THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

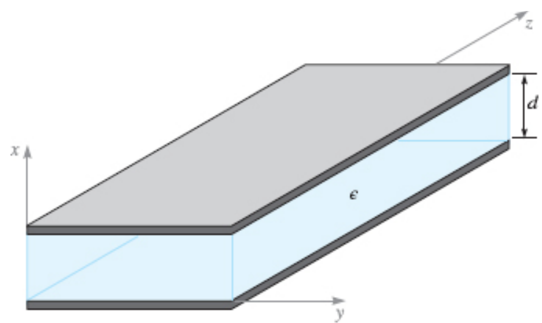
Chapter 13 – Guided Waves

What you need to know!

A compilation of course performance objectives with detailed enabling objectives.

Where formulas are cited, be certain that you can identify each quantity and its units and sketch figures that describe the scenario.

1. Given a parallel-plate waveguide with separation a, calculate the cut-off frequency of the *m*th mode. (§13.1)

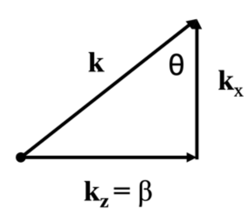


- Recognize that cut-off occurs when the wave must propagate exclusively in the transverse direction (back and forth between the plates or *x* direction) rather than the longitudinal direction (along the waveguide or *z* direction) in order the satisfy the condition that $E_{\text{tan}}(x = 0, x = a) = 0$
- Recognize that $E_{\text{tan}}(x = 0, x = a) = 0$ only if an integer number of half-wavelengths appear in the standing wave., *i.e.*, $a = \frac{m\lambda_c}{2}$, where *c* refers to cut-off.
- Recognize that it follows that $\lambda_{c,m} = \frac{2a}{m}$ and $f_{c,m} = \frac{c}{\lambda_{c,m}} = \frac{cm}{2a}$ where we understand that *c* can refer to either cut-off or the speed of light depending upon the context
- Recognize that the waveguide cannot support the *m*th mode *below* that mode's cut-off; no propagation occurs in either the transverse or longitudinal direction
- Recognize that there is no energy flow in the *z* or longitudinal direction *at* cut-off, only back and forth in the *x* or transverse direction
- Recognize that energy flows in both the transverse and longitudinal direction at frequencies *above* cut-off

2. Given a parallel-plate waveguide with separation *d* through which a wave with frequency *f* is applied, determine which modes are propagating. (§13.2)

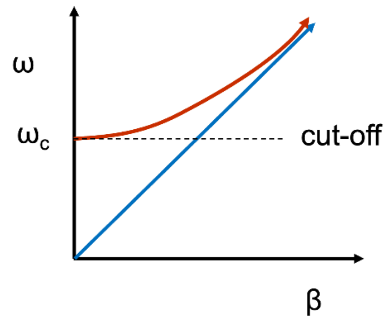
   - Recall that $\lambda_{c,m} = \frac{2a}{m}$ and $f_{c,m} = \frac{c}{\lambda_{c,m}} = \frac{cm}{2a}$
   - Recognize that as *n* increases, so will $f_{c,m}$
   - Find *m* such that $f_{c,m} < f$

3. For each propagating mode, calculate: the angle of reflection, θ, the group velocity, v_g, the phase velocity, v_p, the group delay, and the guide wavelength, λ_g. (§13.3 and §13.4)
   - Recognize that the wave number of the plane wave that is bouncing down the waveguide whatever the mode is $k_0 = 2\pi/\lambda$ where λ is the free space wavelength
   - Recognize that k_0 can be resolved into a transverse component, $k_{c,m}$, that is determined by the separation between the plates, and a longitudinal component k_z
   - Recognize that the *vector diagram* that relates k_0, $k_{c,m}$, and k_z reveals that
     - $k_{c,m} = \frac{2\pi}{\lambda_{c,m}}$
     - $k_{z,m} = \sqrt{k_0^2 - k_{c,m}^2}$
     - $\theta = \cos^{-1}\frac{k_{c,m}}{k_0}$
     - $\lambda_{g,m} = \frac{2\pi}{k_{z,m}}$

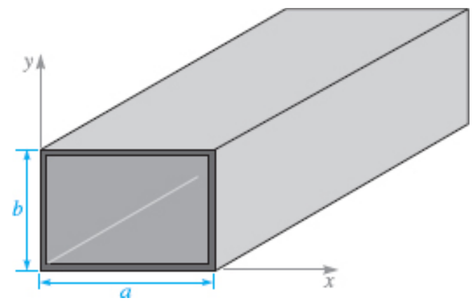


   - Recognize that $\lambda_{g,m}/2$ is the spacing between maxima or minima in the standing waves observed along a waveguide with a mismatched load
   - Recognize that the effective or group velocity of the wave is proportional to the projection of the wave vector **k** into the *z* or longitudinal direction, *i.e.,* $v_g = \frac{\partial\omega}{\partial\beta} = c\sin\theta$ where $\beta = k_z$
     - To derive this, recognize that $\frac{\partial\omega}{\partial\beta} = 1/\frac{\partial\beta}{\partial\omega} = 1/\frac{\partial k_z}{\partial\omega}$
   - Recognize that the group delay is given by $\tau = \frac{\Delta z}{v_g}$
   - Recognize that $v_p = \frac{\omega}{\beta} = \frac{2\pi f\lambda}{2\pi\sin\theta} = \frac{c}{\sin\theta}$
   - Recognize that $v_p v_g = c$ in air or free space
   - Recognize that *c* needs to be replaced with $v = c/\sqrt{\epsilon_r\mu_r}$ if $\epsilon_r, \mu_r \neq 1$

- In light of the above, interpret the difference between the plots of ω vs. β for conventional TEM transmission lines (blue) and waveguides (red).



4. Apply knowledge of the parallel-plate waveguide to the TE or TM_{m0} modes of rectangular waveguide. (§13.3 and §13.4)



- Recognize that the parallel-plate waveguide can be converted to a hollow pipe by adding vertical walls at $y = 0$ and b
- Recognize that such an enclosed structure has many practical advantages over parallel-plate waveguide
- Recognize that standing waves may form in both the x and y directions
- Recall that we use m and n to count the number of half-cycles in the x and y directions, respectively
- The *vector diagram* that relates k_0, $k_{c,m}$, and k_z includes a fourth vector, $k_{c,n}$, that is aligned with the y-axis
- The TE or TM_{m0} modes of rectangular waveguide are identical to the TE or TM_m modes of parallel-plate waveguide