

Chapter 9 – Time Varying Fields and Maxwell’s Equations

What you need to know!

A compilation of course performance objectives with detailed enabling objectives.

Where formulas are cited below, be certain that you can identify each quantity and its units and sketch figures that describe the scenario.

Engineering Communications

In general, we expect you to:

- be able to read and master complicated technical documents such as textbook chapters and application notes using a structured approach such as SQ3R in order to be both efficient and effective.

Survey – Question – Read – Recite – Review

- be good engineering communicators and to present problem solutions that are clearly laid out with good use of freehand sketches, bullet points, and equations to communicate:
 - your understanding of the problem,
 - your strategy for solving it,
 - your execution of that strategy, and,
 - your result with units clearly indicated.

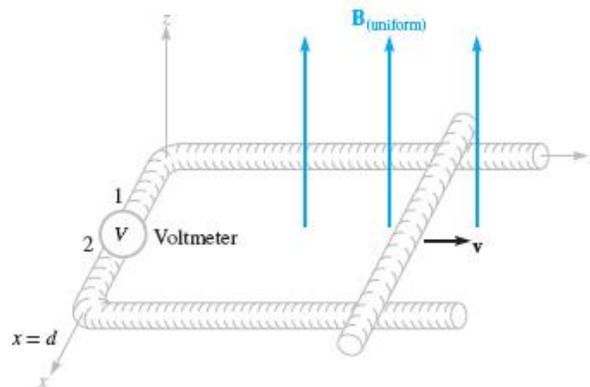
Time Varying Fields and Maxwell’s Equations

This chapter reviews one of the most important properties of electromagnetic fields: A time-varying electric field gives rise to a time-varying magnetic field, and vice versa.

Upon completion of this chapter, you should be able to:

1. Give Faraday’s law in customary, integral, and point form, and determine the EMF associated with a time-varying magnetic field, a time-constant flux, & a moving closed path. (§9.1)
 - Recall the customary form of Faraday’s Law: $emf = -N d\Phi/dt$
 - Recall the integral form of Faraday’s Law: $emf = \oint \mathbf{E} \cdot d\mathbf{L} = -d/dt \int \mathbf{B} \cdot d\mathbf{S}$
 - Recall the point form of Faraday’s Law: $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$
 - Recall that force on a charge Q is given by $\mathbf{F} = Q (\mathbf{v} \times \mathbf{B})$
 - Recall that the motional electric field intensity is given by $\mathbf{E}_m = \mathbf{v} \times \mathbf{B}$
 - Recall that including both the transformer and the motional emf yields
$$emf = \oint \mathbf{E} \cdot d\mathbf{L} = -\int \partial\mathbf{B}/\partial t \cdot d\mathbf{S} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{L}$$
 - Recall that $\nabla \times \mathbf{A} = \mathbf{B}$ where \mathbf{A} is the magnetic vector potential.

This objective is completely focused on problem scenarios that involve a closed loop that experiences an EMF due to a changing magnetic flux. See, for example, the figure below. The key is to be able to determine *whether* the magnetic flux enclosed by the loop is changing with time and *why*.



An example illustrating the application of Faraday's law to the case of a constant magnetic flux density B and a moving path. The shorting bar moves to the right with a velocity v , and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.

Possible causes of a changing magnetic flux include:

- changes in the area of the loop with time,
- changes in the magnetic flux density with time, or
- changes in the direction of the magnetic flux with respect to the normal to the loop with time.

More generally, possible scenarios include various combinations of the following parameters:

- loop geometry (constant or time-dependent)
- loop location (fixed, translating, or rotating)
- flux density (fixed, time-dependent, or location-dependent)

Recall that changing the net flux by using a switch to open or close a circuit in a manner that increases or decreases the net area of a loop will *not* generate an emf because no force has been applied (or energy supplied) to the system.

