THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

A Supplement to

Chapter 12 –Plane Wave Reflection and Dispersion

in W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, McGraw-Hill, 2019, pp. 409-455.

The purposes of this supplement are:

- to assist the reader in identifying key points to be recognized as they apply the SQ3R (Survey, Question, Read, Recite, Review) process to reading and reviewing the chapter and
- to provide comments and supplemental information that fill in apparent gaps in the textbook.

Introduction

Because infinite unbounded media do not exist in real life, accounting for reflection and transmission when plane waves impinge upon boundaries between material media at various angles and the nature of dispersive media, *i.e.,* media in which the velocity of propagation varies with frequency, are critical next steps in our understanding of propagating electromagnetic waves.

This chapter considers: 1) reflection of uniform plane waves from a single interface at normal incidence, 2) the concept of the *standing wave ratio*, 3) wave reflection from multiple interfaces at normal incidence, 4) how to describe plane wave propagation in general directions, 5) plane wave reflection at oblique incidence angles, 6) total reflection and total transmission of obliquely incident waves, 7) wave propagation in dispersive media and 8) pulse broadening in dispersive media.

Key issues include: 1) the manner in which the amplitude and phase of the reflected and transmitted waves account for boundary conditions at the interfaces between the media, 2) the reasons why the velocity of propagation in a medium will vary with frequency and the implications for propagation of modulated signals, including pulses.

12.1 Reflection of Uniform Plane Waves at Normal Incidence

The section considers the simplest possible case: a uniform plane wave *normally incident* on a single interface or boundary. The results apply directly to impedance-matching problems.

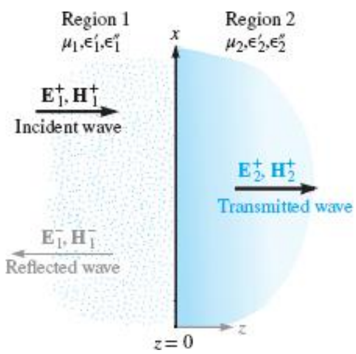

FIGURE 12.1 - A plane wave incident on a boundary establishes reflected and transmitted waves having the indicated propagation directions. All fields are parallel to the boundary, with electric fields along *x* and magnetic fields along *y*.

12.1.1 Reflected and Transmitted Waves at a Boundary

This section:

- establishes a convention for describing a wave travelling in a given direction in a given region
- notes that:
  - the ratio of the electric and magnetic fields associated with each wave must equal the intrinsic impedance of the wave
  - the electric and magnetic field boundary conditions identified in the previous chapter must be satisfied at each interface
  - it is not possible to meet both conditions with only an incident and transmitted wave; a reflected wave must be introduced

Comments

In this section, incidence is normal and fields are parallel to the interface so only continuity of electric and magnetic field strength needs to be considered.

In a later section, incidence is oblique and fields are not parallel to the interface (both normal and parallel components) so both continuity of field strength and continuity of flux density must be considered.

12.1.2 Reflection and Transmission Coefficients

This section:

- applies the intrinsic impedance conditions and the boundary conditions to yield a set of simultaneous equations in terms of the amplitude and phase of the electric and magnetic fields associated with each wave
- solves the simultaneous equations to yield expressions for the reflection and transmission coefficients Γ and τ for both electric and magnetic field strength

Comments

ELEC 311 students should be able to perform the derivation in this section without reference, *i.e.*, from memory.

12.1.3 Total Reflection: Standing Wave Ratio

This section:

- considers total reflection of a forward travelling wave from a perfect electrically conducting (PEC) plane
- considers the implications of forward and reverse travelling waves with the same frequencies and identical amplitudes propagating in a given medium at the same time
- introduces the notion of the *superposition* of such forward and reverse travelling waves forming a *standing wave* in which:
  - the zero crossings lie at fixed locations at half-wavelength intervals, and,
  - the standing wave pattern can be described in terms of: 1) the location of the zero crossing closest to a reference point, *e.g.*, an interface between two media, and 2) the ratio of the voltage maxima and minima, *i.e.*, the standing wave ratio, which is infinite in this case.

Comments

None.

12.1.4 Partial Reflection and Power Reflectivity

This section:

- considers *partial* reflection of a forward travelling wave from the interface between two regions composed of perfect dielectrics
- considers the implications of forward and reverse travelling waves with the same frequencies but different amplitudes propagating in a given medium at the same time
- introduces the notion of the *superposition* of such forward and reverse travelling waves forming a *standing wave* in which:
  - the voltage minima lie at fixed locations at half-wavelength intervals, and,
  - the standing wave pattern can be described in terms of: 1) the location of the voltage minima closest to a reference point, *e.g.*, an interface between two media, and 2) the ratio of the voltage maxima and minima, *i.e.*, the standing wave ratio, which may lie between 1 (no reflected wave) to infinity (total reflection)
- derives expressions for the power reflection and transmission coefficients in terms of the voltage reflection and transmission coefficients

Comments

In follow-on sections, we shall use conservation of power in conjunction with the power reflection and transmission coefficients to solve problems involving multiple interfaces.

