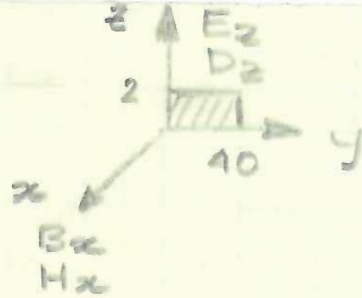
D9.1 Within a certain region, $\epsilon = 10^{-11}$ F/m and $\mu = 10^{-5}$ H/m. If $B_x = 2\times10^{-4}\cos(10^5 t)\sin(10^{-3}y)$ T: (*a*) use $\nabla\times\mathbf{H} = \epsilon\,\partial\mathbf{E}/\partial t$ to find **E**; (*b*) find the total magnetic flux passing through the surface $x = 0$, $0 < y < 40$ m, $0 < z < 2$ m, at $t = 1\ \mu$s; (*c*) find the value of the closed line integral of **E** around the perimeter of the given surface.

Answers: (*a*) $-20{,}000\sin(10^5 t)\cos(10^{-3}y)\mathbf{a}_z$ V/m; (*b*) 0.318 mWb; (*c*) −3.19 V



ϵ, μ

(a) Find $\bar{E}$

$B_x = 2\times10^{-4}\cos 10^5 t \sin 10^{-3}y$ T

$H_x = \dfrac{2\times10^{-4}}{10^{-5}}\cos 10^5 t \sin 10^{-3}y$ A/m

$= \dfrac{B_x}{\mu}$

$$\nabla\times\bar{H} = \begin{vmatrix} x & y & z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & \cancel{H_y} & \cancel{H_z} \end{vmatrix} = \frac{\partial}{\partial z}H_x\hat{y} - \frac{\partial}{\partial y}H_x\hat{z}$$

H_x IS NOT A FUNCTION OF y

$= -2\times10\cos 10^5 t\ 10^{-3}\cos 10^{-3}y\ \hat{z}$ A/m²

$= -2\times10^{-2}\cos 10^5 t\cos 10^{-3}y\ \hat{z}$ A/m²

$= \dfrac{\partial\bar{D}}{\partial t} = \epsilon\dfrac{\partial\bar{E}}{\partial t}$ A/m²

$\dfrac{\partial\bar{E}}{\partial t} = -\dfrac{2\times10^{-2}}{10^{-11}}\cos 10^5 t\cos 10^{-3}y\ \hat{z}$ V/m·s

$\displaystyle\int\frac{\partial\bar{E}}{\partial t}dt = -\frac{2\times10^{-2}}{10^5\,10^{-11}}\sin 10^5 t\cos 10^{-3}y\ \hat{z}$ V/m

$\bar{E} = -2\times10^4\sin 10^5 t\cos 10^{-3}y\ \hat{z}$ V/m

✓

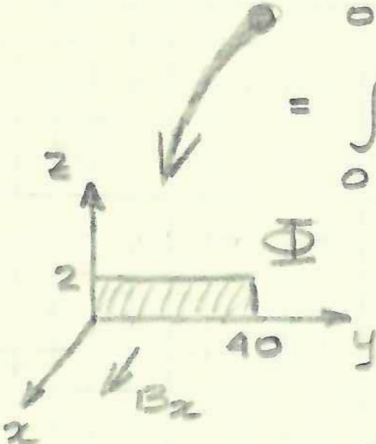
(b) FIND Φ PASSING THROUGH

$x = 0, \ 0 < y < 40\,m, \ 0 < z < 2\,m$

@ $t = 1\,\mu s$.

$$\Phi = \int_0^2 \int_0^{40} \bar{B} \cdot d\bar{S} \quad (\text{Wb})$$

$$= \int_0^2 \int_0^{40} 2 \times 10^{-4} \cos 10^5 t \ \sin 10^{-3} y \ dy\,dz$$



$= \cos 10^5 10^{-6}$

$= \cos 0.1$

$= 0.995$

$$= 2 \times 2 \times 10^{-4} \times 0.995 \int_0^{40} \sin 10^{-3} y \ dy$$

$$= -\frac{2 \times 2 \times 10^{-4} \times 0.995}{10^{-3}} \cos 10^{-3} y \Big|_0^{40}$$

