THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W1

Example Problems
Chapter 12 – Plane Wave Reflection and Dispersion

*The purpose of these nine example problems is to help you master some of fundamental techniques used to analyze the behaviour of plane waves at boundaries between material media. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count.*

12.2   The plane $z = 0$ defines the boundary between two dielectrics. For $z < 0$, $\epsilon_{r1} = 9$, $\epsilon_{r1}'' = 0$, and $\mu_1 = \mu_0$. For $z > 0$, $\epsilon_{r2} = 3$, $\epsilon_{r2}'' = 0$, and $\mu_2 = \mu_0$. Let $E_{x1}^+ = 10 \cos(\omega t - 15z)$ V/m and find (a) $\omega$; (b) $\langle S_1^+ \rangle$; (c) $\langle S_1^- \rangle$; (d) $\langle S_2^+ \rangle$.

Given: A description of the problem geometry, the constitutive parameters of the materials, and a partial description of the incident wave.

Sought: (a) $\omega$; (b) $\langle S_1^+ \rangle$; (c) $\langle S_1^- \rangle$; (d) $\langle S_2^+ \rangle$.

Strategy:  1. Recognize that for a lossless material, velocity $v$ can be calculated in terms of permittivity and permeability, or in terms of ω and β.
2. Recognize that $\beta$ here is 15 and that knowing β and $v$ is enough to calculate ω.
3. Recognize that given η in each region, one can calculate Γ.
4. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.

*12.3  A uniform plane wave in region 1 is normally incident on the planar boundary separating regions 1 and 2. If $\epsilon_1'' = \epsilon_2'' = 0$, while $\epsilon_1' = \mu_{r1}^3$ and $\epsilon_1' = \mu_{r2}^3$, find the ratio $\epsilon_{r2}'/\epsilon_{r1}'$, if 20 percent of the energy in the incident wave is reflected at the boundary. There are two possible answers.

Given: A description of the problem geometry and the fraction of energy reflected from the boundary.

Sought: the ratio $\epsilon_{r2}'/\epsilon_{r1}'$

Strategy:  1. Recognize that given η in each region, one can calculate Γ.
2. Recognize that given μ and $\epsilon$, one can calculate η.
3. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.
4. Set up the equations that relate the unknown quantities and solve using algebra.

12.8 A wave starts at point $a$, propagates 1 m through a lossy dielectric rated at 0.1 dB/cm, reflects at normal incidence at a boundary at which $\Gamma = 0.3 + j0.4$, and then returns to point $a$. Calculate the ratio of the final power to the incident power after this round trip, and specify the overall loss in decibels.

Given: A description of the problem geometry, attenuation in the first media, and the reflection coefficient at the boundary.

Sought: The ratio of the final power to the incident power, expressed in dB.

Strategy: 1. Recognize that loss has three components: loss while travelling in the forward direction, return loss at the boundary $= 1 - |\Gamma|^2$, and loss while travelling in the backward direction.
2. Recall how to calculate total attenuation given attenuation in dB/cm and length in cm.
3. Recall how to express power ratios in dB.

12.9 Region 1, $z < 0$, and region 2, $z > 0$, are both perfect dielectrics ($\mu = \mu_0$, $\epsilon'' = 0$). A uniform plane wave traveling in the $\mathbf{a}_z$ direction has a radian frequency of $3 \times 10^{10}$ rad/s. Its wavelengths in the two regions are $\lambda_1 = 5$ cm and $\lambda_2 = 3$ cm. What percentage of the energy incident on the boundary is (a) reflected; (b) transmitted? (c) What is the standing wave ratio in region 1?

Given: A description of the problem geometry, the constitutive parameters of the materials, and a description of the incident wave.

Sought: The percentage of the energy incident on the boundary is (a) reflected; (b) transmitted. (c) The standing wave ratio in region 1?

Strategy: 1. Recognize that given $\lambda$, one can calculate $\beta$.
2. Recognize that given $\beta$ and $\omega$, one can calculate $v$.
3. Recognize that given $v$ and $\mu$, one can calculate $\epsilon$.
4. Recognize that given $\mu$ and $\epsilon$, one can calculate $\eta$.
5. Recognize that given $\eta$ in each region, one can calculate $\Gamma$.
6. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.
7. Recall the relationship between $s$ and $|\Gamma|$.