2. Give expressions for the time-variation version of Ampère's law in both point and integral form and calculate the magnitude of the displacement current in practical scenarios. (§9.2)

- Recall the point form of Ampère's law, $\nabla \times \mathbf{H} = \mathbf{J}_c + \partial \mathbf{D} / \partial t$
- Recall the integral form of Ampère's law, $\oint \mathbf{H} \cdot d\mathbf{L} = \int (\mathbf{J}_c + \partial \mathbf{D} / \partial t) \cdot d\mathbf{S}$
- Recognize that the magnitude of the displacement current is given by

$$|J_D| = \left| \frac{\partial \mathbf{D}}{\partial t} \right| = \epsilon_r \epsilon_0 \left| \frac{\partial \mathbf{E}}{\partial t} \right|$$

- Recall that including displacement current resolves a contradiction in the continuity of current condition that occurs when only conduction current is accounted for. Otherwise, $\nabla \cdot \mathbf{J} = \nabla \cdot (\nabla \times \mathbf{H}) = 0$ rather than the correct form, $\nabla \cdot \mathbf{J} = -\partial \rho / \partial t$.

3. Give Maxwell's equations in point and integral form and the constitutive relations or auxiliary equations, explain the significance of the Helmholtz theorem to these results, and describe the relationship between the SI and previous systems of electromagnetic units. (§9.3 and §9.4)

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\partial \mathbf{B} / \partial t \\ \nabla \times \mathbf{H} &= \mathbf{J}_c + \partial \mathbf{D} / \partial t \end{aligned} \quad \begin{aligned} \mathbf{D} &= \epsilon_r \epsilon_0 \mathbf{E} \\ \mathbf{B} &= \mu_r \mu_0 \mathbf{H} \\ \mathbf{J} &= \sigma \mathbf{E} \end{aligned}$$

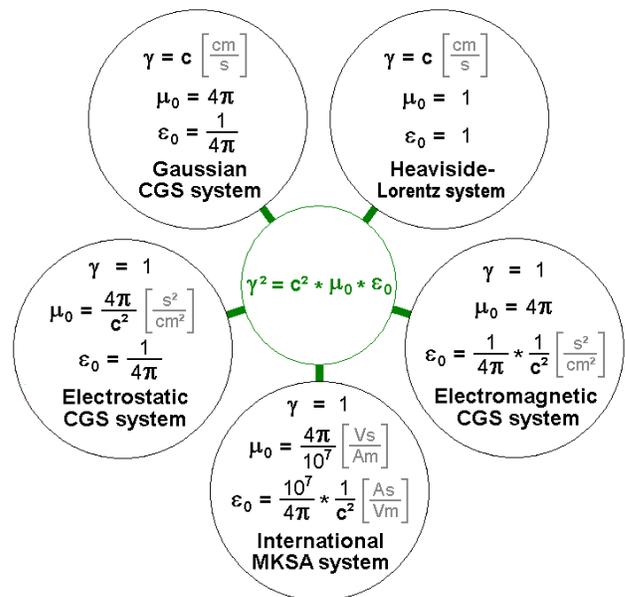
$$\begin{aligned} \oint_S \mathbf{D} \cdot d\mathbf{S} &= \int_v \rho \, dv \\ \oint_S \mathbf{B} \cdot d\mathbf{S} &= 0 \\ \oint_C \mathbf{H} \cdot d\ell &= \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S} \\ \oint_C \mathbf{E} \cdot d\ell &= \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \end{aligned}$$

- The Helmholtz theorem (or the fundamental theorem of vector calculus) tells us that any vector field can be resolved into irrotational and rotational components with former completely specified by the divergence of the field and the latter completely specified by the curl of the field.
- The general form of Maxwell's equations in point form is therefore predictable and inevitable with the right-hand side simply being experimental observations.
- Maxwell's equations are completely concerned with the describing the *form* of the electromagnetic field and are silent concerning force and energy.

- SI (formerly known as rationalized mks) units are based on the fundamental units of metres, kilograms and seconds while the various cgs systems of units (Gaussian, electrostatic, electromagnetic and Lorentz-Heaviside) are based on the fundamental units of centimetres, grams and seconds.

