THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W1

Strategies for the Drill Problems for Chapter 13 – Guided Waves

The purpose of the nine drill problems from Chapter 13 is to help you master fundamental techniques used to analyze guided waves. (Problem D13.4, 13.11, 13.12 are not assigned.)

Answers should be short and to the point. It is especially critical to use sketches to explain your solution here. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

The numerical answers below are from the text. Do you agree with them?

D13.1 Parameters for the planar transmission line shown in Figure 13.2 are $b = 6$ mm, $d = 0.25$ mm, $t = 25$ mm, $\sigma_c = 5.5 \times 10^7$ S/m, $\epsilon' = 25$ pF/m, $\mu = \mu_0$, and $\sigma/\omega\epsilon' = 0.03$. If the operating frequency is 750 MHz, calculate: (a) α; (b) β; (c) Z_0.

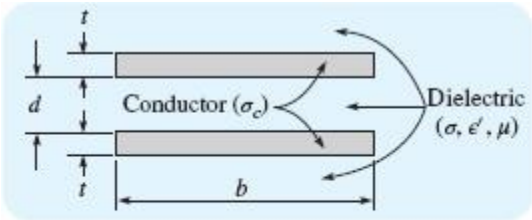

Figure 13.2 The geometry of the parallel-plate transmission line.

Strategy

Given: The dimensions and materials of a parallel-plate transmission line, and the operating frequency.

Sought: (a) α; (b) β; (c) Z_0.

Steps:

1. (A sketch of the problem geometry is already provided.)
2. Choose the appropriate formulas for L, C, R, and G in terms of the dimensions and materials of a parallel-plate transmission line, and the operating frequency from the table from Ramo *et al.*
3. Recognize that $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$
4. Recognize that $Z = R + j\omega L$ and $Y = G + j\omega C$.
5. Recognize that $\gamma = \alpha + j\beta = \sqrt{ZY}$ and $Z_0 = \sqrt{Z/Y}$.

Consilium est demonstratum.

Answers: (a) 0.47 Np/m; (b) 26 rad/m; (c) 9.3∠0.7°Ω

D13.2 The dimensions of a coaxial transmission line are a = 4 mm, b = 17.5 mm, and c = 20 mm. The conductivity of the inner and outer conductors is 2×10^7 S/m, and the dielectric properties are $\mu_r = 1$, $\epsilon_r' = 3$, and $\sigma/\omega\epsilon' = 0.025$. Assume that the loss tangent is constant with frequency. Determine: (*a*) L, C, R, G, and Z_0 at 150 MHz; (*b*) L and R at 60 Hz.

Strategy

Given: The dimensions and materials of a coaxial transmission line.

Sought: (*a*) L, C, R, G, and Z_0 at 150 MHz; (*b*) L and R at 60 Hz

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Choose the appropriate formulas for L, C, R, and G in terms of the dimensions and materials of a parallel-plate transmission line, and the operating frequency from the table from Ramo *et al.* R_{AC} is dependent on the skin depth at the frequency of interest and the consequent reduction in the effective cross-sectional area of the conductors.
3. Recognize that $Z = R + j\omega L$ and $Y = G + j\omega C$.
4. Recognize that $\gamma = \sqrt{ZY}$ and $Z_0 = \sqrt{Z/Y}$.

Consilium est demonstratum.

Answers: (*a*) 0.30 μH/m, 113 pF/m, 0.27 Ω/m, 2.7 mS/m, 51 Ω; (*b*) 0.36 μH/m, 1.16 m Ω/m

D13.3 The conductors of a two-wire transmission line each have a radius of 0.8 mm and a conductivity of 3×10^7 S/m. They are separated by a center-to-center distance of 0.8 cm in a medium for which $\epsilon_r' = 2.5$, $\mu_r = 1$, and $\sigma = 4 \times 10^{-9}$ S/m. If the line operates at 60 Hz, find: (*a*) δ; (*b*) C; (*c*) G; (*d*) L; (*e*) R.

Strategy

Given: The dimensions and materials of a two-wire transmission line.

Sought: (*a*) δ; (*b*) C; (*c*) G; (*d*) L; (*e*) R.

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Recognize that $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$.
3. Choose the appropriate formulas for L, C, R, and G from the table from Ramo *et al.*

Consilium est demonstratum.

Answers: (*a*) 1.2 cm; (*b*) 30 pF/m; (*c*) 5.5 nS/m; (*d*) 1.02 μH/m; (*e*) 0.033 Ω/m

D13.5 Determine the wave angles θ_m for the first four modes (m = 1, 2, 3, 4) in a parallel-plate guide with d = 2 cm, ϵ_r' = 1, and f = 30 GHz.

