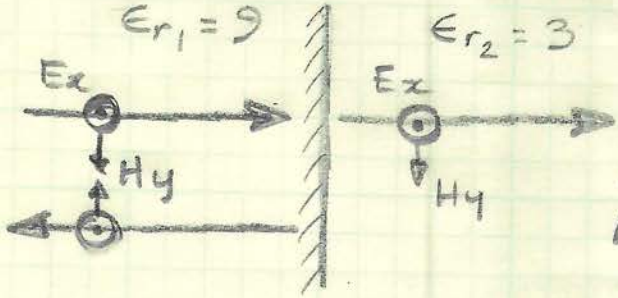
12.2 The plane $z = 0$ defines the boundary between two dielectrics. For $z < 0, \epsilon_{r1} = 9, \epsilon''_{r1} = 0$, and $\mu_1 = \mu_0$. For $z > 0, \epsilon_{r2} = 3, \epsilon''_{r2} = 0$, and $\mu_2 = \mu_0$. Let $E^+_{x1} = 10\cos(\omega t - 15z)$ V/m and find (*a*) ω; (*b*) $\langle S^+_1 \rangle$; (*c*) $\langle S^-_1 \rangle$; (*d*) $\langle S^+_2 \rangle$.



• R1 AND R2 ARE BOTH PERFECT DIELECTRICS

a.

IN R1, $\beta = \omega\sqrt{\mu\epsilon} = 15$

$\eta_1 = \frac{120\pi}{\sqrt{9}}$ $z=0$ $\eta_2 = \frac{120\pi}{\sqrt{3}}$

$\sqrt{\mu\epsilon} = \frac{\sqrt{\epsilon_R}}{c}$

$= 10^{-8}$ rad/m

$\omega = 15/\sqrt{\mu\epsilon}$

$= 15 \times 10^8$ rad/s

$= 1.5 \times 10^9$ rad/s

b.

Direction

$\langle S^+_1 \rangle$ = TIME-AVERAGED POWER OF FWD TRAVELLING WAVE IN REGION 1

Region

$\langle S^+_1 \rangle = \frac{1}{2\eta_1}|E^+_{x10}|^2 = \frac{10^2}{2 \times 40\pi} = 0.398\ W/m^2$

(in the z direction)

c.

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{\sqrt{3}} - \frac{1}{3}}{\frac{1}{\sqrt{3}} + \frac{1}{3}} = 0.2679 = \frac{E^-_{x10}}{E^+_{x10}}$

$\langle S^-_1 \rangle = \frac{1}{2\eta_1}|E^-_{x10}|^2 = \frac{2.679^2}{2 \times 40\pi} = 0.0286\ W/m^2$

(in the −z direction)

d. TWO APPROACHES

① CALCULATE τ

② CALCULATE E^+_{x20}

③ CALCULATE $\langle S^+_2 \rangle$

LOTS OF WORK!

OR

APPLY CONSERVATION OF POWER!

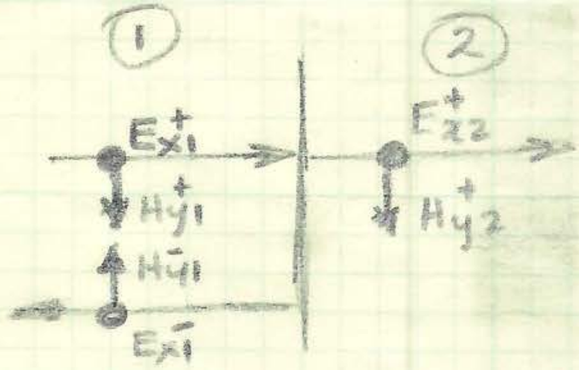
$\langle S^+_2 \rangle = \langle S^+_1 \rangle - \langle S^-_1 \rangle$

$= 0.398 - 0.0286$

$= 0.3694\ W/m^2$

(in the +z direction)

12.3 A uniform plane wave in region 1 is normally incident on the planar boundary separating regions 1 and 2. If $\epsilon_1'' = \epsilon_2'' = 0$, while $\epsilon_1' = \mu_{r1}^3$ and $\epsilon_2' = \mu_{r2}^3$, find the ratio $\epsilon_{r2}'/\epsilon_{r1}'$, if 20 percent of the energy in the incident wave is reflected at the boundary. There are two possible answers.