- The accompanying graphic compares five different systems of electromagnetic units based upon the metric system. Conversion of units between these systems and SI (also known as rationalized MKS) is not straightforward due to fundamental differences in their formulation.

Electromagnetic systems of units



$$\begin{aligned} 1 \text{ N} &= 1 \text{ kgm/s}^2 = 10^5 \text{ gcm/s}^2 = 10^5 \text{ dyn} \\ 1 \text{ J} &= 1 \text{ Ws} = 1 \text{ Nm} = 10^5 \text{ dyn} * 10^2 \text{ cm} = 10^7 \text{ erg} \\ c &\approx 2,998 * 10^8 \text{ m/s} = 2,998 * 10^{10} \text{ cm/s} \end{aligned}$$

4. Demonstrate the relationship between Maxwell's equations in point and integral form, and the boundary conditions imposed on field strength and flux density across material boundaries. (See §9.3 and §9.4. This will be important when we match fields across boundaries in Chapter 12.)

- Recognize that the *divergence theorem*

$$\iiint_V (\nabla \cdot \mathbf{F}) dV = \oiint_S \mathbf{F} \cdot d\mathbf{S}$$

links the divergence equations in \mathbf{D} and \mathbf{B} , *i.e.*, the flux densities, to the corresponding closed surface integrals.

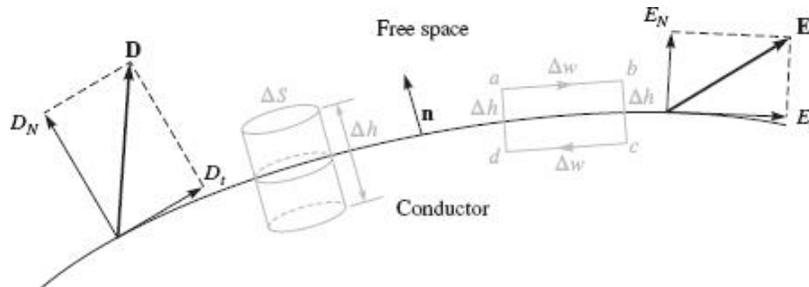
- Recognize that these closed surface integrals can be used to deduce the continuity of the *normal component of the flux density* across a material interface. (See the figure below.)

- Recognize that *Stokes' theorem*

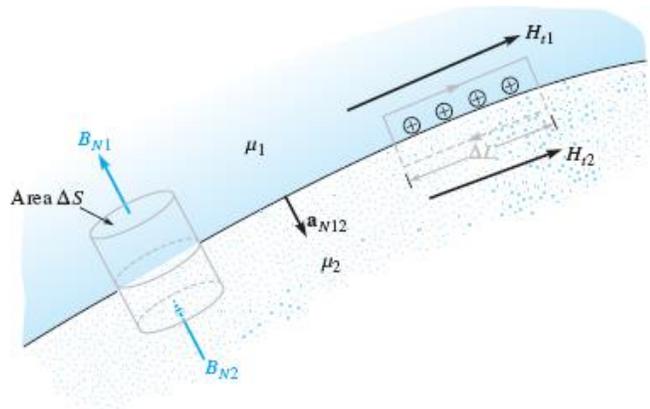
$$\oint_C \mathbf{F} \cdot d\boldsymbol{\ell} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

links the curl equations in \mathbf{E} and \mathbf{H} , *i.e.*, the field strengths, to the corresponding closed line integrals

- Recognize that these closed surface integrals can be used to deduce the continuity of the *tangential component of the field strength* across a material interface. (See the figure below.)



An appropriate closed path and gaussian surface are used to determine boundary conditions at a boundary between a conductor and free space; $E_t = 0$ and $D_N = \rho_s$.



A gaussian surface and a closed path are constructed at the boundary determine the boundary conditions $B_{N1} = B_{N2}$ and $H_{t1} - H_{t2} = K$, the component of the surface current density directed into the page.