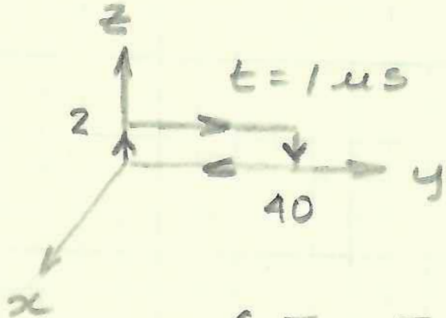
$$= -0.398 \times -0.0008$$

$$\Phi = \boxed{0.318 \ m\text{Wb}}$$

NOTES:

- B_x IS PERPENDICULAR TO THE SURFACE THAT LIES IN THE Y-Z OR $x = 0$ PLANE SO $\bar{B} \cdot d\bar{S}$ REDUCES TO $B_x \, dy\,dz$

(c) $\oint \bar{E} \cdot d\bar{\ell}$



BECAUSE $\bar{E}$ HAS ONLY AN E_z COMPONENT,
$\bar{E} \cdot d\bar{\ell} = 0$ ON THE HORIZONTAL LEGS

$$\oint \bar{E} \cdot d\bar{\ell} = \int_0^2 E_z(y,t)\, dz, \quad y = 0$$

$$+ \int_2^0 E_z(y,t)\, dz, \quad y = 40$$

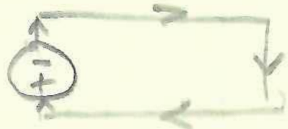
$$= -2 \times 10^4 \sin 10^5 t \left(2 - 2 \cos 40 \times 10^{-3}\right)$$

emf $= -3.19$ V @ $t = 1\,\mu s$

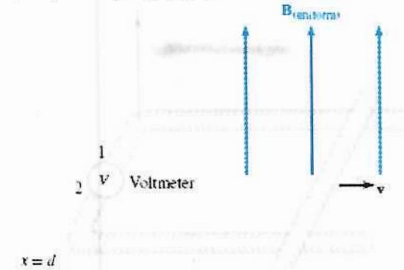
→ IF WE ASSUME THAT THE CONTOUR IS TRAVERSED IN THE CLOCKWISE DIRECTION.

IF WE ASSUME THAT THE $\hat{x}$ IS THE NORMAL TO THE SURFACE $x = 0$, THEN THE CONTOUR IS TRAVERSED IN THE COUNTERCLOCKWISE DIRECTION.

REVERSING THE CONTOUR DIRECTION REVERSES THE REFERENCE DIRECTION FOR VOLTAGE.



D9.2 With reference to the sliding bar shown below, let $d = 7$ cm, $\mathbf{B} = 0.3\mathbf{a}_z$ T, and $\mathbf{v} = 0.1\mathbf{a}_y e^{20y}$ m/s. Let $y = 0$ at $t = 0$. Find: (*a*) $v(t = 0)$; (*b*) $y(t = 0.1)$; (*c*) $v(t = 0.1)$; (*d*) V_{12} at $t = 0.1$.



$= 0.1\,\hat{a}_y\, e^{20y}$ m/s

Figure 9.1 An example illustrating the application of Faraday's law to the case of a constant magnetic flux density **B** and a moving path. The shorting bar moves to the right with a velocity **v**, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.

Answers: (*a*) 0.1 m/s; (*b*) 1.12 cm; (*c*) 0.125 m/s; (*d*) −2.63 mV

(a) $v = 0.1\, e^{20y}$ m/s ; $y = 0$ @ $t = 0$

$$v(t=0) = 0.1\, e^{0} = 0.1 \text{ m/s} \checkmark$$

(b) $y(t = 0.1)$

$$v = \frac{dy}{dt} = 0.1\, e^{20y} \text{ m/s}$$

$$10\, e^{-20y}\, dy = dt$$

$$10 \int e^{-20y}\, dy = \int dt$$

$$\frac{10}{-20} e^{-20y} = t + C$$

$$-\frac{1}{2} e^{-20y} = t - \frac{1}{2} \quad ; \quad (y=0,\ t=0)$$

FOR $t = 0.1$

$$-\frac{1}{2} e^{-20y} = -0.4$$

$$e^{-20y} = 0.8$$

$$-20y = \ln(0.8)$$

$$y = -\frac{1}{20} \ln(0.8)$$

$$= 0.0112 \text{ m} \checkmark$$

(c) $v(t=0.1)$

WE FOUND THAT $y(t=0.1) = 0.0112\ m$

$v = 0.1\, e^{20 \cdot 0.112}\ m/s$

$= 0.125\ m/s$ ✓

(d) V_{12} @ $t = 0.1$, $v = 0.125\ m/s$

$y = 0.0112\ m$

$\bar{\Phi} = \bar{B} \cdot \bar{A}$ ($\bar{B}$ is constant)