BECAUSE THE WAVE IS NORMALLY INCIDENT, THE RESULTS ARE POLARIZATION INDEPENDENT.

$$\epsilon_1 = \mu_{r1}^3 \qquad \epsilon_2' = \mu_{r_2}^3$$

IF $|\Gamma|^2 = 0.2$

THEN $|\Gamma| = 0.447$

$\Gamma = \pm 0.447$

$$\eta_1 = \sqrt{\frac{\mu_{r_1}}{\mu_{r_1}^3}}\,\eta_0 \qquad \eta_2 = \sqrt{\frac{\mu_{r_2}}{\mu_{r_3}^3}}\,\eta_0$$

$$= \frac{\eta_0}{\mu_{r_1}} \qquad = \frac{\eta_0}{\mu_{r_2}}$$

$$\Gamma = \pm 0.447 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{\eta_0}{\mu_{r2}} - \frac{\eta_0}{\mu_{r_1}}}{\frac{\eta_0}{\mu_{r2}} + \frac{\eta_0}{\mu_{r_1}}}$$

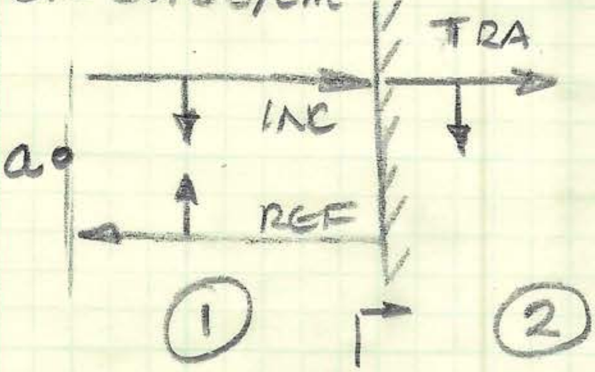
$$= \frac{\mu_{r_1} - \mu_{r2}}{\mu_{r_1} + \mu_{r_2}} = \pm 0.447$$

$$\frac{\mu_{r2}}{\mu_{r1}} = \frac{1 \mp 0.447}{1 \pm 0.447} = \{0.382, 2.62)$$

$$\frac{\epsilon_{r2}}{\epsilon_{r1}} = \left(\frac{\mu_{r2}}{\mu_{r_1}}\right)^3 = \{0.056, 17.9\}$$

12.8 A wave starts at point *a*, propagates 1 m through a lossy dielectric rated at 0.1 dB/cm, reflects at normal incidence at a boundary at which $\Gamma = 0.3 + j0.4$, and then returns to point *a*. Calculate the ratio of the final power to the incident power after this round trip, and specify the overall loss in decibels.

$\alpha = 0.1\ \text{dB/cm}$



WE AREN'T GIVEN ANY DETAILS ABOUT η_1 OR η_2 (BUT WE DON'T NEED THEM)

$\Gamma = 0.3 + j0.4$

$|\Gamma| = \sqrt{0.3^2 + 0.4^2}$

$= 0.5$

$|\Gamma|^2 = 0.25$

$\alpha = 0.1\ \text{dB/cm}$

$= 10\ \text{dB/m}$

$= 1.152\ \text{Np/m}$

DURING THE FORWARD PASSAGE THROUGH REGION 1, THE WAVE IS ATTENUATED BY 10 dB.

UPON REFLECTION, THE POWER IS REDUCED BY 6 dB

DURING THE REFLECTED PASSAGE THROUGH REGION 2, THE WAVE IS AGAIN ATTENUATED BY 10 dB.

$$10 \log \frac{P_{final}}{P_{inc}} = -10 - 6 - 10 = -26\ \text{dB}$$

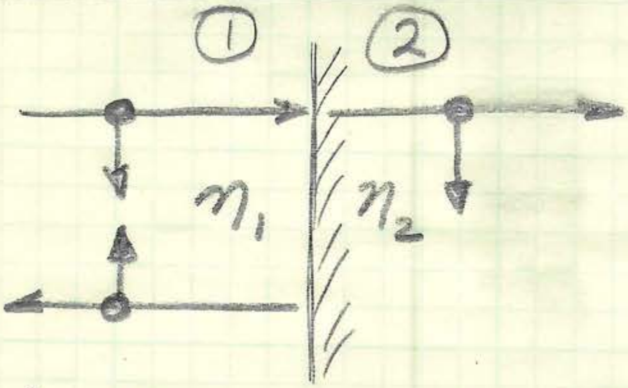
$$= \frac{1}{10} \times \frac{1}{4} \times \frac{1}{10} = \frac{1}{400}$$

$$= 2.5 \times 10^{-3}$$

$1\ \text{Np} = 8.686\ \text{dB}$

TOPS

12.9 Region 1, $z < 0$, and region 2, $z > 0$, are both perfect dielectrics ($\mu = \mu_0$, $\epsilon'' = 0$). A uniform plane wave traveling in the $\mathbf{a}_z$ direction has a radian frequency of 3×10^{10}rad/s. Its wavelengths in the two regions are $\lambda_1 = 5$ cm and $\lambda_2 = 3$ cm. What percentage of the energy incident on the boundary is (*a*) reflected; (*b*) transmitted? (*c*) What is the standing wave ratio in region 1?



