THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields & Waves
2025 W1

Example Problems for
Chapter 9 – Time-Varying Fields and Maxwell's Equations

The purpose of these nine example problems is to help you master some of fundamental techniques used to analyze time-varying fields and apply Maxwell's equations. Many of the concepts will be critical to our study of plane waves, transmission lines and guided waves.

Try these problems before we review the solutions in class. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count.

9.X Justify or refute the claim in the caption of Fig. 9.2 that "An apparent increase in flux linkages does not lead to an induced voltage when one part of a circuit is simply substituted for another by opening the switch. No indication will be observed on the voltmeter."

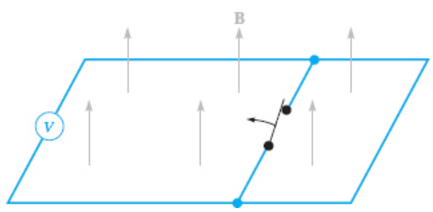

Fig. 9.2

9.3 Given $\mathbf{H} = 300\mathbf{a}_z \cos(3 \times 10^8 t - y)$ A/m in free space, find the emf developed in the general $\mathbf{a}_\phi$ direction about the closed path having corners at (*a*) (0, 0, 0), (1, 0, 0), (1, 1, 0), and (0, 1, 0); (*b*) (0, 0, 0) (2π, 0, 0), (2π, 2π, 0), and (0, 2π, 0).

9.5 The location of the sliding bar in Figure 9.5 is given by $x = 5t + 2t^3$, and the separation of the two rails is 20 cm. Let $\mathbf{B} = 0.8x^2\mathbf{a}_z$ T. Find the voltmeter reading at (*a*) $t = 0.4$ s; (*b*) $x = 0.6$ m.

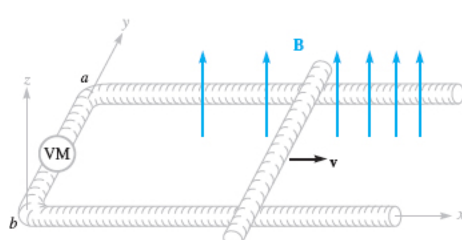

Fig. 9.5

9.11 Let the internal dimensions of a coaxial capacitor be $a = 1.2$ cm, $b = 4$ cm, and $l = 40$ cm. The homogeneous material inside the capacitor has the parameters $\epsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m, and $\sigma = 10^{-5}$ S/m. If the electric field intensity is $\mathbf{E} = (10^6/\rho)\cos 10^5 t\mathbf{a}_\rho$ V/m, find (*a*) $\mathbf{J}$; (*b*) the total conduction current I_c through the capacitor; (*c*) the total displacement current I_d through the capacitor; (*d*) the ratio of the amplitude of I_d to that of I_c, the quality factor of the capacitor.

9.12 The magnetic flux density $\mathbf{B} = B_0 \cos(\omega t)\cos(k_0 z)\,\mathbf{a}_y$ Wb/m^2 exists in free space. B_0 and k_0 are constants. Find (*a*) the displacement current density; (*b*) the electric field intensity; (*c*) k_0.

9.15 Use each of Maxwell's equations in point form to obtain as much information as possible about (*a*) $\mathbf{H}$, if $\mathbf{E} = 0$; (*b*) $\mathbf{E}$, if $\mathbf{H} = 0$.

9.18 The parallel-plate transmission line shown in Figure 9.7 has dimensions $b = 4$ cm and $d = 8$ mm, while the medium between the plates is characterized by $\mu_r = 1$, $\epsilon_r = 20$, and $\sigma = 0$. Neglect fields outside the dielectric. Given the field $\mathbf{H} = 5\cos(10^9 t - \beta z)\mathbf{a}_y$ A/m, use Maxwell's equations to help find (*a*) β, if $\beta > 0$; (*b*) the displacement current density at $z = 0$; (*c*) the total displacement current crossing the surface $x = 0.5d$, $0 < y < b$, $0 < z < 0.1$ m in the $\mathbf{a}_x$ direction.

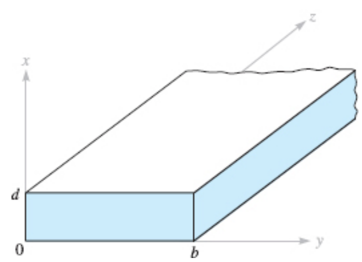

Fig. 9.7

9.22 In a sourceless medium in which $\mathbf{J} = 0$ and $\rho_v = 0$, assume a rectangular coordinate system in which $\mathbf{E}$ and $\mathbf{H}$ are functions only of z and t. The medium has permittivity ϵ and permeability μ. (*a*) If $\mathbf{E} = E_x\mathbf{a}_x$ and $\mathbf{H} = H_y\mathbf{a}_y$, begin with Maxwell's equations and determine the second-order partial differential equation that E_x must satisfy. (*b*) Show that $E_x = E_0\cos(\omega t - \beta z)$ is a solution of that equation for a particular value of β. (*c*) Find β as a function of given parameters.

9.26 Write Maxwell's equations in point form in terms of $\mathbf{E}$ and $\mathbf{H}$ as they apply to a sourceless medium, where $\mathbf{J}$ and ρ_v are both zero. Replace ϵ with μ, μ with ϵ, $\mathbf{E}$ with $\mathbf{H}$, and $\mathbf{H}$ with $-\mathbf{E}$, and show that the equations are unchanged. This is a more general expression of the *duality principle* in circuit theory.