$= Byd$

$-\frac{d\Phi}{dt} = -Bd\frac{dy}{dt} = -Bvd = \text{emf}$

$= -0.3 \times 0.125 \times 0.07$

$= -0.002625\ V$

$= -2.63\ mV$ ✓

D9.3 Find the amplitude of the displacement current density: (*a*) adjacent to an automobile antenna where the magnetic field intensity of an FM signal is $H_x = 0.15\cos[3.12(3\times10^8 t - y)]$ A/m; (*b*) in the airspace at a point within a large power distribution transformer where $\mathbf{B} = 0.8\cos[1.257\times10^{-6}(3\times10^8 t - x)]\mathbf{a}_y$ T; (*c*) within a large, oil-filled power capacitor where $\epsilon_r = 5$ and $\mathbf{E} = 0.9\cos[1.257\times10^{-6}(3\times10^8 t - 5z)]\mathbf{a}_x$ MV/m; (*d*) in a metallic conductor at 60 Hz, if $\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 5.8\times10^7$ S/m, and $\mathbf{J} = \sin(377t - 117.1z)\mathbf{a}_x$ MA/m^2.

Answers: (*a*) 0.468 A/m^2; (*b*) 0.800 A/m^2; (*c*) 0.0150 A/m^2; (*d*) 57.6 pA/m^2

IN EACH CASE, $\bar{J}_D = \dfrac{\partial \bar{D}}{\partial t} = \nabla \times \bar{H} = \dfrac{1}{\mu}\nabla \times \bar{B}$

a. AUTOMOBILE ANTENNA

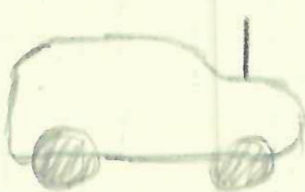
$$H_x = 0.15\cos[3.12(3\times10^8 t - y)] \text{ A/m}$$

$$\nabla \times H_x = \underbrace{\cancel{\frac{\partial H_x}{\partial z}}}_{0}\hat{y} - \frac{\partial H_x}{\partial y}\hat{z}$$

$$= 3.12 \times 0.15 \sin[3.12(3\times10^8 t - y)]\hat{z} \text{ A/m}^2$$

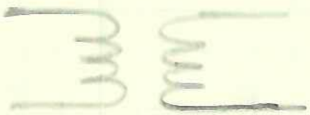
$$= 0.468 \sin[3.12(3\times10^8 t - y)]\hat{z} \text{ A/m}^2$$

$$|J_D| = 0.468 \text{ A/m}^2$$



AUTOMOBILE
ANTENNA

b. POWER DISTRIBUTION TRANSFORMER



$$\bar{B} = 0.8 \cos\left[1.257\times10^{-6}(3\times10^{8}t - x)\right] a_y \, T$$

$$\bar{H} = \frac{\bar{B}}{\mu_0}$$

$$\nabla \times \bar{H} = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cancel{H_x} & H_y & \cancel{H_z} \end{vmatrix} = \frac{\partial}{\partial x} H_y a_z - \cancel{\frac{\partial}{\partial z} H_y a_x}^{\,0}$$

$$J_D = \frac{0.8}{4\pi\times10^{-7}} \, 1.257\times10^{-6} \sin\left[1.257\times10^{-6}(3\times10^{8}t - x)\right] a_y \quad A/m^2$$