12.2 Standing Wave Ratio

This section derives expressions for:

- the relationship between the standing wave ratio s and the voltage reflection coefficient Γ
- the location of the minima z_{max} and minima z_{min} in the standing wave pattern
- the total electric field in the incident region, referred to as region 1 in the text, as the sum of either:
  - the incident and reflected wave, or
  - the travelling and standing wave

Comments

None.

12.3 Wave Reflection from Multiple Interfaces

This section extends the results presented in Section 12.3 to cases involving multiple interfaces.

Comments

The addition of a second interface allows us to realize devices, such as quarter- and half-wave plates, also known as quarter- and half-wave transformers, with important engineering applications.

12.3.1 The Two-Interface Problem

This section:

- defines the essential elements of the two-interface problem
- notes that the single interface problem involves three waves while the two-interface problem involves five waves: incident and net reflected waves in region 1, the net transmitted wave in region 3, and the two counterpropagating waves in region 2.
- acknowledges that the transient solution is a little complicated; our goal is to find the steady-state solution

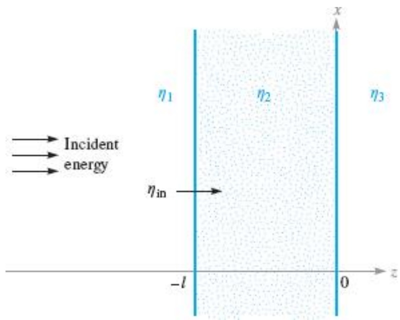

FIGURE 12.4 - Basic two-interface problem, in which the impedances of regions 2 and 3, along with the finite thickness of region 2, are accounted for in the input impedance at the front surface, η_{in}.

Comments

There are two cases of interest:

- in the first case, $\eta_1 > \eta_2 > \eta_3$, *i.e.*, decreasing intrinsic impedance or increasing permittivity (and density)
- in the second case, $\eta_1 = \eta_3 < \eta_2$

12.3.2 Wave Impedance

This section:

- introduces the concept of *wave impedance, i.e.,* the ratio of the total electric and total magnetic field, at a given point, where total means the sum of the forward and backward travelling components
- derives an expression for wave impedance as a function of distance in a medium in which both forward and backward travelling waves propagate
- explains how wave impedance simplifies calculations involving two interfaces

Comments

The expressions in the text correspond to lossless media. Expressions for lossy media are similar but use hyperbolic sine and cosine rather than regular or harmonic sine and cosine.

12.3.3 Special Cases: Half-Wave and Quarter-Wave Layer

This section:

- considers the special cases of half-wave and quarter-wave layers
- shows that if $\eta_2 = \sqrt{\eta_1 \eta_3}$ and $\ell = \lambda/4$, then $\eta_{in} = \eta_1$

- shows that if $\eta_1 = \eta_3$ and $\ell = \lambda/2$, then $\eta_{in} = \eta_1$
- demonstrates that the layers are performing an impedance transformation
- gives various examples of useful applications of such layers

Comments

As noted above, quarter- and half-wave plates, also known as quarter- and half-wave transformers, with important engineering applications in both optics and transmission lines.

12.3.4 The Multilayer Problem: Impedance Transformation

This section:

- shows how the half-wave and quarter-wave transformer concepts can be extended to multiple layers in order to increase the tolerance of the design to deviations from ideal dimensions and material properties

Comments

The layers are designed to operate at a single frequency. The bandwidth of the response is limited. Use of multiple layers can also increase the bandwidth of the response.

12.4 Plane Wave Propagation in General Directions

This section considers how to formally describe an electromagnetic wave that is propagating in an arbitrary or general direction rather than along a coordinate axis

Comments

This formulation is required to set up the problem of plane wave reflection at oblique incidence angles that is considered in the next section

12.5 Plane Wave Reflection at Oblique Incidence Angles

This section:

- determines the relation between incident, reflected, and transmitted angles, and
- derives reflection and transmission coefficients that are functions of the incident angle and wave polarization.