BOTH MEDIA ARE LOSSLESS.

$\omega = 3\times10^{10}$ rad/s

$\lambda_1 = 5$cm $\qquad \lambda_2 = 3$ cm

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

FOR LOSS LESS MEDIA

$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{c}{\omega}\,\frac{2\pi}{\lambda} = \frac{3\times10^{8}}{3\times10^{10}} \times \frac{2\pi}{\lambda}$$

λ	$\sqrt{\epsilon_r}$	ϵ_r
3 cm	$2\pi/3$	4.39
5 cm	$2\pi/5$	1.55

$$\eta_2 = \frac{120\pi \times 3}{2\pi} = 180\,\Omega$$

$$\eta_1 = \frac{120\pi \times 5}{2\pi} = 300\,\Omega$$

$$\Gamma = \frac{180-300}{180+300} = \frac{-120}{480} = -\frac{1}{4}$$

a.

$$|\Gamma|^2 = \frac{1}{16} = 6.25\times10^{-2} = 6.25\%$$

TOPS

b.

$$|\tau|^2 = 1 - |\Gamma|^2 = 93.75 \times 10^{-2}$$
$$= 93.75\%$$

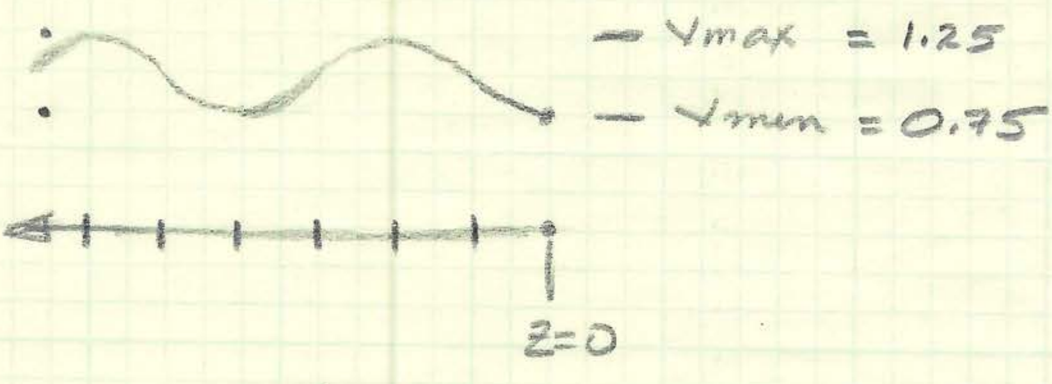
C.

$$s = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{1 + 1/4}{1 - 1/4}$$

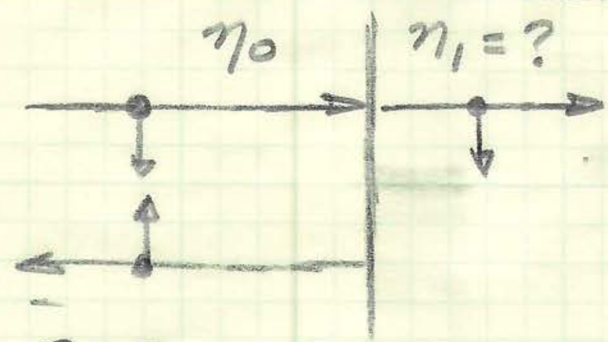
$$= 5/3$$

$$= 1.67$$



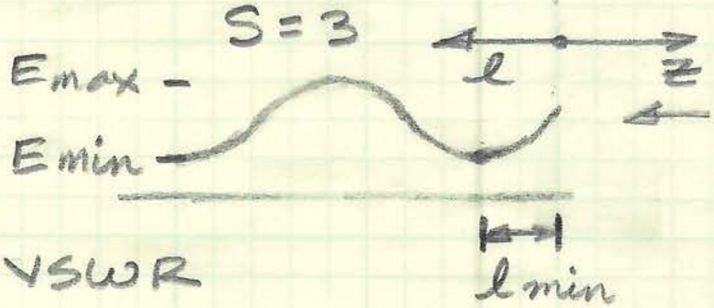
More on VSWR = S
when we study Chapter 10