12.11   A 150-MHz uniform plane wave is normally incident from air onto a material whose intrinsic impedance is unknown. Measurements yield a standing wave ratio of 3 and the appearance of an electric field minimum at 0.3 wavelengths in front of the interface. Determine the impedance of the unknown material.

Given: A description of the problem geometry, the incident wave, and the standing wave.

Sought: The impedance of the unknown material

Strategy:  1. Recall the relationship between $s$ and $|\Gamma|$.
2. Recall the relationship between $\Gamma$ and distance.
3. Recognize that given $\Gamma$ and $\eta$ in one region, one can calculate $\eta$ in the other region.

12.12   A 50-MHz uniform plane wave is normally incident from air onto the surface of a calm ocean. For seawater, $\sigma = 4$ S/m, and $\epsilon_r'=78$. ($a$) Determine the fractions of the incident power that are reflected and transmitted. ($b$) Qualitatively, how (if at all) will these answers change as the frequency is increased?

Given: A description of the problem geometry, the constitutive parameters of the materials, and a description of the incident wave.

Sought: The fractions of the incident power that are reflected and transmitted, and the trend as frequency increases.

Strategy:  1. Recall the expression for the intrinsic impedance of a lossy dielectric.
2. Recognize that given $\eta$ in each region, one can calculate $\Gamma$.
3. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.
4. Recognize how the intrinsic impedance of a lossy dielectric depends on $\sigma$, $\epsilon$, and $\omega$.

12.13  A right-circularly polarized plane wave in air is normally incident from air onto a semi-infinite slab of Plexiglas ($\epsilon_r' = 3.45, \epsilon_r'' = 0,$). Calculate the fractions of the incident power that are reflected and transmitted. Also, describe the polarizations of the reflected and transmitted waves.

Given: A description of the problem geometry, the constitutive parameters of the materials, and a description of the incident wave.

Sought: The fractions of the incident power that are reflected and transmitted, and the polarizations of the reflected and transmitted waves.

Strategy:  1. Recognize that the right circularly polarized wave can be decomposed into TE and TM components that are equal in magnitude and shifted in phase by 90 degrees. At normal incidence only, their reflection and transmission coefficients are calculated using the same expressions.
   2. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.
   3. Recognize that the reflected and transmitted waves will have equal TE and TM components so will both be circularly polarized.
   4. Recognize that the transmitted wave will have the same sense, but the reflected wave will have the opposite sense.

*12.18  A uniform plane wave is normally incident onto a slab of glass ($n = 1.45$) whose back surface is in contact with a perfect conductor. Determine the reflective phase shift at the front surface of the glass if the glass thickness is (a) $\lambda/2$; (b) $\lambda/4$; (c) $\lambda/8$.

Given: A description of the problem geometry, the constitutive parameters of the materials, and a description of the incident wave.

Sought: The reflective phase shift at the front surface of the glass if the glass thickness is (a) $\lambda/2$; (b) $\lambda/4$; (c) $\lambda/8$.

Strategy:  1. Recognize the relationship between refractive index and permittivity.
   2. Recognize that given $\mu$ and $\epsilon$, one can calculate $\eta$.
   3. Recognize that $|\Gamma| = 1$.
   4. Recall the relationship between $\Gamma$ and distance.

12.21 A right-circularly polarized plane wave in air is incident at Brewster's angle onto a semi-infinite slab of Plexiglas ($\epsilon_r' = 3.45, \epsilon_r'' = 0$). (*a*) Determine the fractions of the incident power that are reflected and transmitted. (*b*) Describe the polarizations of the reflected and transmitted waves.

Given: A description of the problem geometry, the constitutive parameters of the materials, and a description of the incident wave.

Sought: The fractions of the incident power that are reflected and transmitted, and the polarizations of the reflected and transmitted waves.

Strategy: 1. Recognize that the right circularly polarized wave can be decomposed into TE and TM components that are equal in magnitude and shifted in phase by 90 degrees. Their angles of reflection and transmission (refraction) are identical but their reflection and transmission coefficients must be calculated separately.
2. Recognize that the fractions of the incident power that are reflected and transmitted are given by $|\Gamma|^2$ and by $1 - |\Gamma|^2$, respectively.
3. Recognize that if the wave is incident at Brewster's angle, the TM component is completed transmitted so the reflected wave has only a TE component and will be linearly polarized. The transmitted wave will have unequal TE and TM components so will be elliptically polarized.