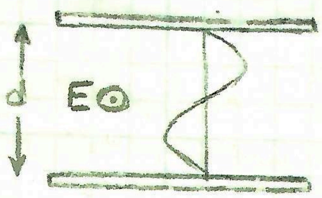
V Guided Waves

1. A TE mode in a parallel-plate waveguide is observed to have two maxima in its electric field pattern between $x = 0$ and $x = d$. What is the value of m?



TWO MAXIMA

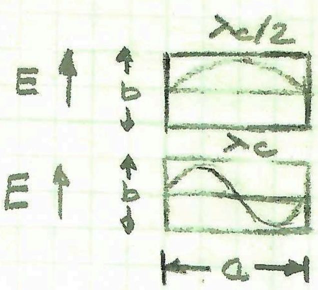
- nulls @ $x = 0, \frac{d}{2}, d$

required to match boundary conditions

$$m = 2$$

m is, in fact, the number of half cycles in the standing wave.

2. The interior of a WR-75 waveguide is 0.75" wide and 0.375" tall. What are the cut-off frequencies of the TE_{10} and TE_{20} modes?



$a = 0.75'' = 19.05\text{mm}$

$b = 0.375'' = 9.525\text{mm}$

FOR THE TE_{10} MODE, $\frac{\lambda_c}{2} = a$ OR $\lambda_c = 2a$.

$$\lambda_{c_{10}} = 38.10\ m = 3.81 \times 10^{-2}\ m$$

$$f_{c_{10}} = \frac{c}{\lambda_{c_{10}}} = \frac{3\times 10^8}{3.81\times 10^{-2}} = \boxed{7.87\text{ GHz}}$$

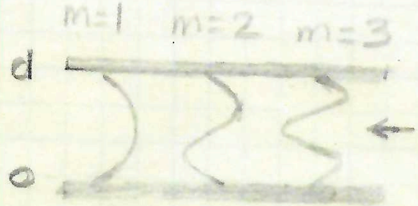
FOR THE TE_{20} MODE, $\lambda_c = a$

$$f_{c_{20}} = \frac{c}{\lambda_{c_{20}}} = \frac{3\times 10^8}{1.57\times 10^{-2}} = \boxed{15.75\text{GHz}}$$

3. Consider a parallel-plate waveguide with $d = 4$ cm, $\varepsilon_r = 1$ and $f = 15$ GHz.

   a. Determine the wave angles θ_m for the first three modes ($m = 1, 2,$ and 3)

   b. What is the maximum frequency at which the guide will operate in the TEM mode only?



$2d$	d	$\frac{2d}{3}$	$\lambda_{c,m}$
8.0 cm	4.0 cm	2.66 cm	$\lambda_{c,m}$
3.75 GHz	7.5 GHz	11.25 GHz	$f_{c,m} = \frac{3\times10^8}{\lambda_{c,m}}$

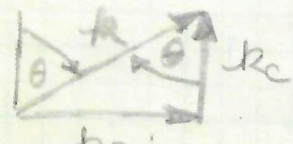
ALL THREE MODES WILL PROPAGATE AT $f = 15$ GHZ

$\lambda_{c,4} = \frac{d}{4} = 2$ cm

$f_{c,4} = 15$ GHZ

$m = 4$ IS CUTOFF (NO PROPAGATION) AT $f = 15$ GHZ.

a) WAVE ANGLES, θ_m



$$\theta_m = \arccos \frac{k_{c,m}}{k} = \arccos \frac{\lambda}{\lambda_{c,m}}$$

λ @ 15 GHZ = 2 cm

m	λ_c	$\frac{\lambda}{\lambda_c}$	$\arccos(\lambda/\lambda_c)$
1	8.0	0.25	75.5°
2	4.0	0.50	60°
3	2.66	0.75	41.2°

WAVE ANGLES θ_m FOR THE FIRST THREE MODES

($\theta_4 = 0° =$ cutoff)

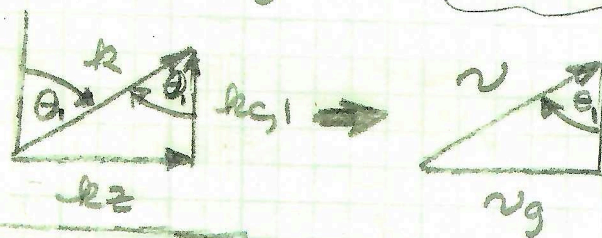
b. THE MAXIMUM FREQUENCY AT WHICH THE GUIDE WILL OPERATE IN THE TEM MODE ONLY IS $f_{c,1} = 3.75$ GHZ.

4. Determine the group velocity of the $m = 1$ (TE or TM mode) in an air-filled parallel-plate waveguide with $d = 1.0$ cm at $f = 15$, 30, and 50 GHz.

THERE ARE TWO WAYS TO CALCUATE THIS.

1. PROJECTION OF v_g

$$\boxed{\bar{k}_z + \bar{k}_{c,1} = \bar{k}}$$



THE ZIGZAG PATH OF THE WAVE IS LONGER THAN THE NET DISTANCE ALONG THE WAVEGUIDE.

$$v_g = v \sin\theta_1$$

$d = 1\text{ cm} = \dfrac{\lambda_{c,1}}{2}$; $\lambda_{c,1} = 2\text{ cm}$, $f_{c,1} = 15\text{ GHz}$

FROM PROBLEM 3, $\theta_1 = \arccos \dfrac{k_{c,1}}{k}$

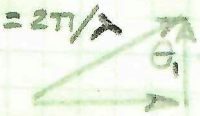
$$= \arccos \frac{\lambda}{\lambda_{c,1}}$$

f (GHz)	λ (cm)	$\frac{\lambda}{\lambda_{c,1}}$	θ_1 (deg)	$\sin\theta_1$	v_g (m/s)
15	2 cm	1	0	0	0 m/s
30	1 cm	0.5	60	0.866	2.59×10^8 m/s
50	0.6 cm	0.3	72.5	0.954	2.86×10^8 m/s

METHOD 2

2. $v_g v_p = v^2$ $\quad$ *where* $v_p = \dfrac{\omega}{\beta} = \dfrac{\omega}{k_z}$

$$v_g = \frac{v^2}{v_p}$$



$k = 2\pi/\lambda$

θ_1

$k_{c,1} = 2\pi/\lambda_{c,1}$

$k_z = 2\pi/\lambda_g$

• HERE, WE DERIVE A SIMPLIFIED EXPRESSION FOR v_p

$$k_z = k \sin\theta_1 = k \frac{\lambda}{\lambda_{c,1}}$$

$$\lambda_g = 2\pi/k_z = 2\pi/k \sin\theta_1$$

$$= \lambda / \sin\theta_1$$

$$v_p = \frac{\omega}{k_z} = \frac{\cancel{2\pi} f \lambda_g}{\cancel{2\pi}} = \frac{f\lambda}{\sin\theta_1} = \frac{v}{\sin\theta_1}$$

$$v_g = \frac{v^2}{v_p} = \frac{v^2 \sin\theta_1}{v} = v \sin\theta_1$$

THIS IS THE SAME EXPRESSION THAT WE DERIVED ON THE PREVIOUS PAGE BUT USING A DIFFERENT APPROACH!

TOPS