THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W1

Drill Problems for
Chapter 9 – Time-Varying Fields and Maxwell's Equations

The purpose of the seven drill problems in Chapter 9 is to help you master fundamental techniques used to analyze time-varying fields and apply Maxwell's equations.

Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit. The numerical answers below are from the text. Do you agree with them?

D9.1 Within a certain region, $\epsilon = 10^{-11}$ F/m and $\mu = 10^{-5}$ H/m. If $B_x = 2\times10^{-4}\cos(10^5 t)\sin(10^{-3}y)$ T: (*a*) use $\nabla\times\mathbf{H} = \epsilon\, \partial\mathbf{E}/\partial t$ to find **E**; (*b*) find the total magnetic flux passing through the surface $x = 0$, $0 < y < 40$ m, $0 < z < 2$ m, at $t = 1\ \mu$s; (*c*) find the value of the closed line integral of **E** around the perimeter of the given surface.

Answers: (*a*) $-20{,}000\sin(10^5 t)\cos(10^{-3}y)\mathbf{a}_z$ V/m; (*b*) 0.318 mWb; (*c*) −3.19 V

D9.2 With reference to the sliding bar shown below, let $d = 7$ cm, $\mathbf{B} = 0.3\mathbf{a}_z$ T, and $\mathbf{v} = 0.1\mathbf{a}_y e^{20y}$ m/s. Let $y = 0$ at $t = 0$. Find: (*a*) $v(t = 0)$; (*b*) $y(t = 0.1)$; (*c*) $v(t = 0.1)$; (*d*) V_{12} at $t = 0.1$.

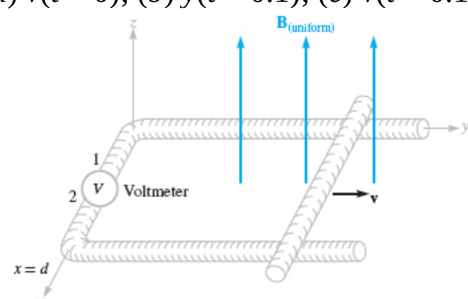

Figure 9.1 An example illustrating the application of Faraday's law to the case of a constant magnetic flux density **B** and a moving path. The shorting bar moves to the right with a velocity **v**, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.

Answers: (*a*) 0.1 m/s; (*b*) 1.12 cm; (*c*) 0.125 m/s; (*d*) −2.63 mV

D9.3 Find the amplitude of the displacement current density: (*a*) adjacent to an automobile antenna where the magnetic field intensity of an FM signal is $H_x = 0.15\cos[3.12(3\times10^8 t - y)]$ A/m; (*b*) in the airspace at a point within a large power distribution transformer where $\mathbf{B} = 0.8\cos[1.257\times10^{-6}(3\times10^8 t - x)]\mathbf{a}_y$ T; (*c*) within a large, oil-filled power capacitor where $\epsilon_r = 5$ and $\mathbf{E} = 0.9\cos[1.257\times10^{-6}(3\times10^8 t - 5z)]\mathbf{a}_x$ MV/m; (*d*) in a metallic conductor at 60 Hz, if $\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 5.8\times10^7$ S/m, and $\mathbf{J} = \sin(377t - 117.1z)\mathbf{a}_x$ MA/m^2.

Answers: (*a*) 0.468 A/m^2; (*b*) 0.800 A/m^2; (*c*) 0.0150 A/m^2; (*d*) 57.6 pA/m^2

D9.4 Let $\mu = 10^{-5}$ H/m, $\epsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$, and $\rho_v = 0$. Find k (including units) so that each of the following pairs of fields satisfies Maxwell's equations:
(*a*) $\mathbf{D} = 6\mathbf{a}_x - 2y\mathbf{a}_y + 2z\mathbf{a}_z$ nC/m^2, $\mathbf{H} = kx\mathbf{a}_x + 10y\mathbf{a}_y - 25z\mathbf{a}_z$ A/m;
(*b*) $\mathbf{E} = (20y - kt)\mathbf{a}_x$ V/m, $\mathbf{H} = (y + 2 \times 10^6 t)\mathbf{a}_z$ A/m.

Answers: (*a*) $k = 15$ A/m^2; (*b*) $k = -2.5\times10^8$ V/(m · s)

D9.5 The unit vector $\mathbf{n} = 0.64\mathbf{a}_x + 0.6\mathbf{a}_y - 0.48\mathbf{a}_z$ is directed from region 2 ($\epsilon_r = 2$, $\mu_r = 3$, $\sigma_2 = 0$) toward region 1 ($\epsilon_{r1} = 4$, $\mu_{r1} = 2$, $\sigma_1 = 0$). If $\mathbf{B}_1 = (\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z) \sin 300t$ T at point P in region 1 adjacent to the boundary, find the amplitude at P of: (*a*) $\mathbf{B}_{N1}$; (*b*) $\mathbf{B}_{t1}$; (*c*) $\mathbf{B}_{N2}$; (*d*) $\mathbf{B}_2$.

Answers: (*a*) 2.00 T; (*b*) 3.16 T; (*c*) 2.00 T; (*d*) 5.15 T

D9.6 The surface $y = 0$ is a perfectly conducting plane, whereas the region $y > 0$ has $\epsilon_r = 5$, $\mu_r = 3$, and $\sigma = 0$. Let $\mathbf{E} = 20 \cos(2\times10^8 t - 2.58z)\mathbf{a}_y$ V/m for $y > 0$, and find at $t = 6$ ns; (*a*) ρ_S at $P(2, 0, 0.3)$; (*b*) $\mathbf{H}$ at P; (*c*) $\mathbf{K}$ at P.

Answers: (*a*) $\rho_S = 0.81$ nC/m^2; (*b*) $\mathbf{H} = -62.3\mathbf{a}_x$ mA/m; (*c*) $\mathbf{K} = -62.3\mathbf{a}_z$ mA/m

D9.7 A point charge Q_1 of $4 \cos(10^8\pi t)$ μC is located at $P_1(0, 0, 1.5)$, whereas $Q_2 = -4 \cos(10^8\pi t)$ μC is located at $P_2(0, 0, -1.5)$, both in free space. Find V at $P(r = 450, \theta, \phi = 0)$ at $t = 15$ ns for θ =: (*a*) 0°; (*b*) 90°; (*c*) 45°.

Answers: (a) 159.8 V; (b) 0; (c) 143 V