12.11 A 150-MHz uniform plane wave is normally incident from air onto a material whose intrinsic impedance is unknown. Measurements yield a standing wave ratio of 3 and the appearance of an electric field minimum at 0.3 wavelengths in front of the interface. Determine the impedance of the unknown material.



$$z_{min} = -0.3\lambda$$
$$= -\frac{1}{2\beta_1}(\varphi + \pi) \qquad (25)$$

$S = 3$



← STANDING WAVE

VSWR

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$S - S|\Gamma| = 1 + |\Gamma|$$

$$S - 1 = |\Gamma|(1+S)$$

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1}$$
$$= 0.5$$

OBSERVATIONS

1. AT E_{max} AND E_{min}, Γ IS REAL

EVERYWHERE ELSE IT IS COMPLEX.

2. $E_{max} = E_o(1+|\Gamma|)$

$E_{min} = E_o(1-|\Gamma|)$

$$E_i(z) = E_o e^{-j\beta z}$$
$$E_r(z) = \Gamma E_o e^{+j\beta z}$$

$$\Gamma(z) = \frac{E_r}{E_i} = \Gamma e^{j2\beta z}$$
$$= \Gamma e^{-j2\beta l}$$

Γ AS A FUNCTION OF z

STRATEGY

1. FIND $|\Gamma|$
2. FIND φ
3. FIND $\Gamma(0) \equiv \Gamma$
4. $\Gamma(0) = \dfrac{\eta_1 - \eta_o}{\eta_1 + \eta_o}$
5. $\eta_1 = \dfrac{1+\Gamma}{1-\Gamma}\eta_o$

TOPS

DETAILS

(1) $|\Gamma| = 0.5$

(2) $$z_{min} = -\frac{1}{2\beta}(\varphi + \pi) = -0.3\lambda$$

$$= -\frac{\lambda}{4\pi}(\varphi + \pi) = -0.3\lambda$$

$$\frac{\varphi}{4\pi} + \frac{1}{4} = 0.3 \qquad \varphi = 4\pi(0.3 - 0.25) = 0.2\pi$$

(3) $$\Gamma(0) = 0.5\, e^{j0.2\pi}$$

(4) $$\Gamma(0) = 0.5\, e^{j0.2\pi} = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0}$$

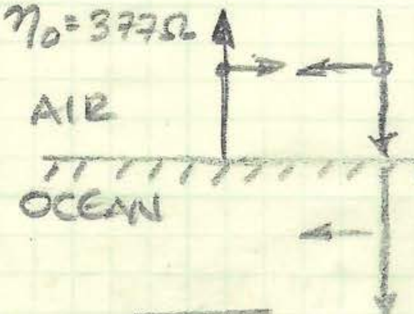
$$\Gamma(0)(\eta_1 + \eta_0) = \eta_1 - \eta_0$$

$$\Gamma(0)\eta_0 + \eta_0 = \eta_1 - \Gamma(0)\eta_1$$

$$\frac{1 + \Gamma(0)}{1 - \Gamma(0)}\eta_0 = \eta_1 = 641 + j503\ \Omega$$

TOPS

12.12 A 50-MHz uniform plane wave is normally incident from air onto the surface of a calm ocean. For seawater, $\sigma = 4$ S/m, and $\epsilon_r' = 78$. (*a*) Determine the fractions of the incident power that are reflected and transmitted. (*b*) Qualitatively, how (if at all) will these answers change as the frequency is increased?



$\epsilon_R = 1$

$\sigma = 0$ S/m

$\frac{\sigma}{\omega\epsilon} = 0$ PERFECT DIELECTRIC

$\epsilon_r = 78$

$\sigma = 4$

$\frac{\sigma}{\omega\epsilon} = 18.4$ GOOD CONDUCTOR

$$= \frac{4}{2\pi(50\times10^6)78\,(8.854\times10^{-12})}$$

$$\eta_1 = \sqrt{\frac{\pi f \mu}{\sigma}}(1+j)$$

(GOOD CONDUCTOR APPROXIMATION)

$$\Gamma = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} = \frac{\sqrt{\pi f \mu/\sigma}\,(1+j) - \eta_0}{\sqrt{\pi f \mu/\sigma}\,(1+j) + \eta_0}$$

$$\sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\pi \times 50\times10^6 \times 4\pi\times10^{-7}}{4}} = 7.02$$