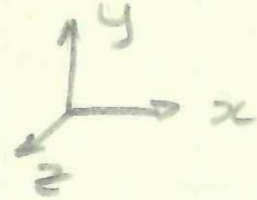
$$|J_D| = 0.800 \, A/m^2$$

c. OIL-FILLED POWER CAPACITOR

① GIVEN $\bar{E}$

② $\bar{D} = \epsilon \bar{E}$

③ $J_D = \dfrac{\partial \bar{D}}{\partial t}$



$$\bar{E} = 0.9 \cos\left[1.257\times10^{-6}(3\times10^{8}t - 5z)\right] a_x \ \frac{MV}{m}$$

$$\frac{\partial \bar{D}}{\partial t} = -0.9\times5\times8.85\times10^{-12}\times1.257\times10^{-6}\times3\times10^{8}$$
$$\times \cos\left[1.257\times10^{-6}(3\times10^{8}t - 5z)\right] a_x \ MA/m^2$$

$$= -1.50\times10^{-8}\cos\left[1.257\times10^{-6}(3\times10^{8}t - 5z)\right] a_x \ MA/m^2$$

$$\left|\frac{\partial \bar{D}}{\partial t}\right| = 1.50\times10^{-8}\ MA/m^2 = 0.0150\ A/m^2$$

d. METALLIC CONDUCTOR $\epsilon = \epsilon_0$ $\sigma = 5.8\times10^{7}\,S/m$ $\mu = \mu_0$ $f = 60\,Hz$

$$\bar{J}_c = \sin(377t - 117.1z)\, a_x \ MA/m^2$$

$$\frac{J_c}{J_D} = \frac{\sigma}{\omega\epsilon} \qquad |J_D| = \frac{\omega\epsilon}{\sigma}|J_c|$$

$$|J_D| = \frac{2\pi\times60\times8.85\times10^{-12}}{5.8\times10^{7}}\times 1 \ MA/m^2$$

$$= 5.75\times10^{-17}\ MA/m^2$$

$$= 5.75\times10^{-11}\ A/m^2 = 57.5\ pA/m^2$$

D9.4 Let $\mu = 10^{-5}$ H/m, $\epsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$, and $\rho_v = 0$. Find k (including units) so that each of the following pairs of fields satisfies Maxwell's equations:
(*a*) $\mathbf{D} = 6\mathbf{a}_x - 2y\mathbf{a}_y + 2z\mathbf{a}_z$ nC/m², $\mathbf{H} = kx\mathbf{a}_x + 10y\mathbf{a}_y - 25z\mathbf{a}_z$ A/m;
(*b*) $\mathbf{E} = (20y - kt)\mathbf{a}_x$ V/m, $\mathbf{H} = (y + 2 \times 10^6 t)\mathbf{a}_z$ A/m.

Answers: (*a*) $k = 15$ A/m²; (*b*) $k = -2.5\times10^8$ V/(m · s)

$\bar{D} = \epsilon \bar{E}$
$\bar{B} = \mu \bar{H}$

(a) THIS CASE CORRESPONDS TO STATIC FIELDS.

$\nabla \times \bar{E} = 0$ AND $\nabla \times \bar{H} = 0$ REGARDLESS OF THE VALUE OF k

$\nabla \cdot \bar{D} = 0$ REGARDLESS OF THE VALUE OF K

$\nabla \cdot \bar{H} = k + 10 - 25 = 0$

IFF $k = 15 \text{ A/m}^2$

(b) THIS CASE CORRESPONDS TO DYNAMIC FIELDS.

$\nabla \cdot \bar{D} = 0$ AND $\nabla \cdot \bar{B} = 0$ REGARDLESS OF THE VALUE OF k.

$$\nabla \times \bar{E} = \overset{\nearrow 0}{\cancel{\frac{\partial E_x}{\partial z}}}\hat{y} - \frac{\partial E_x}{\partial y}\hat{z} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \frac{\partial H_z}{\partial y}\hat{x} - \overset{\nearrow 0}{\cancel{\frac{\partial H_z}{\partial x}}}\hat{y} = \epsilon \frac{\partial \bar{E}}{\partial t}$$

$-20 = -10^{-5} \times 2 \times 10^6 \quad \text{V/m}^2$ ✓

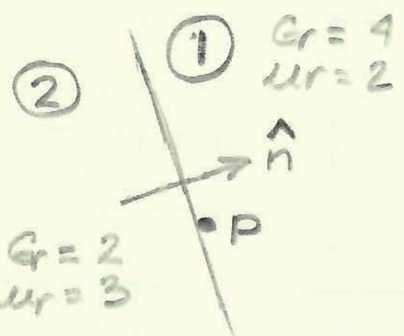
$1 = -k \times 4 \times 10^{-9} \quad \text{A/m}^2$

