THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W1

Strategies for the Drill Problems for Chapter 12 – Plane Wave Reflection and Dispersion

The purpose of the six drill problems from Chapter 12 is to help you master fundamental techniques used to analyze plane wave reflection and dispersion.

Answers should be short and to the point. It is especially critical to use sketches to explain your solution here. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

The numerical answers below are from the text. Do you agree with them?

D12.1 A 1-MHz uniform plane wave with E = 1 V/m is normally incident onto a fresh water lake (ϵ_r' = 78, ϵ_r'' = 0, μ_r = 1). Determine the fraction of the incident power that is (*a*) reflected and (*b*) transmitted. (*c*) Determine the amplitude of the electric field that is transmitted into the lake.

Strategy

Given: The frequency and amplitude of a uniform plane wave, and the constitutive parameters of regions 1 and 2

Sought: The fraction of the incident power that is (*a*) reflected and (*b*) transmitted, and (*c*) the amplitude of the electric field that is transmitted into the lake.

Steps:

1. Determine the intrinsic impedance in each region.
2. Calculate Γ at z = 0 in terms of η_1 and η_2.
3. Calculate $|\Gamma|^2$ and $1 - |\Gamma|^2$.
4. Recognize that $E_t = E_i + E_r$ at z = 0 and $\Gamma = E_r/E_i$.
5. Solve for E_t

Consilium est demonstratum.

Answers: (*a*) 0.63; (*b*) 0.37; (*c*) 0.20 V/m

D12.2 What value of s results when $\Gamma = \pm 1/2$

Strategy

Given: Γ

Sought: s

Steps: Recall the relationship between $|\Gamma|$ and s, (and V_{max} and V_{min}, and V_{inc} and V_{ref}.)

Consilium est demonstratum.

Answer: 3

D12.3 A uniform plane wave in air is normally incident on a dielectric slab of thickness $\lambda_2/4$ and intrinsic impedance $\eta_2 = 260\ \Omega$. Determine the magnitude and phase of the reflection coefficient.

Strategy

Given: The thickness in wavelengths and intrinsic impedance of a dielectric slab

Sought: The magnitude and phase of the reflection coefficient at the first interface.

Steps:

1. Recognize the unique relationship between the intrinsic impedance of the slab, the intrinsic impedance of the third region, and η_{in} the wave impedance at $z = 0$ when the slab is a quarter-wave section.
2. Find Γ at $z = 0$ in terms of η_1 and η_{in}.

Alternatively,

1. Find η_{in}, the wave impedance at $z = 0$, in terms of η_2, η_3 and βl
2. Find Γ at $z = 0$ in terms of η_1 and η_{in}.

Alternatively,

1. Find Γ at $z = \lambda_2/4$
2. Find Γ at $z = 0+$
3. Find η_{in}, the wave impedance at $z = 0$, in terms of Γ and η_2
4. Find Γ at $z = 0$ in terms of η_1 and η_{in}.

Consilium est demonstratum.

Answers: 0.356; 180°

D12.4 Consider a 50-MHz uniform plane wave having electric field amplitude 10 V/m. The medium is lossless, having $\epsilon_r = \epsilon_r' = 9.0$ and $\mu_r = 1.0$. The wave propagates in the x, y plane at a 30° angle to the x axis and is linearly polarized along z. Calculate λ_x, λ_y, v_{px}, and v_{py}.

Strategy

Given: The frequency of a uniform plane wave with

Sought: λ_x, λ_y, v_{px}, and v_{py}.

Steps:

1. Find the *magnitude* of *k.*
2. Find the *direction* of k such that the conditions in the problem are met.
3. Recognize that $\lambda = 2\pi/\beta$ and $v = \omega/\beta$ in each direction.

Consilium est demonstratum.

Answers: 2.2 m; 3.9 m; 1.1×10^8 m/s; 2.0×10^8 m/s

D12.5 Light is incident from air to glass (refractive index = 1.45) at Brewster's angle. Determine the incident and transmitted angles. Calculate the reflection coefficient for s-polarized light.

Strategy

Given: the refractive index of glass, angle of incidence is Brewster's angle

Sought: the reflection coefficient for s-polarized light

Steps:

1. Recall the relationship between refractive index and relative permittivity
2. Recall the relationship between Brewster's angle and the refractive indices of regions 1 and 2.
3. Recall the relationship between the angles of incidence and refraction, and the refractive indices of regions 1 and 2. (Snell's Law of Refraction)
4. Recall the relationship between the angles of incidence and reflection. (Snell's Law of Reflection.)
5. Recall the relationship between the reflection coefficient for an s-polarized wave and the angle of incidence and the refractive indices of regions 1 and 2.
6. Recall the alternative terms used to identify an s-polarized wave.

Consilium est demonstratum.

*Answers***:** 55.4°, 34.6°, -0.355

D12.6 An optical fiber link is known to have dispersion β_2 = 20 ps^2/km. A Gaussian light pulse at the input of the fiber is of initial width T = 20 ps. Determine the width of the pulse at the fiber output if the fiber is 15 km long.

Strategy

Given: The dispersion factor, length of the fibre, and the initial pulse width.

Sought: The width of the pulse at the fibre output.

Steps: Recall the relationship between the total dispersion, the initial pulse width and the output pulse width.

Consilium est demonstratum.

Answers: 25 ps