$\eta_1 \ll \eta_0$ SO THE REFLECTION COEFFICIENT WILL APPROACH -1 BUT WITH A PHASE SHIFT.

a.)

$$\Gamma = \frac{7.02(1+j) - 377}{7.02(1+j) + 377} = -0.9628 + j0.0359$$

$|\Gamma|^2 = 0.928$ = FRACTION OF INCIDENT POWER THAT IS REFLECTED

$|\tau|^2 = 1 - |\Gamma|^2 = 0.072$ = FRACTION OF INCIDENT POWER THAT IS TRANSMITTED

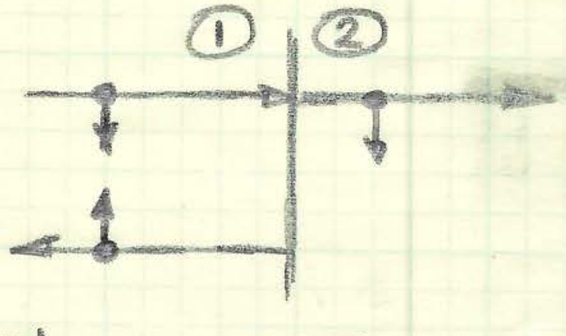
---

b) AS ω INCREASES, THE LOSS TANGENT WILL DECREASE AND η_1 WILL INCREASE.

AS η_1 INCREASES, Γ WILL DECREASE AND $|\Gamma|^2$ WILL DECREASE.

12.13 A right-circularly polarized plane wave in air is normally incident from air onto a semi-infinite slab of Plexiglas ($\epsilon_r' = 3.45, \epsilon_r'' = 0,$). Calculate the fractions of the incident power that are reflected and transmitted. Also, describe the polarizations of the reflected and transmitted waves.

cf. PROBLEM 12.21 WHERE THE WAVE IS OBLIQUELY INCIDENT.



THE RCP WAVE CAN BE RESOLVED INTO TE AND TM POLARIZED COMPONENTS BUT Γ & τ WILL BE THE SAME IN BOTH CASES WHEN INCIDENCE IS NORMAL

$\epsilon_{r_1} = 1$ $\qquad \epsilon_{r_2} = 3.45$

$\eta_1 = 120\pi\,\Omega$ $\qquad \eta_2 = \dfrac{120\pi}{\sqrt{3.45}}$

BY INSPECTION:

(1) THE RCP INCIDENT WAVE WILL SPLIT INTO TRANSMITTED AND REFLECTED COMPONENTS

(2) THE TRANSMITTED WAVE WILL ALSO BE RCP BUT WITH REDUCED INTENSITY (SAME TRANSMISSION COEFFIENT (AMPLITUDE & PHASE) FOR BOTH THE TE & TM COMPONENTS SO NO CHANGE IN POLARIZATION STATE.

(3) THE REFLECTED WAVE WILL ALSO BE CP BUT WITH THE OPPOSITE SENSE, I.e., LEFT CIRCULARLY POLARIZED.

THIS IS REQUIRED TO SATISFY BOUNDARY CONDITIONS!

THE ELECTRIC FIELD VECTOR WILL ROTATE IN THE SAME DIRECTION AS BEFORE, BUT THE DIRECTION OF PROPAGATION WILL BE REVERSED.

AS A RESULT, THE POLARIZATION SENSE WILL BE REVERSED AS BOTH ROTATION DIRECTION AND PROPAGATION DIRECTION MUST BE ACCOUNTED FOR WHEN APPLYING THE RIGHT/LEFT HANDED RULE.

FRACTIONS OF THE INCIDENT POWER THAT ARE REFLECTED AND TRANSMITTED.

$$\frac{P_{refl}}{P_{inc}} = |\Gamma|^2 \qquad \frac{P_{tran}}{P_{inc}} = |\tau|^2$$

(AS OBSERVED AT THE INTERFACE.)