$k = -2.5 \times 10^8 \text{ V/m·s}$ ✓

D9.5 The unit vector $\mathbf{n} = 0.64\mathbf{a}_x + 0.6\mathbf{a}_y - 0.48\mathbf{a}_z$ is directed from region 2 ($\epsilon_r = 2$, $\mu_r = 3$, $\sigma_2 = 0$) toward region 1 ($\epsilon_{r1} = 4$, $\mu_{r1} = 2$, $\sigma_1 = 0$). If $\mathbf{B}_1 = (\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z) \sin 300t$ T at point P in region 1 adjacent to the boundary, find the amplitude at P of: (*a*) $\mathbf{B}_{N1}$; (*b*) $\mathbf{B}_{t1}$; (*c*) $\mathbf{B}_{N2}$; (*d*) $\mathbf{B}_2$.

Answers: (*a*) 2.00 T; (*b*) 3.16 T; (*c*) 2.00 T; (*d*) 5.15 T

$|B_1| = 3.74\ \mathrm{T}$



$\bar{B}_1 = a_x - 2a_y + 3a_z \sin 300t$ T

FIND $|\bar{B}_{N1}|$ $|\bar{B}_{N2}|$ @ P

$|\bar{B}_{t1}|$ $|\bar{B}_2|$

$|B_{N1}| = \bar{B}_1 \cdot \hat{n} = |0.64 - 1.2 - 1.44|\ \mathrm{T}$

$= 2\ \mathrm{T}$ ✓

$|B_{t_1}| = |\bar{B} - B_{N1}|$

OR $|B|^2 - |B_{N1}|^2 = |B_{t_1}|^2$

$|B_{t_1}| = \sqrt{|B|^2 - |B_{N_1}|^2} = \sqrt{14 - 4} = \sqrt{10} = 3.16\ \mathrm{T}$ ✓

$\bar{B}_{N1} \equiv \bar{B}_{N2}$ SO $|B_{N2}| = 2\mathrm{T}$ ✓

$|\bar{B}_2| = \sqrt{|B_{N2}|^2 + |B_{t2}|^2} = \sqrt{4 + |B_{t2}|^2}$

H_t IS CONTINUOUS ACROSS THE BOUNDARY, SO

$|H_{t_1}| = \dfrac{3.16}{2} = |H_{t2}| \Rightarrow |B_{t2}| = \dfrac{3.16}{2} \times 3$

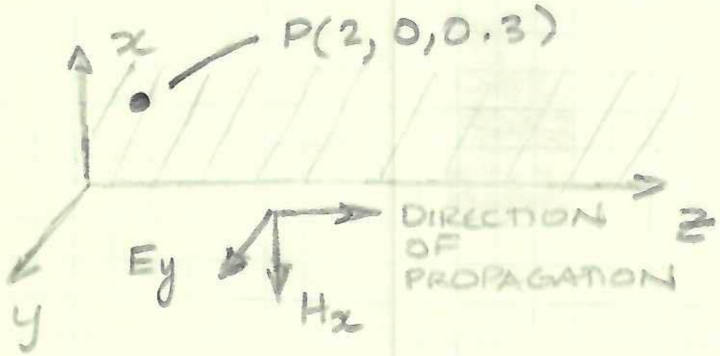
$= 4.74\ \mathrm{T}$

$\Rightarrow |\bar{B}_2| = \sqrt{4 + (4.74)^2}$

$= 5.14\ \mathrm{T}$

D9.6 The surface $y = 0$ is a perfectly conducting plane, whereas the region $y > 0$ has $\epsilon_r = 5$, $\mu_r = 3$, and $\sigma = 0$. Let $\mathbf{E} = 20\cos(2\times10^8 t - 2.58z)\mathbf{a}_y$ V/m for $y > 0$, and find at $t = 6$ ns; (*a*) ρ_S at $P(2, 0, 0.3)$; (*b*) $\mathbf{H}$ at P; (*c*) $\mathbf{K}$ at P.