Strategy

Given: The dimensions and materials of a parallel-plate guide and the operating frequency.

Sought: The wave angles θ_m for the first four modes (m = 1, 2, 3, 4)

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Sketch that right triangle that relates k, k_c, and k_z to θ.
3. Find the free-space wavelength at each frequency and use this to find k.
4. Find the cut-off wavelength of the m = 1 (TE or TM) mode and use this to find $k_{c,m}$.
5. Find θ_m in terms of k and $k_{c,m}$ at the operating frequency.

Consilium est demonstratum.

Answers: 76°; 60°; 41°; 0°

D13.6 A parallel-plate guide has plate spacing d = 5 mm and is filled with glass (n = 1.45). What is the maximum frequency at which the guide will operate in the TEM mode only?

Strategy

Given: The dimensions and materials of a parallel-plate guide.

Sought: The maximum frequency at which the guide will operate in the TEM mode only.

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Recognize that the maximum frequency at which the guide will operate in the TEM mode only is also the cut-off frequency of the m = 1 mode.
3. Recognize the standing wave of the m = 1 mode at cut-off has one half-cycle so $\lambda_{c,1}/2 = d$ or $\lambda_{c,1} = 2d$.
4. Recognize that the wavelength within a material is given by $\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{\lambda_0}{n}$ where λ0 is the free space wavelength and the velocity of a wave within a material is given by $v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n}$. Accordingly, the frequency of a wave with given wavelength is $f = \frac{v}{\lambda}$ and $f_{c,1} = \frac{v}{\lambda_{c,1}}$.

Consilium est demonstratum.

Answers: 20.7 GHz.

D13.7 A parallel-plate guide having $d = 1$ cm is filled with air. Find the cut-off wavelength for the $m = 2$ mode (TE or TM).

Strategy

Given: The dimensions and materials of a parallel-plate guide.

Sought: The cut-off wavelength for the $m = 2$ mode (TE or TM).

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Recognize that the $m = 2$ mode corresponds to a standing wave with two half-cycles so $d = 2(\lambda_{c,2}/2)$ and $\lambda_{c,2} = d$.

Consilium est demonstratum.

Answer: 1 cm.

D13.8 Determine the group velocity of the $m = 1$ (TE or TM) mode in an air-filled parallel-plate guide with $d = 0.5$ cm at $f =$ (a) 30 GHz, (b) 60 GHz, and (c) 100 GHz.

Strategy

Given: The dimensions and materials of a parallel-plate guide.

Sought: The group velocity of the $m = 1$ (TE or TM) mode at three frequencies.

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Sketch that right triangle that relates k, k_c, and k_z to θ.
3. Find the cut-off wavelength of the $m = 1$ (TE or TM) mode and use this to find k_c.
4. Find the free-space wavelength at each frequency and use this to find k.
5. Find θ in terms of k and k_c at each frequency.
6. Find the group velocity in terms of the free space velocity and θ at each frequency.

Consilium est demonstratum.

Answers: (a) 0; (b) 2.6×10^8 m/s; (c) 2.9×10^8 m/s

D13.9 A TE mode in a parallel-plate guide is observed to have three maxima in its electric field pattern between $x = 0$ and $x = d$. What is the value of m?

Strategy

Given: The number of maxima in its electric field pattern between $x = 0$ and $x = d$.

Sought: The value of m

Steps:

1. Sketch the problem geometry as an aid to understanding.
2. Recognize that m = the number of half cycles in the standing wave pattern = the number of electric field maxima.

Consilium est demonstratum.

Answer: 3

D13.10 Specify the minimum width, a, and the maximum height, b, of an air-filled rectangular guide so that it will operate in a single mode over the frequency range 15 GHz < f < 20 GHz.

Strategy

Given: The frequency range over which an air-filled rectangular guide will pass a single mode.

Sought: The minimum width, a, and the maximum height, b, of the rectangular guide.

Steps:

1. Sketch the problem geometry, including the standing waves associated with the TE_{10} and TE_{01} modes as an aid to understanding.
2. Recognize that the lower frequency of 15 GHz corresponds to the cut-off frequency of the TE_{10} mode where $\lambda_{c,10} = 2a$.
3. Recognize that the upper frequency of 20 GHz corresponds to the cut-off frequency of the TE_{01} mode where $\lambda_{c,01} = 2b$.
4. Recognize that the cut-off frequency of the TE_{20} mode would be 30 GHz where $\lambda_{c,20} = a$ so is irrelevant to this problem.

Consilium est demonstratum.

Answers: 1 cm; 0.75 cm