$$= 1 - |\Gamma|^2$$

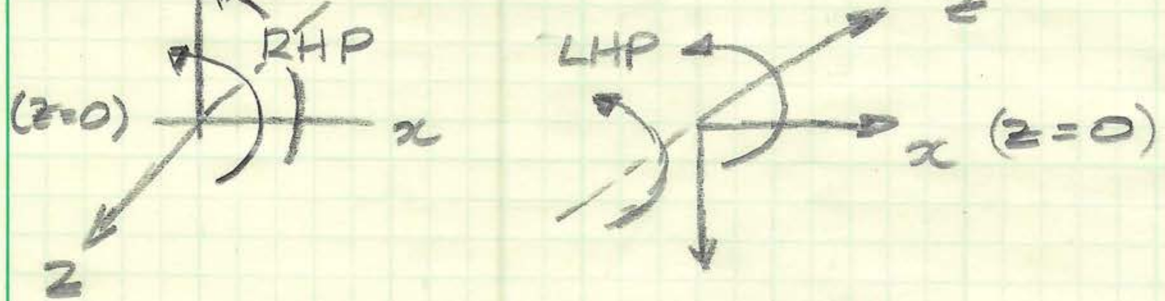
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{120\pi}{\sqrt{3.45}} - 120\pi}{\frac{120\pi}{\sqrt{3.45}} + 120\pi}$$

$$= \frac{\frac{1}{\sqrt{3.45}} - 1}{\frac{1}{\sqrt{3.45}} + 1} = -0.3001$$

$$|\Gamma|^2 = 0.09 = 9\%$$

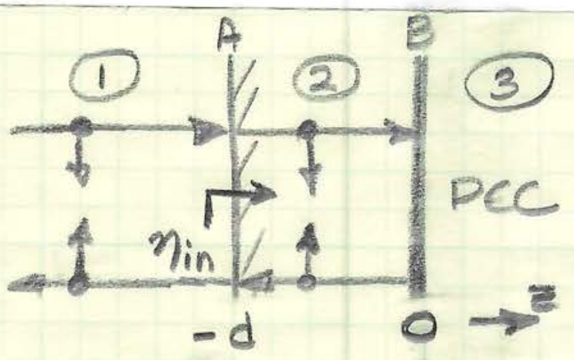
$$1 - |\Gamma|^2 = 0.91 = 91\%$$

APPLY RIGHT AND LEFT-HAND RULE AS APPROPRIATE.



SAME DIRECTION OF ROTATION BUT OPPOSITE POLARIZATION SENSE.

12.18 A uniform plane wave is normally incident onto a slab of glass ($n = 1.45$) whose back surface is in contact with a perfect conductor. Determine the reflective phase shift at the front surface of the glass if the glass thickness is (*a*) $\lambda/2$; (*b*) $\lambda/4$; (*c*) $\lambda/8$.



THIS IS A TWO-INTERFACE PROBLEM WHERE THE INTRINSIC IMPEDANCE OF REGION 3

$$\eta_3 = 0\ \Omega$$

$n_1 = 1 = \sqrt{\epsilon_{r_2}}$ $\quad n_2 = 1.45 = \sqrt{\epsilon_{r_2}}$

$$\eta_1 = 120\pi \quad \eta_2 = \frac{120\pi}{1.45} \quad \eta_3 = 0\ \Omega$$

FROM CONSERVATION OF POWER, WE ALREADY KNOW THAT

$$|\Gamma|^2 = 1.0$$

BECAUSE ALL POWER IS REFLECTED FROM THE PEC AND REGIONS 1 AND 2 ARE BOTH LOSSLESS.

HOWEVER, THE PHASE OF Γ @ INTERFACE A BETWEEN REGIONS 1 AND 2 IS UNKNOWN.

FIRST, THE 'INTUITIVE' ANSWERS:

a. • HALF-WAVE TRANSFORMER

IF $z = -d = \frac{\lambda}{2}$, $\eta_{in} = \eta_3 = 0\ \Omega$

AND $\Gamma = -1$ $\quad \Gamma = \frac{0 - \eta_0}{0 + \eta_0} = -1$

IF $z = -d = \frac{\lambda}{4}$, $\eta_{in} = \frac{1}{\eta_3} = \infty\ \Omega$ (OPEN CIRCUIT)

AND $\Gamma = 1$ $\quad \Gamma = \frac{\infty - \eta_0}{\infty + \eta_0} = 1$ $\quad$ BUT $\eta_2 \neq \sqrt{\eta_1 \eta_3}$

b. • QUARTER-WAVE TRANSFORMER

(IS THIS RESULT TRUE?)

SECOND, THE GENERAL ANSWER.