In particular, it:

- defines the *plane of incidence* that contains the directions of propagation of all three waves
- distinguishes linearly polarized incident waves in which the electric field vector is either parallel or perpendicular to the plane of incidence, *i.e.*, TM (Transverse Magnetic) and TE (Transverse Electromagnetic) polarizations, also known as *p*- or *s*-polarization, respectively
- gives Snell's laws of reflection and transmission
- derives expressions for the reflection and transmission coefficients for both TM and TE polarized incident waves by applying the appropriate boundary conditions at the interface between the two regions and accounting fort the intrinsic impedance of the material in each region

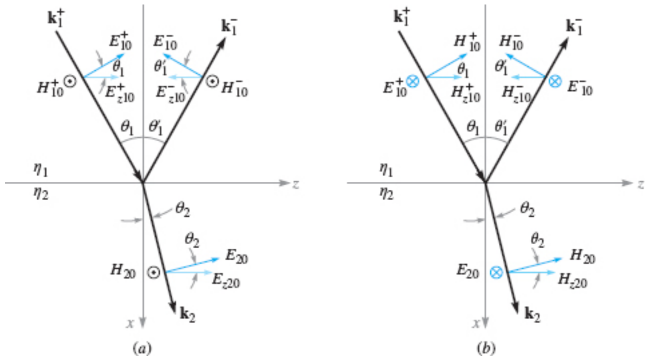

FIGURE 12.7 - Geometries for plane wave incidence at angle θ_1 onto an interface between dielectrics having intrinsic impedances η_1 and η_2. The two polarization cases are shown: (*a*) p-polarization (or TM), with **E** in the plane of incidence; (*b*) s-polarization (or TE),

Comments

Snell's laws of reflection and transmission can also be derived using Fermat's principle, which states that the path taken by a ray between two given points is the path that can be traversed in the least time (principle of least time).

Any arbitrarily polarized incident wave can be decomposed into TM- and TE-polarized components with appropriate amplitude and phase, per Section 9.5.

The notion that Maxwell's equations divide into two independent sets corresponding to TM- and TE-polarized waves when the plane of incidence is the *x*-*z* plane and the problem geometry is uniform in the *y* direction so $\frac{\partial}{\partial y} = 0$ allows us to reduce *many* three-dimensional problems to two-dimensional ones, as here.

One of the implications of this independence is that a TM-polarized incident wave will always result in TM-polarized reflected and transmitted waves when the problem geometry is uniform in the direction perpendicular to the plane of incidence; polarization transformation from TM to TE or vice versa cannot occur.

A corollary is that polarization transformation from TM to TE or vice versa *can* occur if the problem geometry is *not* uniform in the direction perpendicular to the plane of incidence; such transformation is referred to as *depolarization* of the incident wave.

12.6 Total Reflection and Total Transmission of Obliquely Incident Waves

This section:

- looks for special combinations of media, incidence angles, and polarizations that lead to:
  - *total internal reflection* when the wave is directed onto a region with a *lower* dielectric constant than the incident region
  - *total transmission* when the wave is directed onto a region with a *higher* dielectric constant than the incident region
- identifies the angle at which:
  - total internal reflection occurs as the *critical angle*
  - total transmission occurs as *Brewster's angle* or the *polarization angle*
- notes that:
  - total internal reflection is the basis for both fibre optics and dielectric waveguides
  - total transmission is of great interest in both optics and radiowave propagation

Comments

The polarization of the reflected wave when a circularly polarized wave is directed onto a region with a higher dielectric constant than the incident region is of particular interest in radiowave propagation:

- *above* Brewster's angle, the reflected wave is elliptically polarized but has the *opposite* sense to the incident wave
- at Brewster's angle, the reflected wave is TE (linearly) polarized
- *below* Brewster's angle, the reflected wave is elliptically polarized but has the *same* sense to the incident wave

12.7 Wave Propagation in Dispersive Media

This section:

- considers the mechanisms that may cause complex permittivity to vary with frequency
- introduces the $\omega - \beta$ diagram as a way to visual the effects of dispersion
- shows how $\omega - \beta$ plot allows us to distinguish between:
  - the *phase velocity* (also referred to as the carrier velocity in the text) given by ω/β and
  - the *group velocity* (also referred to as the envelope velocity in the text) given by $\partial\omega/\partial\beta$

The *group velocity dispersion* of the medium is, to the first order, the rate at which the slope of the ω-β curve changes with frequency.

Comments

Dispersion may also occur in waveguides (as we shall see in Chapter 13) and in the ionosphere.

12.8 Pulse Broadening in Dispersive Media

This section:

- considers how the width of a Gaussian pulse is broadened when the pulse propagates through dispersive media
- introduces the dispersion parameter β_2 and its use in predicting pulse broadening
- comments on the phenomenon of *chirping* which is associated with the spectral components of a pulse spreading out in time as they propagate at different velocities

Comments

When the electromagnetic pulses due to lightening discharges give rise to very low frequency signals propagating along Earth's magnetic field lines, they also undergo dispersion. The resulting signals are referred to as *whistlers*.