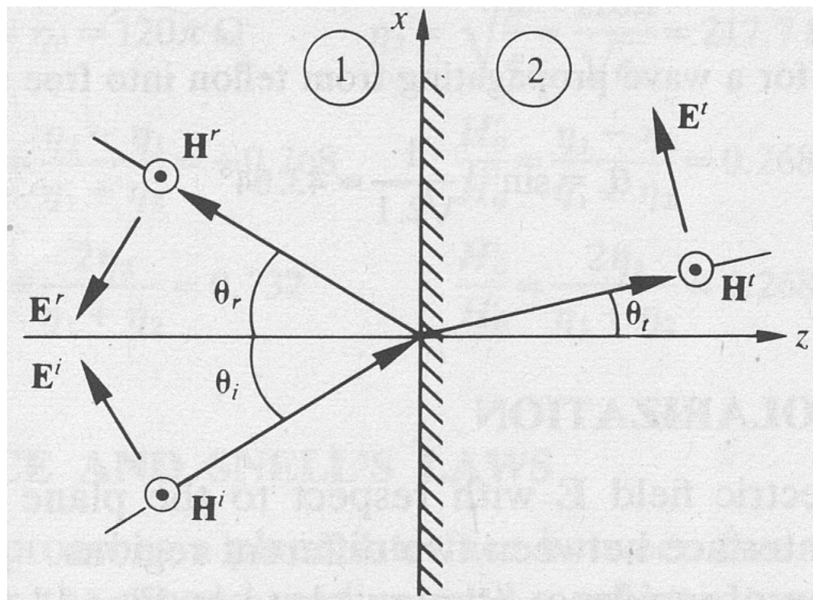
- For normal incidence, $\theta_i = \theta_t = 0^\circ$ and the expressions reduce to those of the previous section.

- Does the reflected wave ever drop to zero for TE polarization?

- $Ans.$ No. Prove this.

4 Parallel or TM Polarization

- For parallel or p or TM polarization, $\mathbf{E}$ is entirely within the plane of incidence.



- At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

  and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \; .$$

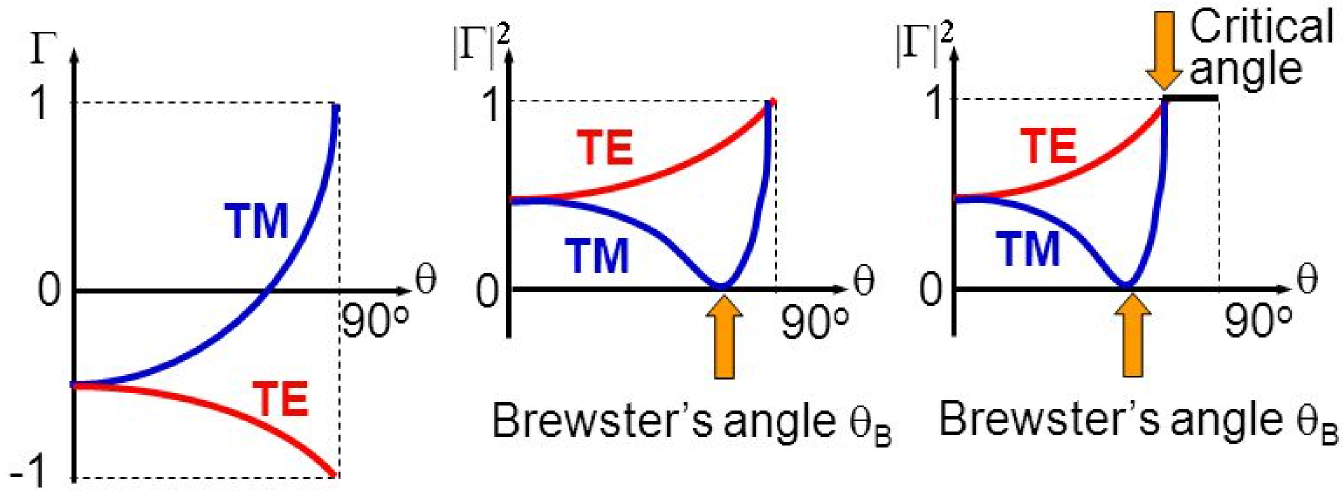
- If $\mu_1 = \mu_2$, will there be a particular angle of incidence for which there is no reflected wave?