- INTRINSIC IMPEDANCE IS THE RATIO OF E/H FOR A SINGLE WAVE PROPAGATING IN A GIVEN MEDIUM

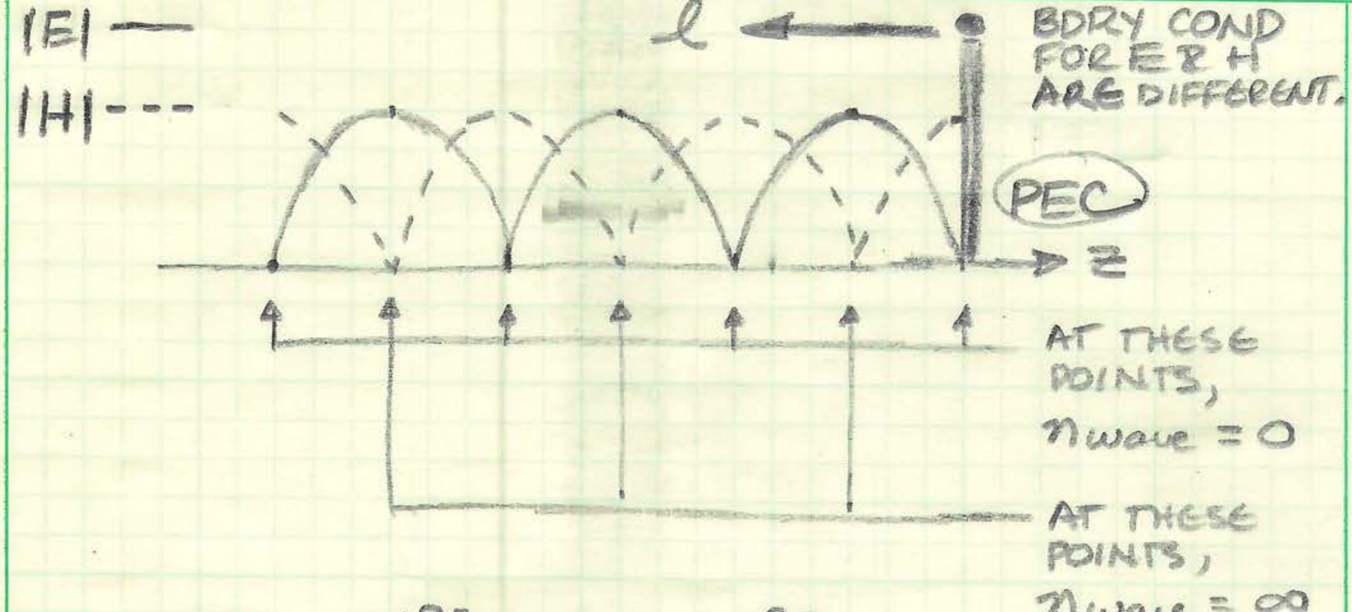
- WAVE IMPEDANCE IS THE RATIO $E_{fwd} + E_{refl}$ TO $H_{fwd} + H_{refl}$ WHEN BOTH FORWARD TRAVELLING AND REFLECTED WAVES ARE PRESENT.

- BECAUSE FORWARD TRAVELLING WAVES EVOLVE AS $\exp(-j\beta z)$ AND REFLECTED WAVES EVOLVE AS $\exp(+j\beta z)$, THEY SUPERIMPOSE TO FORM A STANDING WAVE

- BECAUSE THE STANDING WAVES FOR THE ELECTRIC AND MAGNETIC FIELD ARE GENERALLY NOT IN PHASE WITH EACH OTHER, THE RATIO

$$\eta_{wave} = \frac{E_{0+} e^{-j\beta z} + E_{0-} e^{+j\beta z}}{H_{0+} e^{-j\beta z} + H_{0-} e^{-j\beta z}}$$

WILL VARY RAPIDLY WITH CHANGES IN z.



$$E(z) = E_o e^{-j\beta z} - E_o e^{+j\beta z}$$

$$= -2j\, E_o \sin \beta z$$

$$H(z) = H_0 e^{-j\beta z} + H_0 e^{+j\beta z}$$

$$= 2 H_0 \cos \beta z$$

$$\eta_{wave} = \frac{E(z)}{H(z)} = -j \frac{E_0}{H_0} \tan \beta z$$

$$= -j\, \eta \tan \beta z$$

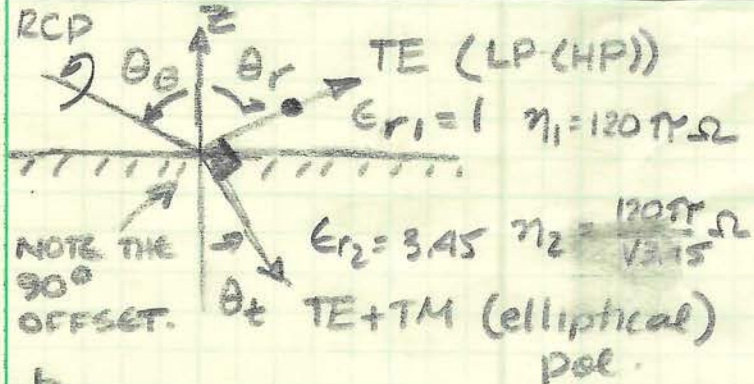
IF WE MEASURE THE DISTANCE BACKWARDS l RATHER THAN THE Z-COORDINATE,

$$\eta_{wave} = j\, \eta \tan \beta l$$

WHERE η IS THE INTRINSIC IMPEDANCE OF THE MEDIUM.