Answers: (*a*) $\rho_S = 0.81$ nC/m^2; (*b*) $\mathbf{H} = -62.3\mathbf{a}_x$ mA/m; (*c*) $\mathbf{K} = -62.3\mathbf{a}_z$ mA/m



@ t = 6 ns

(a) WITHIN THE PEC, $\bar{E}$ AND $\bar{D}$ ARE 0 SO THE BOUNDARY CONDITION

$$D_{N1} - D_{N2} = \rho_s$$

YIELDS

$$\rho_s = D_{N1} = \epsilon E_{N1} \quad C/m^2$$

$$= 5 \times 8.85 \times 10^{-12} \times 20 \cos(2\times10^8 \times 6\times10^{-9} - 2.58 \times 0.3)$$

$$= 0.806 \text{ nC/m}^2$$

(b) WE RECOGNIZE THAT $\bar{E}$ IS A COMPONENT OF A PLANE WAVE AND THE REGION $y>0$ IS LOSSLESS WITH $Z_0 = 120\pi\sqrt{\frac{3}{5}} = 292\Omega$

$$\therefore \bar{H} = -\frac{E}{Z_0}\cos(2\times10^8 t - 2.58z)\,\hat{a}_x$$

$$= \frac{-20}{292}\cos(2\times10^8 t - 2.58z)\,\hat{a}_x \quad A/m$$

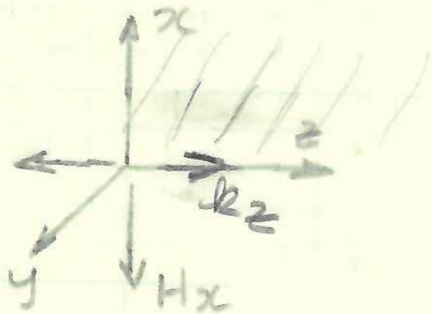
$$= -62.4\,\hat{a}_x \text{ mA/m}$$

✓

(c) $\bar{K} \times \hat{a}_n = \bar{H}_{t_1}$ (42) IN THE TEXT.

IF $\hat{a}_n = \hat{a}_y$

$62.4\ mA/m\ a_z \times a_y = -62.4\ a_x\ \frac{mA}{m} = \bar{H}_{t_1}$



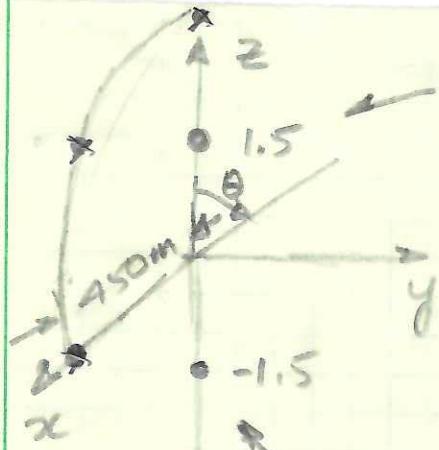
$K = \boxed{62.4\ \hat{a}_z\ mA/m}$

a_x	a_y	a_z	a_x
0	0	+62.4	0
0	1	0	0

(THE SIGN OF $\bar{K}$ IN THE TEXTBOOK IS NOT CORRECT IF WE ASSUME $\hat{a}_n \equiv \hat{a}_y$)

D9.7 A point charge Q_1 of $4\cos(10^8\pi t)$ μC is located at $P_1(0, 0, 1.5)$, whereas $Q_2 = -4\cos(10^8\pi t)$ μC is located at $P_2(0, 0, -1.5)$, both in free space. Find V at $P(r = 450, \theta, \phi = 0)$ at $t = 15$ ns for $\theta =$: (*a*) 0°; (*b*) 90°; (*c*) 45°.