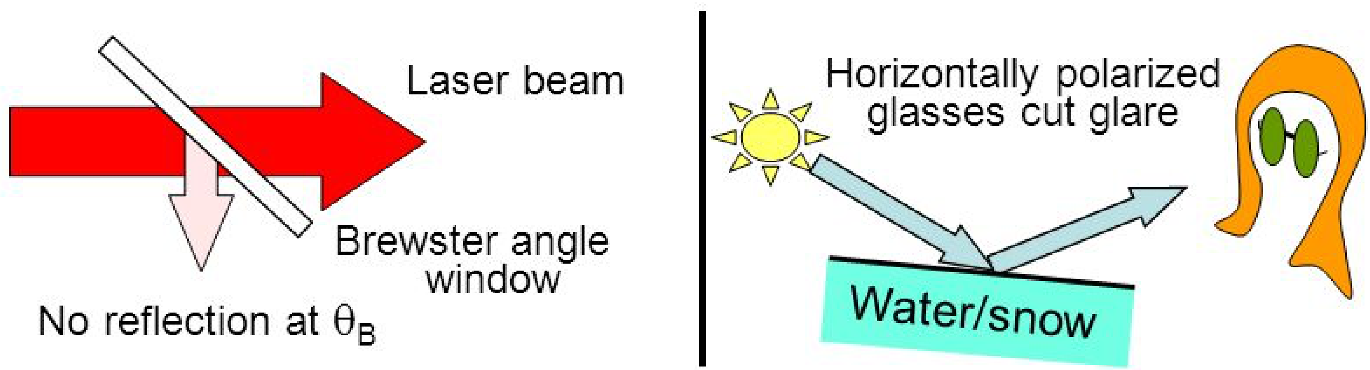
- $Ans.$ Yes, this angle is given by the Brewster angle,

$$\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} \; .$$

- Be very careful not to confuse η with n, the refractive index!

Brewster Angle (no reflection, total transmission)





Simplifications for Pure Dielectrics

- Considerable simplifications are possible for the common case of nonmagnetic dielectric media, i.e., $\mu_1 = \mu_2 = \mu_0$.
- In particular:

$$\Gamma_{TE}(E) = -\Gamma_{TE}(H) = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

and

$$\Gamma_{TM}(E) = -\Gamma_{TM}(H) = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

5 Standing Waves

- The incident and reflected waves will superimpose and create an interference pattern.
- The simplest case occurs when waves travelling through a perfect dielectric ($\sigma_1 = \alpha_1 = 0$) in the $+z$ direction are normally incident upon a perfect conductor.
- The combination of horizontally polarized incident and reflected waves may be written

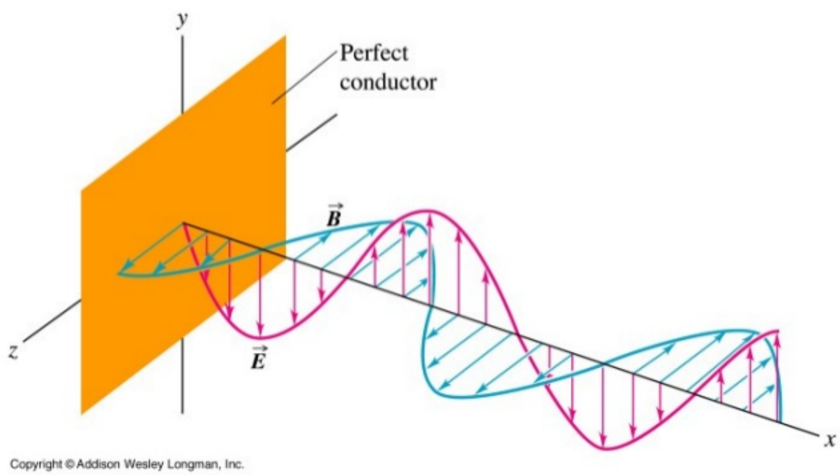
$$\mathbf{E}(z,t) = [E_0^i e^{j(\omega t - \beta z)} + E_0^r e^{j(\omega t + \beta z)}]\hat{x} = e^{j\omega t}(E_0^i e^{-j\beta z)} + E_0^r e^{j\beta z}\hat{x}.$$

- Because $\eta_2 = 0$, $E_0^r / E_0^i = -1$, and

$$\mathbf{E}(z,t) = e^{j\omega t}(E_0^i e^{-j\beta z)} - E_0^i e^{j\beta z}\hat{x} = -2jE_0^i \sin\beta z \, e^{j\omega t}\hat{x}$$

- Taking the real part yields

$$\mathbf{E}(z,t) = 2E_0^i \sin\beta z \sin\omega t\,\hat{x}.$$



A **standing** electromagnetic wave does not propagate along the x-axis; instead, at every point on the x-axis the ***E*** and ***B*** fields simply oscillate.

Wave Impedance

- At each point along the direction of propagation, the *wave impedance* is the ratio between the electric and magnetic fields.
- Because the standing waves that correspond to the electric and magnetic fields are effectively an interference pattern between the forward and backward travelling waves, the wave impedance is a periodic function with period $= \lambda/2$.

References

[1] W. H. Hayt and J. A. Buck, *Engineering Electromagnetics*, 9th ed., McGraw-Hill, 2019.

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