NEXT STEPS?

12.21 A right-circularly polarized plane wave in air is incident at Brewster's angle onto a semi-infinite slab of Plexiglas ($\epsilon_r' = 3.45, \epsilon_r'' = 0$). (*a*) Determine the fractions of the incident power that are reflected and transmitted. (*b*) Describe the polarizations of the reflected and transmitted waves.



WITHOUT DOING ANY CALCULATIONS, WE CAN DRAW THE SKETCH ON THE LEFT AND DRAW THE FOLLOWING CONCLUSIONS:

b.

- THE RCP INCIDENT WAVE CAN BE DECOMPOSED INTO TE AND TM POLARIZED COMPONENTS OF EQUAL MAGNITUDE THAT ARE OFFSET IN PHASE BY 90°
- IF THE INTERFACE IS IN THE HORIZONTAL PLANE, AND $\hat{z}$ IS VERTICAL, THEN
  - TE (TRANSVERSE ELECTRIC) → HORIZONTAL POL. p-polarization
  - TM (TRANSVERSE MAGNETIC) → VERTICAL POL. s-polarization
- AT BREWSTER'S ANGLE, TM POLARIZATION IS COMPLETELY TRANSMITTED (NO REFLECTION)
- THUS, THE REFLECTED WAVE IS TE-POLARIZED
- THUS, THE TRANSMITTED WAVE HAS BOTH TM & TE POLARIZED COMPONENTS AND IS ELLIPTICALLY POLARIZED

a.

RCP → TE → REF ? $= |\Gamma_{TE}|^2$

RCP → TE → TRA ? $= |\Gamma_{TM}|^2$

RCP → TM → REF 0% $= |\Gamma_{TM}|^2$

RCP → TM → TRA 100% $= |\tau_{TM}|^2$

$REF = |\Gamma|^2$

$TRA = |\tau|^2$

EQUAL POWER

(SEE ABOVE)

(SEE ABOVE)

SUM TO 100%

THE FRACTION OF INCIDENT POWER THAT IS REFLECTED IS

$$\frac{|\Gamma_{TE}|^2 + \cancel{|\Gamma_{TM}|^2}^{\,0}}{2} = \frac{|\Gamma_{TE}|^2}{2} \leftarrow TE = s\text{-polarization}$$

THE FRACTION OF INCIDENT POWER THAT IS TRANSMITTED IS

$$\frac{|\tau_{TE}|^2 + |\tau_{TM}|^2}{2} = \underbrace{1 - \frac{|\Gamma_{TE}|^2}{2}}_{\text{CONSERVATION OF POWER}}$$

TM = p-polarization

(WE WOULD, HOWEVER, NEED TO CALCULATE τ_{TE} AND τ_{TM} IF WE WANTED TO ESTIMATE THE POLARIZATION STATE OF THE TRANSMITTED WAVE BUT THAT'S AN <u>ELEC 411</u> TOPIC.)

$$\theta_B = \arctan\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \sqrt{3.45} = 61.7^\circ$$

$$= \theta_i = \theta_r \qquad \text{BREWSTER'S ANGLE}$$

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$= \frac{\frac{1}{\sqrt{3.45}} \times 0.4741 - 0.8805}{\frac{1}{\sqrt{3.45}} \times 0.4741 + 0.8805}$$

$$= -0.5505$$

$$\frac{|\Gamma_{TE}|^2}{2} = 0.1505 = 15\%$$

$$1 - \frac{|\Gamma_{TE}|^2}{2} = 0.8495 = 85\%$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin\theta_t = \sin\theta_i \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

$$= \frac{0.8805}{\sqrt{3.45}}$$

$$= 0.4740$$

$$\theta_t = 28.3^\circ$$

$$\cos\theta_t = 0.8805$$

$$\cos\theta_i = 0.4741$$

$$\frac{\eta_1}{\eta_0} = 1 \quad \frac{\eta_2}{\eta_0} = \frac{1}{\sqrt{3.45}}$$