Answers: (a) 159.8 V; (b) 0; (c) 143 V



$Q_1 = 4\cos(10^8\pi t)\,\mu C$

$t = 15$ ns

$Q_2 = -4\cos(10^8\pi t)\,\mu C$

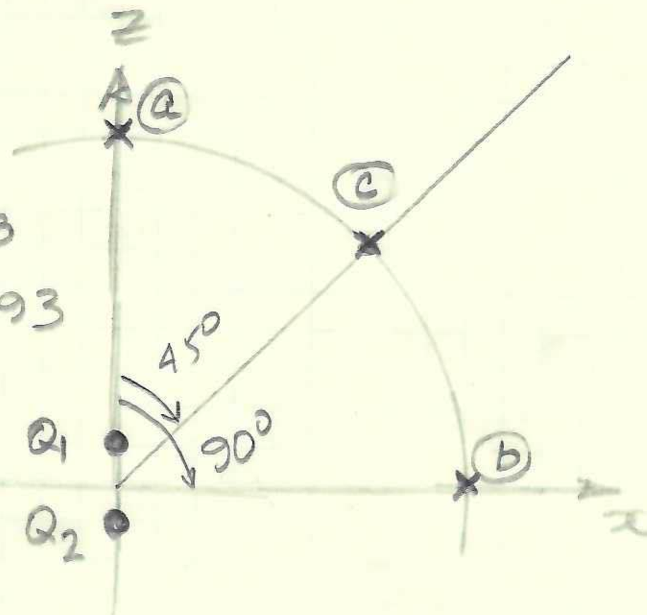
$$V(x,y,z) = \frac{Q_1(t')}{4\pi\epsilon R_1} + \frac{Q_2(t')}{4\pi\epsilon R_2}$$

$t' = t - R/v$

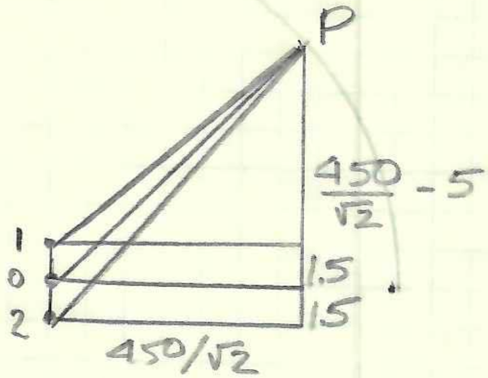
NOTE THAT
THE THREE OBSERVATION POINTS ALL LIE IN THE UPPER QUADRANT OF THE x-z PLANE

	R_1	R_2
a.	445	455
b.	450.028	450.028
c.	446.4785	453.3493

all in m



GEOMETRY FOR CASE C: $\theta = 45^\circ$



$$P1 = \sqrt{\frac{450}{2}^2 + \left(\frac{450}{\sqrt{2}} - 1.5\right)^2}$$

$$= 448.94\, m$$

$$P2 = \sqrt{\frac{450}{2}^2 + \left(\frac{450}{\sqrt{2}} + 1.5\right)^2}$$

$$= 451.06\, m$$

CASE A $\theta = 0^\circ$

$$V(0,0,450) =$$

$$\frac{\cancel{4} \cos(10^8 \pi (t - 448.5/3 \times 10^8))}{\cancel{4} \pi \epsilon_0\, 445}$$

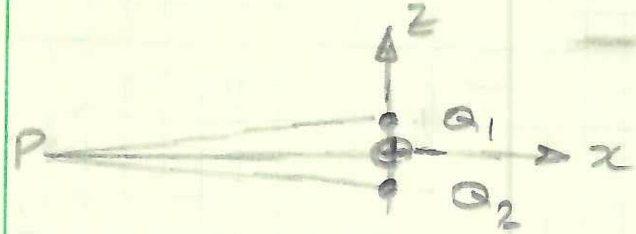
$$- \frac{\cancel{4} \cos(10^8 \pi (t - 451.5/3 \times 10^8))}{\cancel{4} \pi \epsilon_0\, 455}$$

$$= \frac{1}{\pi \epsilon_0} \Big[\cos(10^8 \pi (15 \times 10^{9} - 448.5/3 \times 10^8)$$

$$- \cos(10^8 \pi (15 \times 10^{9} - 451.5/3 \times 10^8) \Big]$$

$$= \boxed{159.9\ V}\ \checkmark$$

CASE B. $\theta = 90°$



THE PATH LENGTHS FROM Q_1 AND Q_2 TO THE X-Y PLANE ($\theta = 90°$) ARE IDENTICAL. AS A RESULT, THE TWO TERMS CANCEL AND $V = 0$.

CASE C $\theta = 45°$ (see fig on previous pg.)

$$V\left(\frac{450}{\sqrt{2}}, 0, \frac{450}{\sqrt{2}}\right) =$$

$$\frac{4\cos(10^8\pi(t - 448.94/3\times10^8))}{4\pi\epsilon_0 448.94}$$

$$-\frac{4\cos(10^8\pi(t - 451.06/3\times10^8))}{4\pi\epsilon_0 448.94}$$

$$= 143.2\ V. \checkmark$$